Czechoslovak Mathematical Journal

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On semigroups which are unions of completely 0-simple subsemigroups

*Czechoslovak Mathematical Journal*, Vol. 16 (1966), No. 1, 63–69

Persistent URL: http://dml.cz/dmlcz/100710

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Чехословацкий математический журнал, т. 16 (91) 1966, Прага

# ON SEMIGROUPS WHICH ARE UNIONS OF COMPLETELY 0-SIMPLE SUBSEMIGROUPS

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(Received December 12, 1964)

## 1.

The Wedderburn-Artin and the Noether Structure Theorems give satisfactory characterizations of semisimple associative rings. In the paper KERTÉSZ-STEINFELD [4] there are given some other characterizations of these rings. Rees' well known Theorem for completely 0-simple semigroups plays the same rôle as the second Wedderburn-Artin Theorem does for simple rings. In his paper [5] SCHWARZ studied among others the semigroups without proper radical which are unions of their 0-minimal left ideals. The purpose of this paper is to give some equivalent conditions for semigroups with zero which are analogous to the first Wedderburn-Artin Theorem, Noether Theorem and other characterizations of semisimple rings. We shall prove that for a semigroup $S$ with zero the following conditions are equivalent:

$S$ is a 0-direct union of two-sided ideals which are completely 0-simple subsemigroups of $S$;

$S$ is regular and the union of its 0-minimal left ideals;

$S$ is regular and the union of its 0-minimal quasi-ideals. (See Theorem 15.)

This characterization is in a close connection with Chapter 6 of CLIFFORD-PRESTON'S book [3]. The basic ideas for this Chapter are to be found in Schwarz's paper [5] and a great deal of it is devoted to theorems of this type.

In section 2 we mention Lemmas and Theorems needed in the proof of Theorem 15 and Corollary 16.

## 2.

We use the terminology of Clifford-Preston's book [2] and we cite some results from it without proof.

**Theorem 1.** ([2], Theorem 2.48.) *Let $S$ be a 0-simple semigroup. Then $S$ is completely 0-simple if and only if it contains at least one 0-minimal left ideal and at least one 0-minimal right ideal.*

**Corollary 2.** ([2], Corollary 2.49.) *A completely 0-simple semigroup is the union of its 0-minimal left (right) ideals.*

**Theorem 3.** ([2], Theorem 2.51.) *A completely 0-simple semigroup is regular.*

**Theorem 4.** ([2], Theorem 1.17.) *The following two conditions on a semigroup $S$ are equivalent:*

(i) *$S$ is regular, and any two idempotents of $S$ commute with each other;*
(ii) *$S$ is an inverse semigroup (i.e. every element of $S$ has a unique inverse in $S$).*

**Theorem 5.** ([2], Theorem 3.9.) *The following conditions on a semigroup $S$ with 0 are equivalent:*

($\alpha$) *$S$ is a completely 0-simple inverse semigroup;*
($\beta$) *$S$ is a Brandt semigroup.*

A subset $\mathfrak{k} \neq \square$ of a semigroup $S$ is called a *quasi-ideal* of $S$ if $\mathfrak{k}S \cap S\mathfrak{k} \subseteq \mathfrak{k}$.

We have

**Lemma 6.** (Cf. [8], Lemma 2.) *Let $e$ be an idempotent element, $\mathfrak{l}$ a left ideal, $\mathfrak{r}$ a right ideal of a semigroup $S$ with 0. Then $\mathfrak{r} \cap \mathfrak{l}$, $e\mathfrak{l}$ and $\mathfrak{r}e$ are quasi-ideals of $S$.*

**Theorem 7.** ([8], Satz 6.) *Let $\mathfrak{l}$ be a 0-minimal left ideal, $\mathfrak{r}$ a 0-minimal right ideal of a semigroup $S$ with 0. If $(e \neq 0)$ is an idempotent element of $\mathfrak{l}$ (or $\mathfrak{r}$), then $e\mathfrak{l}$ (or $\mathfrak{r}e$) is a 0-minimal quasi-ideal of $S$.*

**Theorem 8.** ([8], Satz 1.) *Let $\mathfrak{r}$ be a 0-minimal right ideal and $\mathfrak{l}$ a 0-minimal left ideal of a semigroup $S$ with 0. Then the meet $\mathfrak{r} \cap \mathfrak{l}$ is either zero or a 0-minimal quasi-ideal of $S$.*

It is easy to prove the following assertion.

**Lemma 9.** *Every left (right) ideal $\neq 0$ of a regular semigroup $S$ with 0 contains at least one idempotent element $\neq 0$.*

Proof. Let $\mathfrak{l}$ be a left ideal of $S$; $a \in \mathfrak{l}$, $a \neq 0$. By regularity there is an $x \in S$ such that $a = axa$. Clearly $xa \neq 0$ is an idempotent and $xa \in Sa \subseteq S\mathfrak{l} \subseteq \mathfrak{l}$.

Analogously for the right ideals of $S$.

Let $A$ be any subset of the semigroup $S$ with zero. We shall say that $A$ is *nilpotent* if for some integer $k \geqq 1$ the relation $A^k = 0$ holds. The union of all nilpotent left ideals of $S$ is called the *radical* of $S$. (Cf. [5], Definition 3.2.).

**Lemma 10.** *If $S$ is a semigroup with radical 0 then 0 is the unique nilpotent right ideal of $S$.*

Proof. If $\mathfrak{r} \neq 0$ were a nilpotent right ideal of $S$ then (the two-sided ideal) $\mathfrak{r} \cup S\mathfrak{r} \neq 0$ would be a nilpotent ideal of $S$.

Lemma 9 implies the following two corollaries.

**Corollary 11.** *A regular semigroup $S$ with 0 has zero radical.*

**Corollary 12.** *Every 0-minimal left (right) ideal $\mathfrak{l}(\mathfrak{r})$ of a regular semigroup $S$ with 0 is of the form $\mathfrak{l} = Se\,(\mathfrak{r} = fS)$ with $e^2 = e\,(f^2 = f)$.*

We shall prove two results which are analogous to two known theorems in ring theory.

**Theorem 13.** (Cf. Artin-Nesbitt-Thrall [1], Corollary 5.4.B.) *Let $S$ be a semigroup with radical 0. Then $Se\,(e^2 = e)$ is a 0-minimal left ideal if and only if $eS$ is a 0-minimal right ideal of $S$.*

Proof. Let $Se\,(e^2 = e)$ be a 0-minimal left ideal of $S$; then, in view of Theorem 7, $eSe$ is a 0-minimal quasi-ideal of $S$. Let $\mathfrak{r}$ denote a right ideal of $S$ with $0 \subset \mathfrak{r} \subseteqq eS$. Hence $e\mathfrak{r} = \mathfrak{r}$. Lemma 10 implies that $e\mathfrak{r} \,.\, e\mathfrak{r} = \mathfrak{r}^2 \neq 0$ and thus $e\mathfrak{r}e \neq 0$. By Lemma 6, $e\mathfrak{r}e\ (\subseteqq eSe)$ is a quasi-ideal of $S$ and so $e\mathfrak{r}e = eSe$ holds. Hence $e \in e\mathfrak{r}e \subseteqq e\mathfrak{r} = \mathfrak{r}$ which implies $eS \subseteqq \mathfrak{r}$. Therefore $\mathfrak{r} = eS$; q.e.d.

**Theorem 14.** (Cf. [7], Satz 7.) *Let $S$ be a semigroup with radical 0. Then we can write every 0-minimal quasi-ideal $\mathfrak{k}$ of $S$ in the form $\mathfrak{k} = \mathfrak{l} \cap \mathfrak{r}$, where $\mathfrak{l}$ is a 0-minimal left ideal and $\mathfrak{r}$ is a 0-minimal right ideal of $S$.*

Proof. The 0-minimality of the quasi-ideal $\mathfrak{k}$ and Lemma 6 imply $S\mathfrak{k} \cap \mathfrak{k}S = 0$ or $\mathfrak{k}$.

Let the first case be assumed. If $S\mathfrak{k} \leqq 0$, then $\mathfrak{k}$ is a left ideal of $S$ with $\mathfrak{k}^2 = 0$, which is impossible. If $S\mathfrak{k} \neq 0$, then since $\mathfrak{k}S\mathfrak{k} \subseteqq S\mathfrak{k} \cap \mathfrak{k}S = 0$,

$$S\mathfrak{k} \,.\, S\mathfrak{k} = 0 \quad (S\mathfrak{k} \neq 0)$$

holds, which contradicts the assumption concerning the radical.

Thus $S\mathfrak{k} \cap \mathfrak{k}S = \mathfrak{k}$ must hold. It is sufficient to prove that $S\mathfrak{k}$ is a 0-minimal left ideal of $S$. Let $\mathfrak{l}$ be a left ideal of $S$ satisfying

$$0 \subset \mathfrak{l} \subseteqq S\mathfrak{k}\,. \tag{1}$$

The relation $S\mathfrak{l} \cap \mathfrak{k}S \subseteq S\mathfrak{k} \cap \mathfrak{k}S \subseteqq \mathfrak{k}$, the 0-minimality of $\mathfrak{k}$ and Lemma 6 imply that either

$$S\mathfrak{l} \cap \mathfrak{k}S = 0 \tag{2}$$

or

$$S\mathfrak{l} \cap \mathfrak{k}S = \mathfrak{k} \tag{3}$$

holds. From (2) we obtain $\mathfrak{k}\mathfrak{l} \subseteqq S\mathfrak{l} \cap \mathfrak{k}S = 0$, i.e., $\mathfrak{k}\mathfrak{l} = 0$. Hence $S\mathfrak{k} \,.\, \mathfrak{l} = 0$, whence, in view of (1),

$$\mathfrak{l}^2 \leqq 0$$

follows. This is a contradiction to our hypothesis. From (3) we get $\mathfrak{k} \subseteqq S\mathfrak{l} \subseteqq \mathfrak{l}$. Hence $S\mathfrak{k} \subseteqq S\mathfrak{l} \subseteqq \mathfrak{l}$. This and (1) imply $\mathfrak{l} = S\mathfrak{k}$ and $S\mathfrak{k}$ is a 0-minimal left ideal of $S$, q.e.d.

We shall say that the semigroup $S$ with 0 is the 0-*direct union* of its ideals $\mathfrak{a}_\alpha$ $(\alpha \in A)$ if $S = \bigcup_{\alpha \in A} \mathfrak{a}_\alpha$ and $\mathfrak{a}_\alpha \cap (\bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta) = 0$ hold.

The quasi-ideals $\mathfrak{k}_{\lambda\lambda'}$ $(\lambda, \lambda' \in \Lambda)$ of a semigroup $S$ with 0 are said to form a *complete system*, if the following three conditions hold:

1) $\mathfrak{k}_{\lambda\lambda'} = 0$ or $\mathfrak{k}_{\lambda\lambda'}$ is a 0-minimal quasi-ideal of $S$,

2) if $\mathfrak{k}_{\lambda\lambda'} \neq 0$, then it is of the form $e_\lambda S e_{\lambda'}$ for some idempotents $e_\lambda$, $e_{\lambda'}$ $(\lambda, \lambda' \in \Lambda)$,

3) $\mathfrak{k}_{\lambda\lambda'} \neq 0$ implies $\mathfrak{k}_{\lambda'\lambda}\mathfrak{k}_{\lambda\lambda'} \neq 0$ $(\lambda, \lambda' \in \Lambda)$.

This notion is analogous to the notion of the complete system of quasi-ideals introduced by KERTÉSZ-STEINFELD [4] for associative rings.

## 3.

**Theorem 15.** *The following conditions on a semigroups $S$ with 0 and with more than one element are equivalent*:

(A) *$S$ is regular and the union of its 0-minimal left ideals*;

(B) *$S$ is a union of 0-minimal left*[1]) *ideals of the form $Se_\lambda$ $(e_\lambda^2 = e_\lambda;\ \lambda \in \Lambda)$*;

(C) *$S$ is a 0-direct union of two-sided ideals*[2]) *which are completely 0-simple subsemigroups of $S$*;

(D) *$S$ is a union of quasi-ideals which form a complete system*;

(E) *$S$ is regular and the union of its 0-minimal quasi-ideals.*

Proof. (A) implies (B). In view of Corollary 12 every 0-minimal left ideal $\mathfrak{l}_\lambda$ of $S$ is of the form $\mathfrak{l}_\lambda = Se_\lambda$, where $e_\lambda$ is an idempotent $\in \mathfrak{l}_\lambda$.

(B) implies (C)[3]). First, we show that the radical of $S$ is 0. Let $se_\lambda$ $(\neq 0)$ $(se_\lambda \in Se_\lambda)$ be an arbitrary element of the ideal $\mathfrak{m}$ $(\neq 0)$ of $S$. This implies $\mathfrak{m} \cap Se_\lambda \neq 0$. With respect to the 0-minimality of the left ideal $Se_\lambda$ it must hold $e_\lambda \in Se_\lambda \subseteqq \mathfrak{m}$. Thus $\mathfrak{m}$ cannot be nilpotent and the radical of $S$ is indeed 0.

As the left ideals $Se_\lambda$ $(\lambda \in \Lambda)$ are 0-minimal

$$\text{either } Se_\lambda\, Se_{\lambda'} = 0 \text{ or } Se_\lambda Se_{\lambda'} = Se_{\lambda'} \tag{4}$$

holds. It is easy to see that the relations $\equiv$ defined by

$$Se_\lambda \equiv Se_{\lambda'} \Leftrightarrow Se_\lambda Se_{\lambda'} = Se_{\lambda'} \tag{5}$$

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[1]) If in conditions (A), (B) "0-minimal left ideals" is replaced by "0-minimal right ideals", one obtains conditions equivalent to the original (A)—(E).

[2]) In Section 3 of his paper [6] Schwarz proves some similar decomposition theorems for dual semigroups with radical 0.

[3]) One can prove this part with the help of the theorems in Section 9 of Schwarz [5].

is an equivalence relation in the set of the 0-minimal left ideals $Se_\lambda (\lambda \in \Lambda)$. Let $\mathfrak{a}_\alpha$ denote the union of all the left ideals belonging to the equivalence class $K_\alpha$.

Thus

$$S = \bigcup_{\alpha \in A} \mathfrak{a}_\alpha \tag{6}$$

where $A$ denotes the index set of the different classes.

First, we show that $\mathfrak{a}_\alpha = \bigcup_{Se_\mu \in K_\alpha} Se_\mu$ is a 0-simple two-sided ideal of $S$. (4) and (5) imply

$$\mathfrak{a}_\alpha \mathfrak{a}_\beta = \begin{cases} \mathfrak{a}_\alpha & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta . \end{cases} \tag{7}$$

Hence in view of (6) it follows

$$S\mathfrak{a}_\alpha = \mathfrak{a}_\alpha S = \mathfrak{a}_\alpha \tag{8}$$

Let $\mathfrak{b}$ be an ideal of $\mathfrak{a}_\alpha$. With respect to (6) and (7) $\mathfrak{b}$ is an ideal of $S$ and therefore $\mathfrak{b}^2 \neq 0$ holds. Hence

$$0 \neq \mathfrak{b}^2 \subseteqq \mathfrak{b}\mathfrak{a}_\alpha = \bigcup_{Se_\mu \in K_\alpha} \mathfrak{b} Se_\mu . \tag{9}$$

This implies $\mathfrak{b}Se_\nu \neq 0$ for some $Se_\nu \in K_\alpha$, whence $\mathfrak{b}Se_\nu = Se_\nu \subseteqq \mathfrak{b}$ follows. In view of (5) we obtain

$$\mathfrak{b} \supseteqq \mathfrak{b}\mathfrak{a}_\alpha \supseteqq Se_\nu \mathfrak{a}_\alpha = \bigcup_{Se_\mu \in K_\alpha} Se_\nu Se_\mu = \bigcup_{Se_\mu \in K_\alpha} Se_\mu = \mathfrak{a}_\alpha$$

establishing the 0-simplicity of $\mathfrak{a}_\alpha$.

As $\mathfrak{a}_\alpha$ is a 0-simple ideal of $S$,

$$\mathfrak{a}_\alpha \cap \Big( \bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta \Big) = 0 \text{ or } \mathfrak{a}_\alpha$$

holds. The second case implies $\mathfrak{a}_\alpha \subseteqq \bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta$. By multiplication by $\mathfrak{a}_\alpha$ we obtain in view of (7)

$$\mathfrak{a}_\alpha^2 \subseteqq \mathfrak{a}_\alpha \Big( \bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta \Big) = \bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\alpha \mathfrak{a}_\beta = 0 ,$$

which is a contradiction. Thus for every $\mathfrak{a}_\alpha$ $(\alpha \in A)$

$$\mathfrak{a}_\alpha \cap \Big( \bigcup_{\alpha \neq \beta \in A} \mathfrak{a}_\beta \Big) = 0$$

must hold and therefore (6) is a 0-direct union.

If $Se_\mu$ is a 0-minimal left ideal contained in $\mathfrak{a}_\alpha$, then $e_\mu S$ is a 0-minimal right ideal in $\mathfrak{a}_\alpha$. By Theorem 1 $\mathfrak{a}_\alpha$ is a completely 0-simple subsemigroup of $S$.

(C) implies (D). Let $S$ be the 0-direct union of its ideal $\mathfrak{a}_\alpha$ $(\alpha \in A)$, where $\mathfrak{a}_\alpha$ are completely 0-simple subsemigroups of $S$. In view of Theorem 3 $\mathfrak{a}_\alpha$ $(\alpha \in A)$ are regular semigroups, therefore $S$ is itself regular. Corollary 2 implies that every $\mathfrak{a}_\alpha$ $(\alpha \in A)$ is the union of its 0-minimal left ideals and the union of its 0-minimal right ideals. Since $S$ is the 0-direct union of the 0-simple ideals $\mathfrak{a}_\alpha$ $(\alpha \in A)$, all the left (right) ideals of $\mathfrak{a}_\alpha$ $(\alpha \in A)$ are left (right) ideals of $S$. Thus $S$ is the union of its 0-minimal left ideals and the union of its 0-minimal right ideals.

The regularity of $S$ implies that the 0-minimal left ideals of $S$ are of the form $Se_\lambda$ with idempotent elements $e_\lambda \neq 0$ $(\lambda \in \Lambda)$. From Corollary 11 and Theorem 13 we get that $e_\lambda S$ $(\lambda \in \Lambda)$ are the 0-minimal right ideals of $S$. So we can write $S = \bigcup_{\lambda \in \Lambda} Se_\lambda = \bigcup_{\lambda \in \Lambda} e_\lambda S$. Hence since $S^2 = S$,

$$S = S^2 = \Big(\bigcup_{\lambda \in \Lambda} e_\lambda S\Big)\Big(\bigcup_{\lambda' \in \Lambda} Se_{\lambda'}\Big) = \bigcup_{\lambda, \lambda' \in \Lambda} e_\lambda S e_{\lambda'} . \tag{10}$$

In view of Lemma 6 $e_\lambda S e_{\lambda'}$ $(\lambda, \lambda' \in \Lambda)$ are quasi-ideals of $S$ satisfying $0 \subseteqq e_\lambda S e_{\lambda'} \subseteqq e_\lambda S \cap S e_{\lambda'}$. This and Theorem 8 imply that $e_\lambda S e_\lambda$ $(\lambda, \lambda' \in \Lambda)$ are either 0 or 0-minimal quasi-ideals of $S$.

We have to verify only condition 3). Let $e_\lambda S e_{\lambda'} \neq 0$. The product $Se_\lambda S e_{\lambda'}$ is a left ideal $\neq 0$ contained in the 0-minimal left ideal $Se_{\lambda'}$. Hence $Se_\lambda S e_{\lambda'} = Se_{\lambda'}$, whence $e_{\lambda'} S e_\lambda . e_\lambda S e_{\lambda'} = e_{\lambda'} S e_{\lambda'} \neq 0$.

(D) implies (E). We have only to show the regularity of $S$. By supposition $S = \bigcup_{\lambda, \lambda' \in \Lambda} \mathfrak{k}_{\lambda\lambda'} = \bigcup_{\lambda, \lambda' \in \Lambda} e_\lambda S e_{\lambda'}$. Let $a = e_\lambda s e_{\lambda'}$ $(\neq 0)$ be an arbitrary element of $S$.

By 3) the hypothesis $e_\lambda S e_{\lambda'} \neq 0$ implies $e_{\lambda'} S e_\lambda \neq 0$. In view of Lemma 6 the product $e_\lambda s e_{\lambda'} . e_{\lambda'} S e_\lambda$ is a quasi-ideal of $S$. The 0-minimality of the quasi-ideal $e_\lambda S e_\lambda$ implies that either $e_\lambda s e_{\lambda'} . e_{\lambda'} S e_\lambda = 0$ or $e_\lambda s e_{\lambda'} . e_{\lambda'} S e_\lambda = e_\lambda S e_\lambda$ holds.

The first possibility implies $e_\lambda S e_\lambda s e_{\lambda'} . e_{\lambda'} S e_\lambda = 0$. Since the quasi-ideal $e_\lambda S e_\lambda s e_{\lambda'}$ $(\neq 0)$ is contained in the 0-minimal quasi-ideal $e_\lambda S e_{\lambda'}$, we get $e_\lambda S e_\lambda s e_{\lambda'} = e_\lambda S e_{\lambda'}$. Thus $e_\lambda S e_{\lambda'} . e_{\lambda'} S e_\lambda = 0$ holds, in contradiction to condition 3).

Thus we necessarily have $e_\lambda s e_{\lambda'} . e_{\lambda'} S e_\lambda = e_\lambda S e_\lambda$. This implies the existence of an element $e_{\lambda'} t e_\lambda \in e_{\lambda'} S e_\lambda$ with $e_\lambda s e_{\lambda'} . e_{\lambda'} t e_\lambda = e_\lambda$. Hence $e_\lambda s e_{\lambda'} . e_{\lambda'} t e_\lambda . e_\lambda s e_{\lambda'} = e_\lambda s e_{\lambda'}$, or otherwise $a(e_{\lambda'} t e_\lambda)\, a = a$, which says that $a$ is a regular element of $S$. This proves our assertion.

(E) implies (A). Corollary 11 and Theorem 14 imply that we can write every 0-minimal quasi-ideal $\mathfrak{k}_\alpha$ $(\alpha \in A)$ of $S$ in the form $\mathfrak{k}_\alpha = \mathfrak{l}_\alpha \cap \mathfrak{r}_\alpha$, where $\mathfrak{l}_\alpha = Se_\alpha$ $(e_\alpha^2 = e_\alpha)$ is a 0-minimal left ideal and $\mathfrak{r}_\alpha = f_\alpha S$ $(f_\alpha^2 = f_\alpha)$ is a 0-minimal right ideal of $S$. Thus $S = \bigcup_{\alpha \in \Lambda} \mathfrak{k}_\alpha = \bigcup_{\alpha \in \Lambda} [\mathfrak{l}_\alpha \cap \mathfrak{r}_\alpha] \subseteqq \bigcup_{\alpha \in \Lambda} \mathfrak{l}_\alpha \subseteqq S$. Hence $S = \bigcup_{\alpha \in \Lambda} \mathfrak{l}_\alpha = \bigcup_{\alpha \in \Lambda} Se_\alpha$, q.e.d.

Theorems 4, 5 and 15 imply:

**Corollary 16.** *The following four conditions on a semigroup $S$ with* 0 *and with more than one element are equivalent*:

(a) *$S$ is an inverse semigroup and the union of its* 0-*minimal left ideals*;

(b) *$S$ is a* 0-*direct union of ideals which are Brandt subsemigroups of $S$*;

(c) *$S$ is a* 0-*direct union of ideals which are completely* 0-*simple inverse subsemigroups of $S$*;

(d) *$S$ is an inverse semigroup and the union of its* 0-*minimal quasi-ideals.*

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## Резюме

## ПОЛУГРУППЫ КОТОРЫЕ ЯВЛЯЮТСЯ ОБЪЕДИНЕНИЕМ ВПОЛНЕ ПРОСТЫХ ПОЛУГРУПП С НУЛЕМ


ОТО ШТЕЙНФЕЛД (Oto Steinfeld), Будапешт



Целью статьи является изучение условий при которых полугруппа с нулем имеет следующее свойство: Она является объединением вполне простых полугрупп, причём пересечение всяких двух отличных компонент — нуль полугруппы.