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ALGORITMY

34. STUDENT

AN ALGORITHM FOR STUDENT'S t-TEST WITHOUT APPLICATION OF CRITICAL VALUES

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The algorithm suggested in this paper computes the probability that the modulus of Student's test statistic will exceed the modulus of the value t actually observed, i.e.

$$\alpha_n(t) = 1 - \frac{1}{B\left(\frac{n}{2}, \frac{1}{2}\right)\sqrt{n}} \int_{-|t|}^{|t|} \left(1 + \frac{y^2}{n}\right)^{-(n+1)/2} \mathrm{d}y \,, \tag{1}$$

where n is the number of degrees of freedom. Henceforth this probability will be called the significance degree. The significance degree is then compared directly with the significance level chosen in advance, and if it is less than the latter, the observed value is declared significant.

The main advantage of the procedure presented here is that it requires considerably less memory space in comparison with the usual procedure based on the use of a table of critical values. Moreover, it provides us with an exact value of the significance degree which may be needed, e.g., to perform a combination of several independent t-tests.

Formula (1) can be modified using the relation

$$\alpha_n(t) = A_n(x) \,, \tag{2}$$

where

$$x = \left(1 + \frac{t^2}{n}\right)^{-1} , \tag{3}$$

$$A_n(x) = \frac{1}{B\left(\frac{n}{2}, \frac{1}{2}\right)} \int_0^x y^{(n/2)-1}(1 - y)^{-1/2} \,\mathrm{d}y \,. \tag{4}$$

The algorithm is based on the following reccurent relation and initial conditions

$$A_n(x) = A_{n-2}(x) - \frac{\Gamma\left(\dfrac{n-1}{2}\right)}{\Gamma\left(\dfrac{n}{2}\right)\sqrt{\pi}} x^{(n/2)-1}(1-x)^{1/2}, \tag{5}$$

$$A_1(x) = \frac{2}{\pi} \arcsin x^{1/2}, \quad A_2(x) = 1 - (1-x)^{1/2} \tag{6}$$

$$(n > 2, 0 \leqq x \leqq 1).$$

The statements (2), (5), (6) can be proved by differentiation.

Instead of Student's statistic t we use its transform (3). Let us note that if t has the form

$$t = y \Big/ \left(\frac{z}{n}\right)^{1/2}, \tag{7}$$

where y, z are certain sampling characteristics then x can be evaluated directly from y, z in the form

$$x = z/(y^2 + z). \tag{8}$$

The function *STUDENT* computes the significance degree for a two-sided test. Its parameter x is the transform (3) of Student's statistic t which, in practice, will rather be calculated according to (8), and the other parameter n is the number of degrees of freedom.

```
real procedure STUDENT(x, n); value x;
  real x; integer n;
  begin real a, b, c; integer e, d;
        b := sqrt(1 - x); if n > (n ÷ 2) × 2 then go to L1;
        a := 1; c := -x; e := 2; go to L2;
    L1: a := 0·63661977 × arcsin (sqrt(x));
        b := 0·63661977 × sqrt(x) × b;
        c := 0; e := 3;
    L2: for d := e step 2 until n do
        begin a := a - b; c := c + 2 × x;
              b := (b × c)/d
        end;
        STUDENT := a
  end
```

We obtain the result with accuracy of at least about 5 decimal places. In the case of one-sided test the same procedure is used but the resulting significance degree has to be divided by two and, if $t < 0$ (i.e. $y < 0$), substracted from one.

Some check-values: $STUDENT\ (0{\cdot}3, 1) = 0{\cdot}36901$, $STUDENT\ (0{\cdot}25, 10) = 0{\cdot}00027$, $STUDENT\ (0{\cdot}75, 19) = 0{\cdot}02099$.

For checking the performance of the procedure it is also possible to use the table of critical values t. Of course, we must transform the tabled t to x according to (3).

The program has been tested in the symbolic language MOST [2] and implemented in the Biophysical Institute, Faculty of General Medicine, Charles University for the computer ODRA 1013 [3].

[1] *Janko, J.:* Statistical Tables (in Czech), Publ. House of the Czechoslovak Acad. of Sci., Prague 1958.

[2] *Szczepkowicz, J.:* Programming in the autocode MOST 1 (in Polish), Elwro Publication 03-VI-1, Wroclaw.

[3] *Černý, V., Půr, J.:* Programmer's Manual on Automatic Computer ODRA 1013 (in Czech), Kanc. stroje n.p., Hradec Králové 1967.