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Aplikace matematiky, Vol. 26 (1981), No. 1, 57–58

Persistent URL: http://dml.cz/dmlcz/103894

ON 0–1 MEASURE FOR PROJECTORS, II

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(Received May 6, 1979)

As stated in [1], the non-existence of a non-trivial measure with only the values 0 and 1 on the orthocomplemented lattice of projectors in a Hilbert space is a corollary of Gleason's theorem [2]. However, Gleason's theorem is valid only for σ-additive measures and hence this conclusion is not right (this fact is mentioned in [3]). According to [3], the question of an additive $0-1$ measure on projectors in an infinite dimensional Hilbert space is open. In this remark we shall show that the non-existence of such a measure is an easy consequence of the non-existence of this measure in E_3 and this can be demonstrated without Gleason's theorem [4], [5].

The infinite dimensional Hilbert space $\mathscr{H}$ is given as the direct sum

$$\mathscr{H} = \oplus E^l \tag{1}$$

with m summands E^l and for each summand there is an isomorphism φ^l with the space E_3.

Given a projector P in E_3, we denote by $M(P)$ the subspace in $\mathscr{H}$ which is generated by subspaces $\varphi^l(P) \subset E^l$ (if x is a vector in E_3, $M(x)$ is the subspace generated by $\varphi^l(x)$) and we identify the projector with its range.

Now,

(2) if $P \perp Q$ in E_3, then $M(P) \perp M(Q)$ in $\mathscr{H}$ and vice versa,

(3) if $x, y, z \in E_3$ are orthogonal, then $M(x) \vee M(y) \vee M(z) = \mathscr{H}$.

Both the assertions are obvious.

For a $0-1$ measure μ in $\mathscr{H}$ we set

$$\nu(P) = \mu(M(P)) .$$

If μ were non-trivial, then by (2) and (3), ν would be non-trivial measure in E_3, which is impossible.

By [4] or [5] the non-existence of a $0-1$ measure in E_3 is demostrated by giving a finite set of vector $x_1, \ldots, x_n$ such that no measure is possible for the set of projectors $P_1, \ldots, P_n$ generated by $x_1, \ldots, x_n$. Consequently, there is a finite set of projectors in $\mathscr{H}$ for which the definition of a nontrivial $0-1$ measure is impossible. The number of these projectors is independent of the cardinality of $\mathscr{H}$.

Finally, let us mention that in [6] it is unnecessary to consider separately the finite dimensional and the infinite dimensional case when imbedding the lattice of projectors in the Boolean algebra.

References

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Souhrn

0–1 MÍRA PRO PROJEKTORY, II

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Neexistence additivní $0-1$ míry (netriviální) na množině projektorů v nekonečně dimensionálním Hilbertově prostoru je důsledek neexistence takové míry pro projektory v E_3.

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