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Associative rings and the Whitehead property of modules  
[Abstract of thesis]

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to find such an optimum therapy. The simulated annealing algorithm was implemented to solve this task and to enable the system to be used in an interactive way. This part of the system also offers therapy proposals when current therapy scheme is changed. The dialogue is based on an user-friendly communication subsystem, whose description is included. The system is implemented in Turbo Pascal language on personal computer. The learning abilities of the system were tested. The system is able to predict blood sugar value with precision of 2.5 mmol/l after 3 days and 6 days of insulin pump therapy and conventional injection therapy respectively. Nowadays, the system is tested to evaluate its performance in a more detailed way from the medical point of view.

### LATIN SQUARES AND PARTIAL GROUPOIDS

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(27.9.1989, supervisor K.Drbohlav)

The thesis is devoted to the study of the (so called) couples of companions. A couple of companion  $K$  is such a partial algebra  $K = K(o, *)$  that  $K(o)$  and  $K(*)$  are cancellative groupoids,  $a \circ b$  is defined iff  $a * b$  is defined and for any  $a, b \in K$  there exist  $c, d, e, f \in K$ ,  $c \neq b \neq d$ ,  $e \neq a \neq f$  with  $a * b = a \circ c = e \circ b$  and  $a \circ b = a * d = f * d$ . Couples of companions may be obtained in a natural way from pairs of quasigroups with the same underlying set and they are studied in connection with some open questions of the quasigroup theory (see [1]).

A general structural theory of couples of companions is developed using several types of amalgamation-like constructions. The geometrical notion of planarity is shown to be of central meaning here and some equations quantifying "non-planarity" of a couple are presented.

The constructions may be used to produce an algorithm generating all couples of companions of a given order.

### REFERENCES

- [1] A. Drápal, T. Kepka, *Group modifications of some partial groupoids*, *Annals Discr. Math.* **18** (1983), 319-322.

### ASSOCIATIVE RINGS AND THE WHITEHEAD PROPERTY OF MODULES

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In chapter 1, the notion of an orthogonal theory is studied. This notion generalizes the well-known notions of a torsion theory and a cotorsion theory, respectively (Dickson, Salce). It is proved that there is a canonical embedding of the upper semilattice of all orthogonal theories of the  $\text{Tor}^n$  bifunctor into the lower semilattice of all orthogonal theories of the  $\text{Ext}^n$  bifunctor.

The author introduces the notion of a  $\otimes$ -ring ( $\otimes$  is the tensor product bifunctor). He proves that if  $R$  is a (von Neumann) regular ring of cardinality  $\leq \aleph_0$  or a certain

regular ring of cardinality  $\aleph_1$ , then  $R$  is a  $\otimes$ -ring iff  $R$  is simple and completely reducible.

In chapter 2, the notion of a Whitehead property of modules is introduced. A ring is said to be a left Ext-ring if each module has the Whitehead property. The following result is proved: a ring  $R$  is a left artinian left non-singular left Ext-ring iff  $R = S$  or  $R = T$  or  $R = S \oplus T$ , where  $S$  is a completely reducible ring and there is a division ring  $K$  such that the ring  $T$  is Morita equivalent to the ring of all upper triangular matrices of degree 2 over  $K$ .

The main result of chapter 3 states that if  $R$  is a simple countable regular ring such that  $R$  is not completely reducible, then the assertion "every countable module has the Whitehead property" is independent of ZFC + GCH.

### BEHAVIOUR OF FINITE AUTOMATA IN INFINITE ENVIRONMENT

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(27.9.1989, supervisors B.Novák, A.Pultr)

An algebraic theory of formal languages is presented. It can be looked on as a first order axiomatic theory.

The properties of the ordered monoid of all formal languages over a given set are investigated. Such an ordered monoid is called a babylon. Several types of morphisms among babylons are studied. Some of them are standard (homomorphism, substitution), others are new. A relation similar to the Galois connection is introduced; and found useful.

Systems of linear equations in babylons are considered. This leads to a theorem which generalizes both the theorem "an inverse homomorphism preserves regular languages" and the theorem "a two-way automaton accepts a regular language".

A characterization of the behaviour of the finite automaton in the Abelian group with two pebbles is given. This generalizes the folklore case the Abelian group being  $\mathbb{Z}^2$ .

### SOLUTION OF SUBSONIC ROTATIONAL NONVISCOUS FLOW IN THREE-DIMENSIONAL AXIALLY SYMMETRIC CHANNELS

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(28.9.1989, supervisors J.Polásek, M.Feistauer, J. Citavý)

The thesis is concerned with the mathematical study of a stationary, subsonic, generally rotational flow of an ideal fluid in three-dimensional axially symmetric channels. The author formulates the physical situation as a two-dimensional boundary value problem for the stream function which satisfies a nonlinear, elliptic partial differential equation. The essence of the work lies in a profound theoretical analysis of the weak formulation of the mentioned boundary value problem supplemented by some numerical results.

The finite element method is used for the discretization, firmly linear triangular elements. Detailed investigation of the discrete problem properties leads to two