

Abstracts of CSc. theses in mathematics

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ABSTRACTS OF CSc. THESES IN MATHEMATICS

(Candidatus Scientiarum)

defended recently at Charles University, Prague

SEMILINEAR STRUCTURE OF a-LANGUAGES

PHAM HUU UYEN, Vietnam

(March 15, 1990; supervisor M. Chytil)

The chief goal of the thesis is to study the semilinear property of languages with fast context-free approximation introduced by M. P. Chytil (Proc. of MFCS' 1986, LNCS No. 233, Springer-Verlag). It is shown that each language approximated by a sequence of context-free grammars converging with the rate which is a nondecreasing function $\in o(\log^{[4]})$ is a slip language. In other words, the well-known Parikh theorem holds even for classes wider than the class of context-free languages. In addition, some special cases of approximations by context-free languages are studied. It is proved that, unlike approximations by context-free languages, in the case of approximations by bounded context-free languages there exists a gap.

Attention is also paid to the study of full AFL-properties. The concept of uniform closedness under an operation (for classes of a-sequences) is introduced. It is shown that if a class of a-sequences is uniformly closed under full AFL-operations, then languages approximated by sequences belonging to this class form a full abstract family of languages as long as it contains at least one unempty language. Based on that fact, it is shown that $\mathfrak{L}[f]$ —the class of all languages approximated by sequences converging with the rate f —forms a full AFL provided that f is an orderly function $\in o(\log)$. This means that $\mathfrak{L}[f]$ is a full slip AFL for each orderly function $f \in o(\log^{[4]})$.

N-ARY SEMIHEAPS

HAVLAS Josef, Katedra matematiky a desk. geometrie, Stavební fakulta ČVUT, Thákurova 7, 166 29 Praha 6, Czechoslovakia

(April 6, 1990; supervisor K. Drbohlav)

In the thesis, the author sets an aim to generalize the notion of semiheap with regard to the operation defined in the relevant algebra, namely in the set S together with an n -ary operation $S \times S \times \dots \times S \times S \rightarrow S$ denoted by $(x_1, x_2, \dots, x_n) \mapsto [x_1 x_2 \dots x_n]$ for every $x_1, x_2, \dots, x_n \in S$. This n -ary operation subjects to the following laws

$$\begin{aligned} [[x_1 \dots x_n] x_{n+1} \dots x_{2n-1}] &= [x_1 \dots x_{n-1} [x_n \dots x_{2n-1}]] = \\ &= [x_1 \dots x_k [x_{n+k} x_{n+k-1} \dots x_{k+1}] x_{n+k+1} \dots x_{2n-1}], \end{aligned}$$

for every $x_1, \dots, x_{2n-1} \in S$ and for every $k = 1, \dots, (n-2)$.

The theory of n -ary semiheaps is closely related to the theory of semigroups. The author uses the standard terminology similarly as in semigroups and deals with the properties of the bounded partially ordered n -ary semiheaps. Some algebraic properties of the partially ordered n -ary semiheaps are discussed. The central role in the paper is played by Theorem 2.4, in which the properties of the interval

$I = [u, v]$, where u is an operatively increasable and v an operatively decreasable element of the partially ordered n -ary semiheap S .

Further, the author deals with the bounded partially ordered n -ary semiheaps. The weakest definition of the zero element has been chosen and the properties of the minimal ideal J_0 of these n -ary semiheaps are also discussed. If an n -ary semiheap, in addition, is a complete lattice, this minimal ideal J_0 is a complete lattice, too. It is proved that the properties of simple or zero-simple partially ordered n -ary semiheaps are similar to those in simple or zero-simple semigroups, respectively.

Finally, an analogue to Tarski's fixed point theorem is shown in complete partially ordered n -ary semiheaps, considering the fixed points of the first kind. There are also considered the fixed points of the k -th kind, $k = 1, 2, \dots, (n - 1)$.

ISOTONIC REGRESSION AND STOCHASTIC APPROXIMATION

CHARAMZA Pavel, KPMS, Sokolovská 83, 186 00 Praha 8, Czechoslovakia
(May 3, 1990; supervisor J. Dupačová)

In this thesis, the problem of finding the root of some unknown regression function is solved using the isotonic and quasiisotonic regression approach. In the first two chapters, the generalizations of the algorithms for finding these regressions are given, when the data are fitted not only in the least squares or least absolute deviations sense. In the third chapter, the consistency of M -estimates is generalized for somehow dependent random variables arising in the stochastic approximation schemes. This consistency is used in the following two chapters dealing with limit theorems for estimates of the root or extremes of the regression or of a more general function. In Chapter 6, the multidimensional version of the previous approach is discussed. The thesis contains the software (for IBM compatibles) which enables us to compare various modifications of our algorithms together with the standard methods of Robbins Monro type.

PIECEWISE RATIONAL INTERPOLATIONS IN THE PLANE AND THEIR USE FOR SOLVING ELLIPTIC BOUNDARY VALUE PROBLEMS

KOUKAL Stanislav, Kim Ir Senova 33, 616 00 Brno, Czechoslovakia
(June 5, 1990; supervisor M. Zlámal)

At first, the reductions and concentrations of parameters of interpolation polynomials on the triangle are investigated. By the concentrations we mean such a choice of parameters determining an interpolation polynomial on the triangle that as many parameters as possible are prescribed at the vertices and at the center of gravity, while only as many parameters as it is necessary for obtaining the desired smoothness of the polynomial interpolation are prescribed on the sides. We get new interpolations which are combined with the reduced hermite interpolations.

In the second chapter, we construct, by means of these combinations and the rational interpolations on curved biangle (resp. triangle), the piecewise rational interpolations in domains with the boundary formed by an abscissa or a part of a conic section. The main aim of this article is to construct piecewise polynomial interpolations in a parabolic polygon. This result is demonstrated for solving the Dirichlet problem by the finite element method.

The last chapter is devoted to numerical approximations of the solution $u \in W_{2,0}^{(2)}(D)$ of the abstract linear elliptic equation of the second order $Lu = f$. The convergence of the sequence of approximations to the exact solution is proved.

Remark. It should be mentioned that the original version of the thesis was submitted already in 1973. For political reasons, the degree was obtained only in 1990.

REFLEXION FUNCTION AND THE DIRICHLET AND NEUMANN PROBLEMS

DONTOVÁ Eva, Katedra matematiky FJFI, Trojanova 13, 120 00 Praha 2, Czechoslovakia
(June 15, 1990)

The work is devoted to the Dirichlet and Neumann problems for the Laplace's equation on doubly connected regions in the plane. The problems are solved by means of integral equations. Using the global reflection function of one of the two boundary curves the system of two integral equations is reduced to only one equation. The existence and unicity of the solution of that equation are investigated. Some conditions on the boundary of the region which guarantee the existence of the solution of that equation are given. Further a numerical method for solving the integral equation is investigated and a convergence of that method is proved.

METHODS OF GEOMETRICAL MODELLING WITH THE SPECIAL REFERENCE TO THE RULED SURFACES OF REVOLUTION

KOPINCOVÁ Edita, Katedra matematiky a konstruktivní geometrie strojní fakulty ČVUT, Horská 3, 120 00 Praha 2, Czechoslovakia
(June 27, 1990; supervisor L. Drs)

The submitted thesis concerns with the modelling of the geometrical bodies, in particular those formed by the ruled surfaces. A three-dimensional object is created from the basical elements, namely the conical and cylindrical surfaces of revolution and the hyperboloid of revolution of one sheet, by the means of the set operations.

From the symbolic representation is then the geometrical model generated. The methods already known are generalized here by an introduction of the strictly determined surfaces of the second degree.

The thesis contains the algorithms for the evaluation of the curves of intersection of all types of surfaces considered, as well as of the curves needed to picture the bodies in an axonometry. The algorithms, written in the programming language FORTRAN 77, were implemented on the minicomputer HP 1000.

NATURAL BUNDLES AND OPERATORS ON SOME GEOMETRICAL CATEGORIES

SLOVÁK Jan, Matematický ústav ČSAV, Mendlovo nám. 1, 662 82 Brno, Czechoslovakia
(June 27, 1990; supervisor I. Kolář)

Bundle functor generalize the natural bundles in the sense of [Nijenhuis A., *Natural bundles and their general properties*, in Differential Geometry in Honor of K. Yano, Kinokuniya, Tokio, 1972] and present a powerful tool for the study of geometric operations in various geometric categories. We give a proof of the regularity

of bundle functors on certain class of categories over manifolds and a description of all bundle functors on fibred manifolds with fixed dimensions of bases and fibers including sharp estimates of the orders. Further, there are described in the terms of Weil algebras all bundle functors on fibred manifolds with fixed dimension of bases which preserve fibred products. Finally, natural operations with projectable vector fields are discussed.

LIPSCHITZ DISTANCES OF METRIC SPACES

MATOUŠEK Jiří, Katedra kybernetiky a informatiky UK, Malostranské nám. 25, 118 00 Praha 1, Czechoslovakia
(June 28, 1990; supervisor M. Hušek)

Let $(X, d), (Y, \varrho)$ be metric spaces and $f : X \rightarrow Y$ an injective mapping. Put

$$\|f\|_{\text{lip}} = \sup\{\varrho(x, y)/d(x, y); x, y \in X, x \neq y\}, \quad \text{and} \\ \text{dist}(f) = \|f\|_{\text{lip}} \cdot \|f^{-1}\|_{\text{lip}}$$

(distortion of the mapping f). If $\text{dist}(f) \leq C$, then f is called a C -isomorphism. The Lipschitz distance of X from a subspace of Y is defined as follows:

$$\text{dist}(X, \subseteq Y) = \inf\{\text{dist}(f); f : X \rightarrow Y \text{ an injective mapping}\}.$$

The main topics of the thesis are embeddings of finite metric spaces into Euclidean spaces and spaces l_p^n . The following estimates are obtained:

Let $f(n, k) = \sup\{\text{dist}(X, \subseteq E^k); X \text{ } n\text{-point metric space}\}$; then

$$f(n, 1) = O(n), f(n, k) = O(n^{2/k}(\log n)^{3/2}) \quad (k \geq 2), \\ f(n, 2q) = \Omega(n^{1/q}), f(n, 2q - 1) = \Omega(n^{1/q}) \quad (q \geq 1).$$

Some estimates are also obtained for $\sup\{\text{dist}(X, \subseteq l_p); X \text{ } n\text{-point metric space}\}$ and for the number of n -vertex graphs (modulo a prescribed distortion).

For tree metric spaces, the following is proved: any n -point tree metric space can be embedded into l_p with distortion of order $O((\log \log n)^{\min(1/2, 1/p)})$ and this is asymptotically optimal ($p \in (1, \infty)$ is fixed).

Some results about the extendability of Lipschitz mappings are given (for mappings defined on tree metric spaces and on spaces of bounded metric dimension).

Some Ramsey-type questions for mappings of finite metric spaces are studied; e.g., the following theorem is proved: Let X be a finite metric space, and let $\varepsilon > 0$, K be given numbers. Then there exists a finite metric space Y , such that for every mapping $f : Y \rightarrow Z$ (Z arbitrary metric space) with $\text{dist}(f) \leq K$ one can find a mapping $g : X \rightarrow Y$, such that both the mappings g and $f|_{g(X)}$ are $(1 + \varepsilon)$ -isomorphic. If X is isometrically embeddable into Euclidean space, then also Y can be chosen with this property.

RAPIDLY GROWING FUNCTIONS

LOEBL Martin, Katedra aplikované matematiky MFF UK, Malostranské nám. 25, 118 00 Praha 1, Czechoslovakia

(June 28, 1990; supervisor J. Nešetřil)

This thesis is a study of some functions related to problems from various parts of combinatorics. These functions grow extremely rapidly to the infinity. Establishing the large growth rate (majorizing all functions from the Wainer's hierarchy) of a recursive function means, by the fundamental result of Wainer, that it is impossible to prove the statement "the function is total in Peano Arithmetic". The considerations proceed as follows. In the beginning, a large growth rate of the length $L(n)$ of the longest strategy of Hercules in the Hercules versus Hydra game (introduced by L. Kirby and J. Paris) is proved. Further, lower bounds for other functions by relating them to $L(n)$ are obtained. Main results are:

1. A new elementary proof of unprovability of the statement "every strategy of Hercules is a winning strategy",
2. Pathological games examples,
3. Almost complete classification of unprovability of Friedman's type effective versions of the Kruskal's theorem,
4. A new elementary and simple proof of unprovability of the modified Ramsey's theorem.

QUASICONVEX AND PSEUDOCONVEX FUNCTIONS IN MATHEMATICAL PROGRAMMING

AMBROŽ Luděk, Výzkumný ústav strojírenské technologie a ekonomiky, Velfřikova 4, 160 00 Praha 6, Czechoslovakia

(September 18, 1990; supervisor K. Zimmermann)

The purpose of this paper is to investigate special properties of quasiconvex and pseudoconvex functions.

The function $f(x)$ is explicitly quasiconvex (pseudoconvex), if and only if it is locally explicitly quasiconvex (locally pseudoconvex). If the function $f(x)$ is locally quasiconvex and lower semicontinuous, then every path connected component of the set $L^{\leq C}$ is a convex set.

Consider a special class of functions having this property: "In every point, there can be constructed a hyperplane passing through this point and such that this point is a local minimum subject to this hyperplane". Replacing hyperplane by halfspace and local minimum by strict local, global, strict global minimum, further classes of functions are obtained. The paper shows that these functions are closely related to quasiconvex functions. The Hess matrix of a function $f(x)$ at every point x is positive subdefinite on a linear subspace which is orthogonal to the gradient of this function. This condition is necessary for a twice differentiable function to be quasiconvex. It is shown that this condition can be sufficient in special cases. Some conditions equivalent (or stronger) to positive semidefiniteness of the Hess matrix are stated. It is also shown that in the BFGS method it is possible to use the Hess matrix of a quasiconvex function as a starting matrix. This matrix need not be necessarily positively semidefinite.

HERMITE–BIRKHOFF INTERPOLATION

FIALA Jiří, Ústředí výpočetní techniky dopravy, Olšanská 9, 130 66 Praha 3, Czechoslovakia
(September 18, 1990; supervisor I. Marek)

The thesis deals with the Hermite–Birkhoff interpolation. In the first part of this dissertation the problems of existence and uniqueness are discussed.

In the second part a new algorithm for the numerical solution of this problem in the special case is given. The algorithm is based on the calculation of some missing derivatives. Thus the studied problem is reduced to the classical Hermite interpolation. The missing derivatives are calculated from a system of linear equations. Corresponding algorithm for the Hermite interpolation is a generalization of the classical Nevil–Aitken algorithm. Fortran program and numerical results are included.

SPINOR FIELDS ON RIEMANNIAN MANIFOLDS

POKORNÁ Olga, Katedra matematiky MF VŠZ, Kamýcká ul., 160 00 Praha 6, Czechoslovakia
(September 26, 1990; supervisor J. Bureš)

The aim of the thesis is the study of relations among Killing spinors, twistor-spinors and E -spinors on a Riemannian spin manifold. These E -spinors are introduced by Lichnerowicz in C.R. Acad. Sci. Paris, Ser. I, **306** (1988), 381–385. The proof of the existence of E -spinors which are not Killing spinors on the noncompact manifolds $S^2 \times \mathbb{R}^1$ and $\mathbb{H}^2 \times \mathbb{R}^1$ are given here.

INTERPOLATION IN MULTIDIMENSIONAL RANDOM SEQUENCES

BUDINSKÝ Petr, Fakulta strojní, ČVUT, Suchbátarova 4, 160 00 Praha 6, Czechoslovakia
(October 22, 1990; supervisor J. Anděl)

In the theoretical part it is studied how the best linear interpolation and its residual variance can be found. The multidimensional weak stationary random sequences are considered.

In this work the form of the residual variance of the best linear interpolation is derived without finding this interpolation. The respective result is given for the case of any finite number of missing variables. There are given some conditions under which no improvement of the interpolation is reached by adding some variables.

Two methods for finding the best linear interpolation are presented. It is proved that both methods provide the same results in the case of invertible stationary sequences. These methods allow to express simultaneously, using one formula only, the best linear interpolation of all missing variables.

The results are then applied to the interpolation in autoregressive sequences by means of computers. In this case, there are estimated the order and parameters of the autoregression.

ROBUST BAYESIAN TYPE ESTIMATORS

HANOUSEK Jan, Matematicko-fyzikální fakulta UK, Ke Karlovu 3, 121 16 Praha 2, Czechoslovakia

(October 22, 1990; supervisor J. Jurečková)

Let X_1, \dots, X_n be i.i.d. random variables with a distribution P_θ depending on an unknown parameter $\theta \in \Theta \subset R^k$. Moreover, we assume that the parameter space Θ is an open interval. The estimator T_n of θ which is defined as

$$T_n = \frac{\int t \cdot \exp \left\{ -\sum_{i=1}^n \varrho(X_i, t) \right\} \pi(t) dt}{\int \exp \left\{ -\sum_{i=1}^n \varrho(X_i, t) \right\} \pi(t) dt}$$

(whenever both integrals exist) is called *B*-estimator (Bayesian type estimator) generated by the function ϱ . Observe that the *B*-estimator T_n is the Bayes' posterior mean (of the random variable θ having the prior distribution with a density $\pi(\theta)$) in the situation that $c(\theta) \cdot \exp \{-\varrho(x, \theta)\}$ is the density of the observations. With an appropriate choice of ϱ (i.e. such a choice which we would use as generating an *M*-estimator of θ), we obtain a Bayesian type estimator with good robustness properties. We have studied the asymptotic properties of T_n as $n \rightarrow \infty$; namely, we have derived an asymptotic representation of T_n and have found that T_n is asymptotically equivalent to the *M*-estimator generated by the same function ϱ . Some numerical results are illustrated in the location model.

A GENERAL THEORY OF SEMANTICS OF LANGUAGES OF FORMAL LOGIC

PEREGRIN Jaroslav, ORGATEX, Hradec Králové, Czechoslovakia

(October 24, 1990)

The basic feature of the classical Tarskian notion of interpretation is the following: if an expression is a combination of some subexpressions, then its interpretation is as a rule the result of the functional application of the interpretation of one of its subexpressions to the interpretations of the others, or the result of a functional composition of the interpretations of the subexpressions. However, such a functionality does not seem to be really inseparably connected with the concept of interpretation, and, moreover, it seems to exclude certain interesting languages which cannot be interpreted functionally, from our considerations. The aim of the dissertation is therefore (a) to develop a general notion of model theory not based on the principle of functionality; (b) to define the concept of functional interpretation in an exact way; and (c) to characterize the class of theories and languages which can be interpreted functionally.

The resulting general notion of interpretation is based on the concept of compositionality, but not on that of functionality. An interpretation is called applicative, if the interpretation of any complex expression is the result of the functional application of the interpretation of one of its subexpressions to the interpretations of the others; it is called functional, if it is, more generally, the result of a functional composition of the interpretations of the subexpressions. A language can be interpreted applicatively iff its grammatical categories can be ordered in such a way

that there exists an injective assignment of categories to grammatical rules such that any rule is assigned a category greater than any other category handled by the rule. What kinds of theories can be interpreted functionally is shown in the dissertation. Functional languages are shown to be those extensions of the applicative languages, which arise from addition of rules corresponding to lambda-terms over the original rules. The traditional languages of formal logic are then considered from the viewpoint of the problem of functionality.

A NEW METHOD OF PARSING BY RECURSIVE DESCENT
FOR $LR(k)$ GRAMMARS

DRÓZD Januš, Středisko výpočetní techniky ČSAV, Pod vodárenskou věží 2, 182 07 Praha 8, Czechoslovakia

(October 25, 1990; supervisor J. Demner)

This thesis introduces a new parsing method and investigates its properties. The method is applicable to the whole class of $LR(k)$ grammars and creates parsers using the principles of the recursive descent.

The main contribution of the method is that it makes it possible to apply the recursive descent to a larger class than $LL(k)$ without any restriction to the non-left-recursive grammars.

To make the parsing method more feasible, a method of evaluating attributes during the parsing process was developed. The attributes and rules are defined by an L -attribute grammar. Both methods are implemented in a program which converts an L -attribute grammar into the parser evaluating attributes.

AGGREGATION AND DISAGGREGATION IN MARKOV CHAINS

POLÁK Jaroslav, VCUK, MFF UK, Malostranské nám. 25, 118 00 Praha 1, Czechoslovakia

(October 29, 1990; supervisor B. Sekerka)

The aggregation–disaggregation method can be used to compute the stationary distribution of the finite homogeneous Markov chain. The proof of the following theorem is the main result of the thesis.

Theorem. *Let \mathbf{p} be a stationary distribution of a finite homogeneous irreducible Markov chain with the transition probability matrix \mathbf{P} . Let \mathbf{P} have at least one column with all elements positive. Then the iterative aggregation–disaggregation method locally converges to \mathbf{p} , i.e., an open neighborhood U of \mathbf{p} exists such that for all $\mathbf{p}^0 \in U$ the sequence of vectors generated by the iteration method converges to \mathbf{p} .*

RANDOM EIGENVALUE PROBLEMS FOR ORDINARY DIFFERENTIAL
EQUATIONS WITH RANDOM COEFFICIENTS

HÁLA Martin, Stavební fakulta ČVUT, Thákurova 7, 160 00 Praha 6, Czechoslovakia

(December 4, 1990; supervisor P. Mandl)

Eigenvalue problems for ordinary differential equations with random coefficients and deterministic boundary conditions are studied. The coefficients are supposed

to be vectorial processes with sufficiently smooth trajectories, derived from the Gaussian stationary processes with rational spectral densities.

On conditions that the random parts of the coefficients are asymptotically small in probability and near to the white noise and if the centralized equation satisfies proper conditions, the asymptotic normality of the random eigenvalues is derived. The explicit formulas for the variances are given, too.

The method of Ritz and the method of difference equations are presented for an approximate solution of the above mentioned problem. Several illustrating examples concerning eigen-vibrations or bending problems concerning engineering structures are added.

RANDOM BOOLEAN FORMULAS

SAVICKÝ Petr, KKI MFF UK, Malostranské nám. 25, 118 00 Praha 1, Czechoslovakia

(December 5, 1990; supervisor M. Vlach)

Given a Boolean connective α , starting with a set G of Boolean functions, new functions are produced by induction: those at $(n + 1)$ -th level are obtained by substitution of n -th level functions into α . The uniform distribution of probabilities on G is assumed. The problem is considered under which conditions the resulting distributions on the levels converge to the uniform distribution on the set of all Boolean functions. Necessary and sufficient conditions for α linear and sufficient ones for α non-linear are given. Finally, for special connectives α , the asymptotic behaviour of the sequence of distributions is approximated using a power series.

TIME STABILIZATION OF THE ARTIFICIAL COMPRESSIBILITY METHOD FOR THE SOLUTION OF THE TRANSONIC FLOW PROBLEM

KLOUČEK Petr, Matematický ústav UK, Sokolovská 83, 186 00 Praha 8, Czechoslovakia

(December 20, 1990; supervisor J. Nečas)

The thesis is devoted to the mathematical and numerical study of a transonic full potential flow. An artificial time-coordinate is added which results in the equation

$$\ddot{u} - \Delta \dot{u} - \operatorname{div}(\rho \nabla u) = 0,$$

investigated in a bounded domain Ω with Lipschitz-continuous boundary. The prescribed boundary conditions are of the Neumann type and characterize the normal flow through the outer boundary and tangential flow over the solid walls. The thesis deals with the global solvability of the problem and the existence of a generic solution of the transonic potential flow. Further the numerical analysis can be found.

The method of stabilization was applied to the solution of the supercritical flows past the two-dimensional airfoils. A special attention was paid to the description of the implementation of the Kutta–Joukowski condition. This approach was tested on NACA 0012, RAE 2822 and CAST 7 airfoils.