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Miguel Florencio; Pedro José Paúl

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A NOTE ON λ-MULTIPLIER CONVERGENT SERIES

MIGUEL FLORENCIO, PEDRO J. PAÚL, Sevilla

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Summary. A (formal) series Σx_n in a sequentially complete locally convex space (*lcs*) E is said to be λ-multiplier convergent, for λ a sequence space, if $\Sigma\alpha_n x_n$ converges in E for all $\alpha \in \lambda$. In this paper we show that, if $\lambda(\mu(\lambda, \lambda^\beta))$ is a barrelled AK-space, then Σx_n is λ-multiplier convergent if and only if it is weakly λ-multiplier Cauchy. This enables us to give a unified scheme for the previously known results due to Bessaga and Pe-czynski [4], Singer [13], Bennett [3] and Maddox [11]. Besides, we study the problem whether λ_1- and λ_2-multiplier convergences are equivalent for all E for different sequence spaces λ_1 and λ_2, and we obtain a characterization in terms of a density-type relation between λ_1 and λ_2. This relation is defined through a topology on the dual pair (λ, λ^β) which was introduced by Schaefer, namely, the finest topology under which λ is an AK-space.

Keywords: λ-multiplier convergence, barrellednes, AK-space.

AMS Subject Classification: 46A45.

INTRODUCTION

Bessaga and Pełczynski [4] proved that a series Σx_n in a Banach space E is weakly unconditionally Cauchy, i.e., $\Sigma|f(x_n)| < +\infty$ for every $f \in E'$, if and only if $\Sigma\alpha_n x_n$ converges for all $\alpha \in c_0$. This result suggested the following definition to I. Singer [13]: "A series Σx_n is weakly p-unconditionally Cauchy if $\Sigma\alpha_n x_n$ converges for all $\alpha \in l^p(1 < p < \infty)$". For certain Banach spaces, Singer characterized such series as those for which $\Sigma|f(x_n)|^q < +\infty$ (q is the conjugate exponent of p) for all $f \in E'$. More generally, Bennett [3] extended these results to an arbitrary sequentially complete *lcs* E. Recently, Gupta and Kantham [7] and Maddox [11] have studied similar problems for particular sequence spaces. In this paper we deal with general sequence spaces.

In what follows $E(\tau_E)$ stands for a Hausdorff sequentially complete locally convex space and λ for a sequence space containing the space ϕ of all sequences with finite support. $W(E, \lambda)$ denotes the space of all sequences $(x_n)_n$ such that the series Σx_n is λ-multiplier convergent, i.e.

$$W(E, \lambda) := \{(x_n)_n \in E^N : \Sigma\alpha_n x_n \text{ converges in } E \text{ for all } \alpha \in \lambda\} .$$

We are going to use several notions from the theories of α-duality (see [9, § 30] or

[14, Ch. 2]) and β-duality (see [5, 6 or 12]). Let us recall some relevant terms: cs stands for the space of all convergent series endowed with its natural Banach norm $\|\cdot\|_{cs}$. The topological dual of cs is the space bv of all bounded variation sequences and B stands for the unit ball in bv under the dual norm.The β-dual of a sequence space λ is defined as

$$\lambda^\beta := \{(\xi_n)_n : (\xi_n \alpha_n)_n \in cs \text{ for all } \alpha \in \lambda\} ;$$

(λ, λ^β) is a dual pair. The topology on λ given by the family of seminorms

$$p_\xi(\alpha) := \sup_n \{|\sum_{k=1}^{n} \alpha_k \xi_k|\} = \|(\alpha_n \xi_n)_n\|_{cs} =$$

$$= \sup \{|\Sigma \alpha_n \xi_n b_n| : b \in B\} , \quad \xi \in \lambda^\beta$$

is called the $\sigma\gamma(\lambda, \lambda^\beta)$-topology (see [12, Prop. 4] or [5, § 5]) and plays the same role in the β-duality as the Köthe normal topology $v(\lambda, \lambda^\times)$ in the α-duality. Under this topology λ is an AK-space, i.e., the sequence of n-th sections $\{P_n(\alpha)\}_{n=1}^\infty$, where $P_n(\alpha) := (\alpha_1, \ldots, \alpha_n, 0, 0, \ldots)$, converges to α for all $\alpha \in \lambda$.

A CHARACTERIZATION OF λ-MULTIPLIER CONVERGENCE

Our first result provides a characterization of λ-multiplier convergent series for a certain λ. Its proof follows the ideas of the proof given by Bennett [3] for the spaces c_0 and l^p.

Theorem 1. *If λ endowed with Mackey topology $\mu(\lambda, \lambda^\beta)$ is a barrelled AK-space, then Σx_n is λ-multiplier convergent if and only if $(f(x_n))_n \in \lambda^\beta$ for all $f \in E'$.*

Proof. Necessity is clear (cf. [13]): note that if Σx_n is λ-multiplier convergent, $\alpha \in \lambda$ and $f \in E'$, then

$$f(\Sigma \alpha_n x_n) = \Sigma \alpha_n f(x_n) .$$

Conversely, assume that $(f(x_n))_n \in \lambda^\beta$ for all $f \in E'$. Set $T: \phi \to E$, $T(\alpha) := \Sigma \alpha_n x_n$. Then

$$\langle T(\alpha), f \rangle_{(E,E')} = \langle \alpha, (f(x_n))_n \rangle_{(\varphi, \lambda^\beta)} .$$

Now, $T^*: E' \to \omega$ is such that $T^*(f) = (f(x_n))_n \in \lambda^\beta$. Hence, using [8, 8.6.1 and 8.6.5], we obtain that T is $\mu(\phi, \lambda^\beta) - \tau_E$ continuous. On the other hand, $\mu(\lambda, \lambda^\beta)$ induces $\mu(\phi, \lambda^\beta)$ on ϕ: indeed, each absolutely convex $\sigma(\lambda^\beta, \phi)$-compact set A is $\sigma(\lambda^\beta, \lambda)$-bounded since every point α in λ lies in the $\sigma(\lambda, \lambda^\beta)$-closure of a bounded set in ϕ, namely the sequence $\{P_n(\alpha): n = 1, 2, \ldots\}$. Now, since $\lambda(\mu(\lambda, \lambda^\beta))$ is barrelled, A is $\sigma(\lambda^\beta, \lambda)$-relatively compact. Finally, if $\alpha \in \lambda$, then $\{P_n(\alpha): n = 1, 2, \ldots\}$ is a $\mu(\phi, \lambda^\beta)$-Cauchy sequence because $\lambda(\mu(\lambda, \lambda^\beta))$ is an AK-space; therefore, by the continuity of T, $\{TP_n(\alpha): n = 1, 2, \ldots\} = \{\sum_{k=1}^{n} \alpha_k x_k : n = 1, 2, \ldots\}$ is a τ_E-Cauchy sequence.

Q.E.D.

Remarks. (1) Observe that the condition $(f(x_n))_n \in \lambda^\beta$ for all $f \in E'$ means, in other words, that Σx_n is weakly λ-multiplier Cauchy.

(2) Bennett and Kalton [1] and Garling [5, Thm. 11] give some conditions under which $\lambda(\mu(\lambda, \lambda^\beta))$ is barrelled.

(3) If λ is normal then, according to [14, Ch. 2 § 6.(9)], we can state Theorem 1 as follows: "*If $\lambda(\beta(\lambda, \lambda^\times))$ is an AK-space, then $W(E, \lambda) = \{(x_n)_n: (f(x_n))_n \in \lambda^\times$ for all $f \in E'\}$*". In this form, Theorem 1 includes, if we set $\lambda = c_0$, the well-known characterization of weakly unconditionally convergent series given by Bessaga and Pełczynski [4] and Bennett [3]. Taking $\lambda = l^p$ $(1 \leqq p < \infty)$ we obtain the characterization of weakly p-unconditionally convergent series given by Singer [13] and Bennett [3] (both for $p > 1$) and Maddox [11] (for $p = 1$).

Corollary 1.1. *Let $\lambda(\tau)$ be an FK-, AK-space (not necessarily locally convex). Then*

$$W(E, \lambda) = \{(x_n)_n: (f(x_n))_n \in \lambda^\beta \text{ for all } f \in E'\} .$$

Proof. We have to verify that λ satisfies the hypotheses of Theorem 1. To start with, note that $(\lambda(\tau))' = \lambda^\beta$ (the inclusions follow from the property AK and from the Banach-Steinhaus theorem, respectively). Next, if A is a $\sigma(\lambda^\beta, \lambda)$-bounded set, then A is equicontinuous so that the topology $\beta(\lambda, \lambda^\beta)$ is coarser than τ. Therefore $\lambda(\beta(\lambda, \lambda^\beta))$ is an AK-space and, in this case, $(\lambda(\beta(\lambda, \lambda^\beta)))' = \lambda^\beta$, hence $\beta(\lambda, \lambda^\beta) = \mu(\lambda, \lambda^\beta)$. Q.E.D.

Remark. This corollary generalizes a result given by Gupta and Kamthan [7] for the locally convex case, as well as the cases $\lambda = l(p_n)$ and $\lambda = w_0(p)$ given by Maddox [11]. These spaces are defined as follows (see [11] for further references). Let $(p_n)_n$ be a sequence such that $0 < p_n \leqq 1$ for all n, and p a number such that $0 < p \leqq 1$; then

$$l(p_n) := \{(\alpha_n)_n: \Sigma|\alpha_n|^{p_n} < \infty\} ,$$

$$l^\infty(p_n) := \{(\alpha_n)_n: \sup_n |\alpha_n|^{p_n} < \infty\} ,$$

$$w_0(p) := \{(\alpha_n)_n: \lim \frac{1}{n} \sum_{k=1}^{n} |\alpha_k|^p = 0\} .$$

They are FK-, AK-, and normal spaces and we have $(l(p_n))^\times = l^\infty(p_n)$ and $(w_0(p))^\times = \{(\alpha_n)_n: \sum_{r=0}^{\infty} 2^{r/p} \max \{|\alpha_n|: 2^r \leqq k < 2^{r+1}\} < \infty\}$.

If $\lambda(\tau)$ is a K-space then λ^f is defined in the following way [13, 7.2.3]:

$$\lambda^f := \{(f(e_n))_n: f \in \lambda'\} .$$

Then we have the following inclusion-type result (compare [15, 8.2.1)]):

Corollary 1.2. *Let λ_1 and λ_2 be FK-spaces, λ_2 in addition locally convex, λ_1 in addition an AK-space. Then $\lambda_1 \subset \lambda_2$ if and only if $\lambda_2^f \subset \lambda_1^\beta$.*

Proof. $\lambda_1 \subset \lambda_2$ if and only if $(e_n)_n \in W(\lambda_2, \lambda_1)$: note that if $\lambda_1 \subset \lambda_2$ then this inclusion is continuous. By the preceding corollary we obtain that $(e_n)_n \in W(\lambda_2, \lambda_1)$ if and only if λ_2^f is included in λ_1^β. Q.E.D.

Remark. Since $(\bigcap_{0<p<1} l^p)^\beta = l^\infty$, the above result readily yields the following theorem due to Bennett [3]: "*A locally convex FK-space contains $\bigcap_{0<p<1} l^p$ (if and) only if it contains l^1*".

Corollary 1.3. *Let $\lambda_i(\mu(\lambda_i, \lambda_i^\beta))$ $(i = 1, 2)$ be AK-spaces such that $\lambda_2(\mu(\lambda_2, \lambda_2^\beta))$ is sequentially complete, and $\lambda_1(\mu(\lambda_1, \lambda_1^\beta))$ is barreled. Then $\lambda_1 \subset \lambda_2$ if and only if $\lambda_2^\beta \subset \lambda_1^\beta$.*

Proof. Take $E = \lambda_2$ in Theorem 1. λ_2 is an AK-space, therefore $\lambda_1 \subset \lambda_2$ means that $(e_n)_n \in W(\lambda_2, \lambda_1)$, whence $\lambda_1 \subset \lambda_2$ if and only if $(f(e_n))_n \in \lambda_1^\beta$ for all $f \in (\lambda_2)'$. But $(\lambda_2)' = \lambda_2^\beta$, i.e., for each $f \in (\lambda_2)'$ there is a sequence $\alpha \in \lambda_2^\beta$ such that $f(e_n) = \alpha_n$ and, conversely, each sequence $\alpha \in \lambda_2^\beta$ yields an element $f \in (\lambda_2)'$. Hence the condition for f is equivalent to $\lambda_2^\beta \subset \lambda_1^\beta$. Q.E D.

Remark. If λ_2 is a perfect space, then λ_2 satisfies the hypotheses of the preceding corollary [9, § 30].

Corollary 1.4. *Let λ be a perfect space such that $\lambda(\nu(\lambda, \lambda^\times))$ is semi-reflexive (see [9, § 30.4]). Then*

$$W(E, \lambda^\times) = \{(x_n)_n : (f(x_n))_n \in \lambda \text{ for all } f \in E'\} .$$

Remark. Pietsch (see [9, § 44.8]) defined, for a perfect space λ, the space of E-valued sequences

$$\lambda(E) := \{(x_n)_n : \Sigma\alpha_n x_n \text{ converges unconditionally for all } \alpha \in \lambda^\times\} .$$

Bearing in mind that unconditional convergence and bounded multiplier convergence are equivalent in E, and that $\lambda^\times$ is a normal space, we obtain that $\Sigma\alpha_n x_n$ is unconditionally convergent for all $\alpha \in \lambda^\times$ if and only if it converges in the usual sense for all $\alpha \in \lambda^\times$, i.e. $W(E, \lambda^\times) = \lambda(E)$.

Analogously (see [8, 19.4]) one defines

$$\lambda[E] := \{(x_n)_n : (f(x_n))_n \in \lambda \text{ for all } f \in E'\} .$$

Then 1.4 states that if λ is a perfect, semi-reflexive (with its Köthe normal topology) space, then $\lambda(E) = \lambda[E]$ (compare [8, 16.5]).

Observe that if both λ_1 and λ_2 satisfy the hypotheses of Theorem 1, then $W(E, \lambda_1) = W(E, \lambda_2)$ holds for all E if and only if $\lambda_1^\beta = \lambda_2^\beta$. In particular, if we take $\lambda_1 = l^1$ and $\lambda_2 = l(p_n)$, we obtain the following result due to Maddox [11] (recall that $(l(p_n))^\times = (l(p_n))^\beta = l^\infty(p_n)$): "*$W(E, l^1) = W(E, l(p_n))$ holds for all E if and only if* $\inf_n p_n > 0$". On the other hand, it is well-known that a series in E is bounded multiplier convergent if and only if it is subseries summable, or, in our terminology, that $W(E, l^\infty) = W(E, m_0)$ for all E (where, as usual, m_0 stands for the linear span of all sequences of zeros and ones), although l^∞ does not satisfy the hypotheses of Theorem 1, and neither does m_0. In this section we study the general case of two different sequence spaces λ_1 and λ_2. We need the notion of the topology $\tau\, S(\lambda)$ that was introduced by H. H. Schaefer [12]. Namely, $\tau\, S(\lambda)$ is the finest locally convex topology on λ which is consistent with the dual pair (λ, λ^β) and which has the property AK. This topology is given by the family of seminorms

$$\alpha \to p_C(\alpha) := \sup \{|\sum_{k=1}^{n} \alpha_k c_k|: n \in \mathbb{N},\ c \in C\}$$

where C runs through the family $S(\lambda)$ of all absolutely convex $\sigma(\lambda^\beta, \lambda)$-bounded subsets of λ^β such that for all $\alpha \in \lambda$, $\Sigma\alpha_n c_n$ converges uniformly with respect to $c \in C$. Note that, by [5, Prop. 11], $S(\lambda)$ is the family of all absolutely convex $\sigma\gamma(\lambda^\beta, \lambda)$-relatively compact subsets of λ^β.

Theorem 2. *Let λ_1 and λ_2 be sequence spaces. Then the following assertions are equivalent:*

(1) *$W(E, \lambda_1) \subset W(E, \lambda_2)$ for every E,*

(2) *$\lambda_2 \subset \bar{\lambda}_1$, where the closure is taken in $\lambda_1^{\beta\beta}(\tau\, S(\lambda_1))$.*

Proof. (1) $\Rightarrow$ (2) Let us first check that the topology $\tau\, S(\lambda_1)$ can be described by the family of polar seminorms

$$\alpha \to q_C(\alpha) := \sup \{|\Sigma\alpha_n c_n|: c \in C\}$$

as C runs through $S(\lambda_1)$. Indeed, if $\alpha \in \lambda_1$ and $C \in S(\lambda_1)$, then for every $\varepsilon > 0$ we can find an index N such that

$$\sup \{\|(I - P_{n-1})(\alpha c)\|_{cs}: n \geqq N,\ c \in C\} =$$
$$= \sup \{|\sum_{k=n}^{m} \alpha_k c_k|: m \geqq n \geqq N,\ c \in C\} < \varepsilon .$$

Now, if b is in B (the unit ball in bv) then

$$|\sum_{k=n}^{\infty} \alpha_k c_k b_k| = |\langle (I - P_{n-1})(\alpha c), b\rangle_{(cs,bv)}| \leqq \|(I - P_{n-1})(\alpha c)\|_{cs} .$$

Thus $B(C) := \{(b_n c_n)_n \colon b \in B,\ c \in C\}$ is a set of uniform convergence in the sense of Schaefer, hence $acx(B(C))$ is in $S(\lambda_1)$ by [12, Prop. 3]. Therefore, we can write

$$p_C(\alpha) = \sup\{\|(\alpha_n c_n)_n\|_{cs} \colon c \in C\} = \sup\{|\Sigma\alpha_n c_n b_n| \colon c \in C,\ b \in B\} =$$
$$= \sup\{|\Sigma\alpha_n d_n| \colon d \in B(C)\} = q_{B(C)}(\alpha) \leqq q_{acxB(C)}(\alpha)\,.$$

Next, if $C \in S(\lambda_1)$ then by [12, Prop. 3] C is $\sigma(\lambda_1^\beta, \lambda_1)$-relatively compact and, a fortiori, $\sigma(\lambda_1^\beta, \phi)$-relatively compact. So we obtain, as we did in the proof of Theorem 1, that C is $\sigma(\lambda_1^\beta, \lambda_1^{\beta\beta})$-bounded, and we can consider $\tau\, S(\lambda_1)$ as a polar topology defined in $\lambda_1^{\beta\beta}$. Bearing in mind the form of the semi-norms p_C and [12, Prop. 4], one can apply [9, § 18.4.4] to the topologies $\sigma\gamma(\lambda_1^{\beta\beta}, \lambda_1^\beta)$ and $\tau\, S(\lambda_1)$ to deduce that $\lambda_1^{\beta\beta}(\tau\, S(\lambda_1))$ is a complete space and, therefore, that $\bar{\lambda}_1(\tau\, S(\lambda_1))$ is also complete.

Now, take $E = \bar{\lambda}_1(\tau\, S(\lambda_1))$. Then $(e_n)_n \in W(\bar{\lambda}_1, \lambda_1)$ since $\lambda_1(\tau\, S(\lambda_1))$ is an AK-space. Hence, by virtue of (1), $(e_n)_n \in W(\bar{\lambda}_1, \lambda_2)$, i.e. $\Sigma\alpha_n e_n$ converges in $\bar{\lambda}_1$ for all $\alpha \in \lambda_2$. However, in that case we have that $\Sigma\alpha_n e_n = \alpha$ since $\bar{\lambda}_1$ is a K-space, therefore $\alpha \in \bar{\lambda}_1$ for all $\alpha \in \lambda_2$.

(2) ⇒ (1) Let E be a *lcs* space and $(x_n)_n \in W(E, \lambda_1)$. If we take an absolutely convex zero-neighbourhood U in E, it is clear that the series $\Sigma\xi_n f(x_n)$ converges uniformly with respect to $f \in U^0$ for all $\xi \in \lambda_1$. Thus, the set $A := \{(f(x_n))_n \colon f \in U^0\}$ belongs to $S(\lambda_1)$. Now, if $\alpha \in \lambda_2$ then, by (2), we can find $\xi \in \lambda_1$ such that $p_A(\alpha - \xi) < 1/4$, whence

$$\sup\{|\sum_{k=n}^{m}(\alpha_k - \xi_k) f(x_k)| \colon m, n \in \mathbb{N},\ m \geqq n;\ f \in U^0\} < 1/2\,.$$

On the other hand, $\Sigma\xi_n x_n$ converges in E, therefore we can find an index N such that

$$\sup\{|\sum_{k=n}^{m}\xi_k f(x_k)| \colon m, n \in \mathbb{N},\ m \geqq n \geqq N;\ f \in U^0\} < 1/2\,.$$

Then we can find an index N such that

$$\sup\{|\sum_{k=n}^{m}\alpha_k f(x_k)| \colon m, n \in \mathbb{N},\ m \geqq n \geqq N;\ f \in U^0\} < 1\,,$$

i.e., $\sum_{k=n}^{m}\alpha_k x_k \in U$ if $m \geqq n \geqq N$, so that $(x_n)_n \in W(E, \lambda_2)$. Q.E.D.

Corollary 2.1. *Let λ_1 and λ_2 be sequence spaces. Then the equality $W(E, \lambda_1) = W(E, \lambda_2)$ holds for every E is and only if $\lambda_j \subset \bar{\lambda}_i$ where the closure is taken in $\lambda_i^{\beta\beta}(\tau\, S(\lambda_i))$ for $i, j = 1, 2;\ i \neq j$.*

Remark. If λ is a normal space, then $\tau\, S(\lambda)$ is the Mackey topology $\mu(\lambda, \lambda^\times)$ (see [14, Ch. 2, § 4 (16)]).

Corollary 2.2. *Let $\lambda_1 \subset \lambda_2$ be sequence spaces. Consider the following conditions:*

(1) $W(E, \lambda_1) = W(E, \lambda_2)$ *holds for every* E;
(2) $\lambda_2 \subset \bar{\lambda}_1$ (*the closure taken in* $\lambda_1^{\beta\beta}(\tau\, S(\lambda_1))$);
(3) $\lambda_1^\beta = \lambda_2^\beta$ *and* λ_1 *is dense in* $\lambda_2(\beta(\lambda_2, \lambda_2^\beta))$;
(4) $\lambda_2 \subset \bar{\lambda}_1$ (*the closure taken in* $\lambda_1^{\times\times}(\mu(\lambda_1, \lambda_1^\times))$).
Then (3) $\Rightarrow$ (1) $\Leftrightarrow$ (2). *Moreover, if* λ_1 *is normal then* (1) $\Leftrightarrow$ (4).

Example 1. In his important paper [10], Lorentz defined the space of almost convergent sequences by using the idea of Banach limits. A sequence $(\alpha_n)_n$ is said to be almost convergent to s if

$$s = \lim_{p \to \infty} \frac{1}{p} \sum_{k=n}^{n+p-1} \alpha_k \quad \text{uniformly in} \quad n \in \mathbb{N}\,.$$

By ac and ac_0 we denote, respectively, the closed subspaces of $l^\infty(\|\cdot\|_\infty)$ of almost convergent and almost convergent to zero sequences. If $[e]$ stands for the linear span of the sequence $e := (1, 1, \ldots)$ then $ac = [e] \oplus ac_0$. If bs denotes the space of all sequences which have bounded partial sums, then bs is dense in $ac_0(\|\cdot\|_\infty)$ according to Bennett and Kalton [2, Thm. 3], whence $[e] \oplus c_0 \oplus bs$ is dense in ac under the ∞-norm. Since $([e] \oplus c_0 \oplus bs)^\beta = l^1 = ac^\beta$, by using the above corollary we obtain, for every E, that

$$W(E, ac) = W(E, [e] \oplus c_0 \oplus bs)\,.$$

Plainly, the latter space equals $W(E, [e]) \cap W(E, c_0) \cap W(E, bs)$. Now $(x_n)_n \in W(E, bs)$ if and only if $x_n \to 0$ and $\Sigma|f(x_n) - f(x_{n+1})|$ converges uniformly on each equicontinuous subset of E'. (This can be easily deduced from the result about $W(E, l^\infty)$ given in [11] and the usual linear isomorphism between bs and l^∞. Indeed, the proof is almost contained in [11, Thm. 2($\Leftarrow$)].) Finally, according to the remarks made about c_0 after Theorem 1, we obtain: "$(x_n)_n$ *is in* $W(E, ac)$ *if and only if* (i) Σx_n *converges in* E, (ii) Σx_n *is weakly unconditionally Cauchy and* (iii) $\Sigma|f(x_n - x_{n+1})|$ *converges uniformly on each equicontinuous subset of* E'." This result was established by Maddox in [11].

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Souhrn

POZNÁMKA O λ-MULTIPLIKÁTOROVĚ KONVERGENTNÍCH ŘADÁCH

MIGUEL FLORENCIO, PEDRO J. PAÚL

Formální řada Σx_n v sekvenciálně úplném lokálně konvexním prostoru E je λ-multiplikátorově konvergentní (λ je sekvenciální prostor), jestliže $\Sigma \alpha_n x_n$ konverguje v E pro každé $\alpha \in \lambda$. V článku se dokazuje, že v sudovitém AK-prostoru řada Σx_n je λ-multiplikátorově konvergentní právě když je slabě λ-multiplikátorově cauchyovská. Dále se studuje problém ekvivalence konvergence tohoto typu pro různé multiplikátory λ_1 a λ_2.

Резюме

ЗАМЕЧАНИЕ О λ-МУЛЬТИПЛИКАТОРНО СХОДЯЩИХСЯ РЯДАХ

MIGUEL FLORENCIO, PEDRO J. PAÚL

Формальный ряд Σx_n в секвенциально полном локально выпуклом пространстве E называется λ-мультипликаторно сходящимся (λ—некоторое пространство последовательностей), если $\Sigma \alpha_n x_n$ сходится в E для каждого элемента $\alpha \in \lambda$. В статье доказывается, что в бочечном AK-пространстве ряд Σx_n λ-мультипликаторно сходится тогда и только тогда, когда он удовлетворяет слабому λ-мультипликаторному условию Коши. Изучается также проблема эквивалентности сходимостей этого типа, соответствующих разным мультипликатором λ_1 и λ_2.

Authors' address: E. S. Ingenieros Indurstiales, Avda. Reina Mercedes s/n, 41012 - Sevilla, Spain.