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ABSTRACTS OF THESES* IN MATHEMATICS

defended recently at Charles University, Prague

ASYMPTOTIC BEHAVIOR OF DISSIPATIVE EQUATIONS BY THE METHOD OF TRAJECTORIES

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(April 4, 2001; supervisor J. Málek)

Time evolution of dissipative equations is usually studied in terms of the semigroup of solution operators, defined on some suitable space of initial conditions. The method of trajectories is an alternative approach, where as the phase space there serves the space of "trajectories", which are simply the solutions to given equation restricted to the time interval of some fixed length. It turns out that certain problems (existence of global attractor, finiteness of its fractal dimension, construction of exponential attractor) are difficult or impossible to solve directly for solution operators, but it is possible to establish those results first for "trajectories" and then transfer them back to the original phase space.

The thesis consists of three chapters, each of them applying the method of trajectories to a different problem. Chapter 1 is devoted to equations of parabolic type. We develop the trajectory approach in general setting and then apply it to the abstract non-linear dissipative equation and to a class of equations from non-linear fluid mechanics. In all cases, the existence of global attractor with finite fractal dimension and the existence of exponential attractor are established.

Chapter 2 studies the wave equation with non-linear damping. For this equation, the existence of global attractor has been known. Using the method of trajectories, it is shown that the global attractor has finite fractal dimension.

The method of trajectories seems to be well suited also to equations having non-local (in time) terms. In particular, equations with bounded delay are studied in Chapter 3. For certain class of problems, the existence of global attractor with finite fractal dimension is established.

WORK OF THE MATH TEACHER WITH TALENTED STUDENTS IN MATHEMATICS

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The author of this text taught at the secondary grammar school with enhanced teaching of mathematics for many years and worked with mathematically talented

*An equivalent to PhD.

students. The presented thesis discusses in detail the content and way of work with these students.

The text is didactic and methodical, it also includes psychological aspects and at the same time, it is a mathematical text. The difficulty of presented mathematics corresponds to the age of students who are the prime focus of the thesis.

The first chapter characterizes classes in question and individuals from the standpoints of psychology, pedagogy, sociology, didactics, ethics etc. In classes with enhanced teaching of mathematics there is an unordinary concentration of mathematically talented students, therefore, specific forms of work in these classes have been created. The thesis gives a comprehensive survey of these forms, which to the author's mind have never been summarized anywhere else. The forms are divided into compulsory (i.e. given by laws, edicts, orders of the Ministry of Education) and optional.

From the whole spectrum of activities, the written school leaving examinations in the fourth year of classes with enhanced teaching of mathematics is discussed in detail in Chapters 3 to 5. Specifically, their preparation, realization and evaluation is presented. This topic has been chosen because the author has put most of his effort with certain plan and conception into this area.

The author's principles for the preparation of individual problems and the whole written school leaving examination are the main part of this chapter and one of the main parts of the whole thesis. One of the problems from 1999 is used for a detailed analysis of the author's methodology of the preparation of problems for the written school leaving examinations.

In Chapter 6, one mathematical topic which has been elaborated into two similar problems is considered. One of the problems appeared in 1998 in a written school leaving examination, the second one in Mathematical Olympiad, category C in 1996. Solutions to both these problems are given and analyzed.

Chapter 7 is closely linked to the previous one. Again, one mathematical topic is being elaborated into many similar problems which can be used in Mathematical Olympiad or some other competitions.

SHEAVES OF SOLUTIONS TO ELLIPTIC AND PARABOLIC PDE'S AND THEIR PROPERTIES

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The thesis consists of two loosely related parts. Both parts use the theory of harmonic spaces to provide various characteristics of solutions to the second order partial differential equations.

The first part is devoted to discussing various ways of solving the first boundary value problem (known as the Dirichlet problem) on a bounded domain. For

uniformly elliptic operators with smooth coefficients and continuous boundary data it is known that the Perron-Wiener-Brelot solution and the weak solution coincide. A similar result is obtained here for uniformly parabolic operators on a bounded cylindrical domain.

The second part deals with sequences of functions and their convergence properties. In particular, the sets of locally uniform convergence of pointwise convergent sequences will be characterized in this part. Results obtained for sequences of holomorphic functions go back to Hartogs and Rosenthal ([HR]). Here we prove analogous results for many other sheaves of functions, especially for the sheaf of solutions to the second order elliptic PDE's. An essential part of this section was published in Comment. Math. Univ. Carolinae 40, 4 (1999), pp. 665–678.

ESTIMATES AND TESTS OF THE PARETO INDEX

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The main subject of this thesis is an estimation of the Pareto type index of a given distribution, and testing the hypotheses on its value. We first define the heavy tails of the distribution. Let X be a random variable with distribution function $F(x)$; then the right tail $1 - F(x)$ for $x \to \infty$ behave like x^{-m} for some $m > 0$. In ther words, if we have a data sample from this distribution, we can find much more "extreme" observations in other, lighter distributions. With extremes closely coherent extreme value theory and we start with its results.

We propose an estimator of the Pareto index m of a distribution, that competes well with the Hill, Pickands and moments estimators. The above estimators are based only on the extreme observations. We define a new estimator $m_n(x) = -\log P(\bar{X}_n > x)/\log x$ of the Pareto index m. The proposed estimator utilizes all observations and its idea rests in the tail behavior of the sample mean $\bar{X}_n$, having a simple structure under heavy-tailed F. We use that fact that $m_n(x) \to m$ in a construction of a new estimator of the Pareto index m. The observations are divided into k independent samples of sizes n. Then the empirical distribution function based on k sample means is the main estimation tool. The estimator is strongly consistent and asymptotically normal as $k \to \infty$, while n remains fixed.

By a similar technique we build the estimator $m_n^*(x) = -\log P(X_{(n)} > x)/\log x$, where $X_{(n)}$ is a maximum of n observations. Its computation is based on the sample maxima.

Further, we test whether the tail is of the Pareto type with index $m > m_0$ against the alternative that the tail is lighter. Finally we compare the behavior of these estimators and show simulations of random variables of Cauchy and t-distributions.

LIFE AND WORK OF BOHUMIL BYDŽOVSKÝ

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Bohumil Bydžovský was a Czech mathematician. The aim of this work is to describe his life and his work in mathematics and other fields. The first chapter describes his life and his scientific and pedagogical activities and presents his father's and his sons' lives. Bohumil Bydžovský was born in March 14, 1880, in Duchcov. He studied mathematics and physics at the Faculty of Philosophy of the Czech University in Prague. In 1903 he became Doctor of Philosophy. From 1903 to 1909 he was teaching at secondary schools. At that time he started his scientific work and published his first mathematical articles. After his successful habilitation in 1909 he started his lectures at the Czech University in Prague. From 1910 to 1920 he also lectured at the Czech Technical College in Prague. In 1919 he was named Extraordinary Professor and in 1921 Full Professor of mathematics of the Czech University in Prague. For the study year 1930/31 he was elected dean of the Faculty of Sciences. In 1946 he was elected rector of the Charles University for the study year 1946/47. When Karel Engliš gave up the function of the rector after the February events in 1948, Bydžovský was elected rector for the remaining part of the study year 1947/48. In 1957 he stopped his pedagogical activity at the Charles University. He was a member of many scientific societies and a regular participant of various international mathematical congresses. He died in May 6, 1969, in the hospital at Jindřichův Hradec.

All his life he was interested in the issues of the secondary school reforms and he took active part in them. The second chapter describes his activities in this field. The third chapter presents his secondary school textbooks. He wrote textbooks of arithmetics and algebra for the higher classes of secondary schools. In 1910 and 1911 his textbooks were published for the first time. We can say that they were used for more than forty years. His textbooks were strongly influenced by the European movement for the reform of the teaching of mathematics at secondary schools and by Merano program. Bohumil Bydžovský is the author of three university textbooks: *Introduction to Analytic Geometry* (1923), *The Elements of the Theory of Determinants and Matrices and their Use* (1930) and *Introduction to Algebraic Geometry* (1948). The sixth chapter presents the scientific works of Bohumil Bydžovský. During his life he published 61 scientific mathematical articles. The first one was published in 1906 and the last one in 1963. They were published in Czech, French and German. Most of them belong to algebraic geometry. These articles can be divided into four thematic groups. The common feature of most of them is the use of elliptic functions in geometry. The contents of the first and the largest thematic group consists in the theory of algebraic curves. The second thematic part is devoted to birational transformations, especially to

Cremona transformations. The third thematic part are the works about plane configurations. The fourth thematic part are two articles about geodetic curves that belong to differential geometry. The seventh chapter concerns the other works by Bohumil Bydžovský.

MATHEMATICAL AND STATISTICAL ANALYSIS OF SOLVENCY SCREENING METHODS

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The doctoral thesis describes and analyzes several solvency monitoring methods used for property liability insurance companies by state regulators (NAIC) and rating companies (Standard & Poor's, Moody's). The theoretical mathematical model used in financial modeling of free capital (also called risk reserve) development amends these methods. Firstly, there are defined basic terms: free capital, required capital and ruin probability. To quantify the required capital, there is presented a different approach than classical approximation by e.g. standardized normal distribution. There is an approximation employing Markov-inequality. This approximation assumes to be known the three central moments of the random variable representing the change in the free capital and produces a relation for the required capital, which can be broken down into individual required capitals allocated into individual risks. The mathematical model considers multi-line insurance company and further dependent lines of business (respective their loss ratios). Then, the description and theoretical analysis of the methods follow, in order to illustrate their similarities and differences. The final chapter contains numerical analysis of the methods and comparison with results of the mathematical model for a specified insurance company, which offers three different classes of business. The influence of the assumption about dependency or independency different lines of business on the required capital, ruin probability and risk distribution is also presented and analyzed. Moreover, there are financial ratios employed in the mathematical model to evaluate ruin probability.

LIFE INSURANCE OPTIONS: PRICING AND RESERVING

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A lot of companies selling life business offer some kind of embedded or additional option, which allows the policyholder to alter his life cover at some stage of his policy duration or to prolong it after the maturity date.

The paper examines the influence of antiselection on such options where no underwriting is required to increase or prolong the cover. The following issues are dealt with:

- additional premium for the option,
- necessity of an additional reserve for policyholders having possibility to use option,
- necessity of an additional reserve for policies altered or issued by the option without medical underwriting and comparison with reserves of policies with the same parameters, but with medical underwriting.

The proposed method consists in dividing the insured into three categories, which are defined during the medical underwriting process: standard risks, substandard risks and refused risks. A multiple state model is constructed to describe the transitions between this categories, each of them having different mortality rates. Under the assumption that every policyholder in the group of substandard or refused risks takes advantage of the option, the behavior of the group of standard risks (healthy people) is the key parameter for pricing and reserving of the option.

Mathematical formulae for premiums and premium reserves are presented together with numerical illustrations.

MATHEMATICAL METHODS OF FINANCING MOTOR INSURANCE GUARANTEE FUNDS

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Bank guarantee funds have been adopted in many countries to protect bank clients against losses resulting from bank insolvency and to grant that clients will obtain at least some proportion of their deposits. Mathematical theory which gives the answer how to set a price of deposit insurance premium which each bank has to pay to the guarantee fund is based upon theory for pricing common-stock put options.

On the other hand, insurance guarantee funds (GF) have been adopted only in non-life insurance, specially in third-party liability business, to compensate liabilities of insolvent insurance companies. One of the areas where GFs are very common is motor third-party liability insurance (MTPL). These MTPL GFs are much more complex in comparison with the bank GFs as they are created not only to pay compensation for insurers which are not able to fulfill their obligations due to insolvency but also to finance liability claims caused by uninsured or unknown drivers. Such MTPL GFs are financed by contributions of the licensed insurers (members of GF) which are allowed to run MTPL business but the theory upon

which the contributions should be based is not discussed in literature.

Due to democratization of Czech MTPL market since January 1, 2000, twelve insurance companies started to offer MTPL insurance and there also has been established Czech GF. This thesis studies the basic methods of financing motor insurance guarantee funds (GFs) and possible impacts upon economic efficiency of MTPL market. As the insurance market is strongly influenced by asymmetric information, the study of equilibrium and its efficiency is not so trivial and the asymmetric information can even lead to market failure. Therefore, main goals of this thesis are:

(1) fair assessment rule for members' contributions from the viewpoint of their structure coveting the analysis of possible impacts upon economic efficiency,

(2) optimal prediction of ultimate claims in MTPL,

(3) optimal asset management (i.e. investment strategy and construction of bench-mark).

STOCHASTIC ANALYSIS OF FINANCIAL FLOWS

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The thesis is focused on phenomena, which occur in order to coming "news" to the market. It is clear, that the "news" has two representations: one is the "good" and the second is the "bad". So, the news has two states, like so called *dichotomous process* — the Markov process with two states. It suggests us to describe the asset price by a continuous-time process based on Brownian motion with additional dichotomous component in the model. There are two ways how to incorporate the dichotomous process into the log-normal model. It can be added to the *drift* part of the stock price or to the *volatility*.

The problem of option pricing, where the expected rate of returns changes in the Poisson time points with two values, is similar to the problem, where the stock-price is driven by the stochastic equation with the Poisson process component. The martingale properties of the compensated Poison process play an important role there. In the case of dichotomous drift, the rate of return is higher in times of good economic news rather than in times of bad news. The obtained stock price process is an exponential function of sum of two stochastic processes — the Itô process with a constant drift and the *integrated dichotomous process*. The exact form of the density function is derived and the martingale property is studied. Also the compensated integrated dichotomous process is derived. It stays on the same place in the analyzed model as the compensated Poisson process in the above mentioned model.

Many researchers have documented, that the stock return volatility tends to rise following the good and bad news. This is a good reason for considering the model, where the volatility varies with two values σ_- and σ_+. In the stock price model the volatility can be replaced by the dichotomous process, independent of the Brownian motion. It leads to a model with *stochastic volatility*. In the case of dichotomous volatility it is easy to study the option price, because the density function of the integrated dichotomous process is known.

The time continuous model of stock prices is approximated by Euler scheme, which gives the time discrete model $Y_i = D_i + \varepsilon_i$, where ε_i is a white noise and D_i is a Markov chain with two real states. Also the issue of the least squares estimation of parameters, which leads to so called *strict algorithm*, is mentioned. Another least squares estimation method, which is based on minimizing a local sum of squares over a window of samples, leads to so called *local algorithm* of estimation of parameters. The relation between the strict and local methods is explained too. Computer simulations compare both these methods and maximum likelihood estimation method.

A method of technical analysis, which is based on the assumption that price grows if the Markov chain is in the upper state, and price decreases, if it is in the lower state, is proposed. First, the time series is analyzed, the state values and their probabilities are estimated. Then the indicator of buying and indicator of selling are found as an inverse function to the probability of the upper and lower state, respectively. As another application of the studied model, one can use this model to study the business cycle or exchange rate too.

DIRAC OPERATOR IN PARABOLIC CONTACT SYMPLECTIC GEOMETRY

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The thesis takes up the long tradition of study of invariant differential operators on manifolds. We follow the analogy of the Dirac and Twistor operators on manifolds with a given contact (and spin) structure, and we define their counterparts on symplectic manifolds and, inside the invariant parabolic theory, on manifolds with a given contact symplectic structure. In this case, the counterpart of the group $Spin$ being the metaplectic Lie group $Mp(n,\mathbb{R})$ as the twofold covering group of symplectic Lie group $Sp(n,\mathbb{R})$. The rôle of spinor representation is played by the infinite dimensional Segal-Shale-Weil (metaplectic) representation L of $Mp(n,\mathbb{R})$ on the Hilbert space $L^2(\mathbb{R}^n)$. We present the decomposition of the target space of sections $L\otimes\mathbb{C}^{2n}$ onto a direct sum of irreducible $Sp(n,\mathbb{R})$ modules in the language of Verma modules, and also in the language of finite dimensional $U(n)$ modules to specify the representation contents.

POLYNOMIAL SOLUTIONS FOR A CLASS OF HIGHER SPIN EQUATIONS

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The doctoral thesis is concerned with generalizations of the Dirac and Rarita-Schwinger equations for functions with values in more complicated $Spin_n$-modules. The first thing to be understood and studied in such a case is the structure of (generalized) Taylor series, i.e. the behaviour of polynomial solutions of the corresponding homogeneous operators.

In particular, in the thesis, the structure of Taylor series with values in $Spin_n$-modules with the highest weights of the form

$$\left(\tfrac{2k+1}{2},\dots,\tfrac{2k+1}{2},\tfrac{1}{2},\dots,\tfrac{1}{2},\pm\tfrac{1}{2}\right) \tag{1}$$

is considered and described. The results in the thesis were obtained using techniques that come from representation theory only, as opposed to methods needed for derivation of the results received up to then. Our results comprise all previously known cases and have a potential to be extended with use of the same methods.

The calculations are based on the relatively recent results by P. Littelmann on Littlewood-Richardson rule (Littelmann P., *A generalization of the Littlewood-Richardson rule*, J. Algebra **130** (1990), no. 2, 328–368). If we are to describe the Littlewood-Richardson rule in a few words, it transfers the task of decomposing a tensor product of two irreducible representation of a semisimple Lie group into irreducible components to an exercise in calculating the number of ways how to fill certain tables with positive numbers. It turns out that in our case, where we must treat tensor products of irreducible $Spin_n$-modules of the form

$$(r,0,\dots,0)\otimes\left(\tfrac{2k+1}{2},\dots,\tfrac{2k+1}{2},\tfrac{1}{2},\dots,\tfrac{1}{2},\pm\tfrac{1}{2}\right), \tag{2}$$

the Littlewood-Richardson formulas are substantially simplified.

As the result, the thesis contains an explicit description of the decomposition of the kernel of generalized Dirac operators with values in the $Spin_n$-modules of the form (1), both in even and odd dimensions. Examples of re-calculation of the previously known results using the new techniques are provided. They demonstrate how much the task has been simplified.

AFFINE FUNCTIONS

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The thesis follows a long standing symbiosis of potential theory and abstract convex analysis. Throughout the work we consider an abstract framework of a function space on a compact space K. By this we mean a linear subspace of $\mathcal{C}(K)$ (the space of all real-valued continuous functions on K) containing constant functions and separating points of K. The abstract concept of function spaces is a useful tool in potential theory and convex analysis. Namely, the space of all affine continuous functions on a compact convex set and the space of all continuous functions on the closure $\overline{\Omega}$ (Ω is an open bounded subset of $\mathbb{R}^n$) which are harmonic on Ω can be dealt as particular cases of function spaces.

We start with the study of the approximation of semicontinuous $\mathcal{H}$-concave functions by continuous $\mathcal{H}$-concave functions. These results are used for the exploration of the minimum principle for $\mathcal{H}$-concave and $\mathcal{H}$-affine functions. We state and prove several versions of the minimum principle.

The barycentric formula for affine functions on compact convex sets is studied in the next section. A characterization of affine functions satisfying the barycentric formula provides a simplified proof of the Choquet barycentric theorem and the Mokobodzki approximation theorem on affine Baire-one functions.

Relationship between abstract function spaces and compact convex sets is investigated. The key tool is a result on preserving of Borel classes under perfect mapping. It turns out that $\mathcal{H}$-affine functions in a general function space $\mathcal{H}$ may be treated as bounded affine functions on a suitable compact convex subset of a locally convex space. This representation allows us to use methods of convex analysis (namely the Hahn-Banach separation theorem) to obtain results on approximation of $\mathcal{H}$-affine semicontinuous and Baire-one functions by continuous $\mathcal{H}$-affine functions.

The last part of the thesis is devoted to the simplicial function spaces and their connection to the previous results. We prove the existence of a sequence of operators which pointwise approximate the solution of the Dirichlet problem. A relation of the Dirichlet problem for a simplicial function space $\mathcal{H}$ with the descriptive properties of its Choquet boundary is treated at the end of the thesis. An analogue to the well-known characterization of Bauer simplexes is proved in terms of Baire-one functions.

Applications of obtained results in potential theory conclude the thesis.