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# EIGENVALUE ASSIGNMENT VIA STATE OBSERVER FOR DESCRIPTOR SYSTEMS

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The problem of eigenvalue placement via state observer for generalized linear time invariant systems is considered. The main contribute is to whow that even if the well-known Luenberger eigenvalue separation theorem may be easily extended to the generalized case, the solvability of the resulting compensator is not in general assured. It is shown that regularity may be reached by a suitable choice of the feedback matrices, and a numerical procedure to solve the problem is proposed.

## 1. INTRODUCTION

The generalized state space theory for linear systems has been recently of great interest and many recent contributes are found in literature.

The main attraction of this theory is that in encloses systems, that are often found in practice, having static variables (we shall name nondynamic variables) that are proportional to the input or variables that are proportional to some of the input derivatives. These last generate the so called impulsive modes that are related to the system infinite eigenvalues [13].

Correspondingly, in the discrete-time case, the infinite eigenvalues, are related to variables depending at each time on some of the future values of the inputs. The question of solvability, i.e. the existence and the uniqueness of the solution for such kind of systems, has been analyzed in [15].

The problem of controllability and observability and duality for generalized state space systems has been considered by [15], [5], [6], [13]. The difference from the regular state space case is that different concepts of controllability and observability are used in relation to the fact if the nondynamic variables and the infinite eigenvalues are considered or not.

The problem of eigenvalue assignment by state feedback for generalized systems has been considered in different papers for example [5], [1], [10]. The dual problem of the asymptotic state estimation has been considered in [11], [7], [8], [14].

In the present paper, we consider the problem of compensator design via state observer. It is shown that even if the well known Luenberger eigenvalue separation theorem holds in the generalized case, the simple assignment of the system eigenvalues does not in general assure that the derived compensator is solvable. This condition may be assured by a suitable choice of the feedback matrices. This result is proved by a theorem for which a constructive proof is given.

In the next chapter some remarks about the theory of observer design for state space systems are made. In Chapter 3, the problem of compensator design via state observer is developed and the main theorem is presented.

A numerical procedure based on the algorithm presented in [2] is finally proposed.

## 2. OBSERVABILITY AND OBSERVER DESIGN FOR GENERALIZED STATE SPACE SYSTEMS

We consider a generalized continuous-time system of the form

$$\begin{aligned} E\,x'(t) &= A\,x(t) + B\,u(t) \\ y(t) &= C\,x(t)\,, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^q$ and $y(t) \in \mathbb{R}^p$ are respectively the generalized state, the input and the output and $E$, $A$, $B$, $C$ are matrices of appropriate dimensions. We shall refer to $m = \operatorname{rank}[E]$ as the dynamic dimension of (1), while $n$ will be named the algebraic dimension. We assume that the pencil $sE - A$ is regular, that is for some complex $s$ $\det(sE - A) \neq 0$. This condition is known to be necessary and sufficient [15] for the solvability of (1). We assume the following definitions.

System (1) is R-observable (RO) iff (the superscript T denotes the transpose)

$$\operatorname{rank}[sE^{\mathrm{T}} - A^{\mathrm{T}}, C^{\mathrm{T}}] = n\,, \quad \text{for every complex } s\,.$$

This condition is commonly referred as the observability of the finite eigenvalues of the pencil $[sE - A]$ or the "observability at finite".

Another definition is found in [13] (observability of both finite and infinite eigenvalues, see [6]): (1) is strongly observable (SO) if it is RO and

$$\operatorname{rank}[E^{\mathrm{T}}, A^{\mathrm{T}}S_0, C^{\mathrm{T}}] = n\,, \quad \text{where} \quad \operatorname{span}[S_0] = \ker[E^{\mathrm{T}}]\,.$$

System (1) is completely observable (CO) if it is RO and $\operatorname{rank}[E^{\mathrm{T}}, C^{\mathrm{T}}] = n$. If this property holds, one has that also the nondynamic variables of the system may be observed from the output. For a complete exposition the reader is referred to [6]. It is immediate that CO $\Rightarrow$ SO $\Rightarrow$ RO.

By duality, three different concepts of controllability that is the reachability or complete controllability (CC), the strong controllability (SC), and the R-controllability [15] are found [6]. The dual conditions of those given above are in this case obtained in terms of the matrices $(E, A, B)$.

The eigenvalue assignment via proportional state feedback has been considered by many authors. In [1] it proved that the complete assignability is equivalent to SC (or to SO in the dual case). Other contributes are found in [12] and [10]. Algorithms for the eigenvalue assignment have been presented in [4] and [2].

The dual problem, that is the observer design, has been also largely investigated. The hypothesis of CO has been made in [14] and [11]. In both these papers, an $n - p$ order observer is realized. No mention to any observability property is made in [9] where a numerical technique for the observer design is suggested. A distinction between causal and non-causal systems is made in [8] and the two cases are handled separately.

In the present paper, we consider an observer of the following form

$$E\, z'(t) = [A - LC]\, z(t) + B\, u(t) + L y(t)\,, \tag{2}$$

where $L$ is an unknown matrix.

We define $\varepsilon(t) = z(t) - x(t)$ as the state estimation error. It is immediate from (1) and (2) that $\varepsilon(t)$ fulfills the equation

$$E\, \varepsilon'(t) = [A - LC]\, \varepsilon(t)\,. \tag{3}$$

We say that (2) is an asymptotic observer for (1) if $\varepsilon(t)$ converges to zero as $t \to 0$ from every initial condition $\varepsilon(0)$.

We assume that a solution to the asymptotic state estimation is achieved if $[sE - A - LC]$ is a regular pencil, $\varepsilon(t)$ does not exhibit an impulsive behavior and its convergence speed is arbitrarily fast. This condition is assured if the $m$ eigenvalues of $[sE - A - LC]$ are assigned to arbitrary finite positions in the left half plane by a suitable choice of $L$.

We have the following theorem, that is the dual version of that in [1] whose proof is immediately derived from that work.

**Theorem 1.** The assignability of the eigenvalues of $[sE - A - LC]$ with regularity is equivalent to the SO of (1).

In [7] the strong observability hypothesis is replaced by the weaker one of "detectability", namely the unobservable eigenvalues are all contained in the unitary circle. The continuous version of this property is that if there are unobservable eigenvalues, they are contained in the half left plane. In this case, however, an arbitrary convergence speed may not be assured.

The algebraic dimension of the resulting state observer is the same of the original system. However its dynamic dimension is $m$. So the observer (3) is preferable to that one proposed in [14] if $m$ is less than $n - p$. This means that an observer in the form (3) is recommended for systems with many static variables and few output ones.

Finally we note that since the eigenvalues of $[sE - A + LC]$ are all finite, we may separate the static and the dynamic part of the observer (3) as follows.

Since (2) has no infinite eigenvalues, we may find two nonsingular matrices $P$ and $Q$ such that the transformation $z(t) = P\,w(t)$, $M = QEP$, $F = Q[A - LC]\,P$, $G = QB$, $N = QL$ lead to the following representation

$$\begin{aligned} w_1'(t) &= F_{11}\,w_1(t) + F_{12}\,w_2(t) + N_1\,y(t) + G_1\,u(t) \\ 0 &= F_{21}\,w_1(t) + F_{22}\,w_2(t) + N_2\,y(t) + G_2\,u(t) \end{aligned}$$

where $w(t) = [w_1(t)^{\mathrm{T}}, w_2(t)^{\mathrm{T}}]^{\mathrm{T}}$, $w_1(t) \in \mathbb{R}^m$, $w_2(t) \in \mathbb{R}^{n-m}$, and $\operatorname{rank}[F_{22}] = n - m$. Denoting by $S = [F_{11} - F_{12}F_{22}^{-1}F_{21}]$, $T = N_1 - F_{12}F_{22}^{-1}N_2$, $V = G_1 - F_{12}F_{22}^{-1}G_2$, we have

$$\begin{aligned} w_1'(t) &= S\,w_1(t) + T\,y(t) + V\,u(t) \\ w_2(t) &= -F_{22}^{-1}[F_{21}\,w_1(t) + N_2\,y(t) + G_2\,u(t)]\,, \end{aligned}$$

so a regular state space observer of order $m$ is derived. The estimation error is given by $\varepsilon(t) = P\,w(t) - x(t)$ and its convergence speed is determined by the eigenvalues of $S$.

**Remark.** It should be noted that the convergence to zero of the estimation error $\varepsilon(t)$ is assured by the only stability of the finite eigenvalues, so the RO hypothesis would be sufficient. In this case, however, $\varepsilon(t)$ may have an initial impulse for some initial conditions. To eliminate the impulsive modes, we have to remove the infinite eigenvalues and then the SO hypothesis is necessary.

## 3. COMPENSATOR DESIGN

The problem of compensator design has been considered in many previous papers. In [12] a proportional-derivative output feedback is considered and a dynamic compensator is proposed. A compensator may be obtained by the introduction of derivators as suggested by [3]. Here, we consider a compensator that realizes the eigenvalue assignment via a state observer.

It is well-known that, in the regular state space case, the problem is solved by a control law of the form

$$u(t) = K\,z(t) + v(t)\,. \tag{4}$$

Without difficulties, we see that, in the generalized case, the introduction of (4) leads to a resulting system of the form

$$\begin{aligned} E\,x'(t) &= [A + BK]\,x(t) + BK\,\varepsilon(t) + B\,v(t) \\ E\,\varepsilon'(t) &= [A - LC]\,\varepsilon(t)\,. \end{aligned} \tag{5}$$

By assuming the strong controllability and observability of (1), we see that the eigenvalues of (5) (i.e. those of $[sE - A - BK]$ and $[sE - A + LC]$) are all assigned to prespecified positions and regularity is assured by a suitable choice of the matrices $L$ and $K$. So, as in the regular case (eigenvalue separation theorem) the problem reduces to the eigenvalue placement via state feedback for two descriptor systems. We see that, if the original system has impulsive modes, these may be removed.

If strong observability and controllability do not hold, we may eliminate the uncontrollable and unobservable part of the system using for example the algorithm proposed in [2] to assign the reachable and observable eigenvalues only.

It is no surprising to see that the input-output behavior of the global system is equivalent to that of $E\,x'(t) = [A + BK]\,x(t) + B\,v(t)$, $y(t) = C\,x(t)$, since $\varepsilon(t)$ is completely uncontrollable. It is also clear that we may use other ctiteria of choice of $K$ such as the optimization of a performance index as suggested in [16].

Unfortunately, in the generalized state case, some troubles concerning the compensator regularity may arise. We see from (4) and (2) (we assume $v = 0$) that the compensator equation has the form

$$\begin{aligned} E\,z'(t) &= [A + BK - LC]\,z(t) + L\,y(t)\,. \\ u(t) &= K\,z(t) \end{aligned} \tag{6}$$

Unfortunately, even if $[sE - A - BK]$ and $[sE - A + LC]$ are both regular, it may happen that the compensator pencil $[sE - A - BK + LC]$ is singular, as we can see in the following example.

**Example.** Let consider the following descriptor system

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = [1 \;\; 1]\,.$$

Suppose that the matrices $K = [k_1\; k_2]$ and $L = [l_1\; l_2]^{\mathrm{T}}$ are to be chosen such that the eigenvalues of $[sE - A - BK]$ and $[sE - A + LC]$ are $\lambda_1 = -1$ and $\lambda_2 = -2$ respectively, with regularity assumption for both pencils. Among other possible solutions we may choose

$$K = [-1 \;\; -1] \quad \text{and} \quad L = [0 \;\; -1]^{\mathrm{T}}\,.$$

Unfortunately, it is immediately seen that

$$\det(sE - A - BK + LC) = \det \begin{bmatrix} 0 & s + 1 \\ 0 & -1 \end{bmatrix} \equiv 0\,.$$

If we choose $K = [-2, -1]$, as an alternative solution, we have $\det(sE - A - BK + LC) = (s + 2)$ and so the solvability is reached.

In general, compensator design may be achieved in two steps as follows.

Step (i) Choose $L$ such that $[sE - A + LC]$ has the desired eigenvalues and it is regular.

Step (ii) Choose $K$ such that $[sE - A - BK]$ has the pre-specified eigenvalues with the condition that $[sE - A - BK]$ and $[sE - H - BK]$, where $H = A - LC$, are both regular pencils.

The following theorem assures that there is ever a choice for $K$ as required in step (ii).

**Theorem 2.** Let be $[sE - H]$ and $[sE - A]$ regular pencil matrices and let $(E, A, B)$ a SC system. Let $\Lambda$ be an assigned set of $m = \text{rank}\{E\}$ complex numbers constrained under conjugation. Then there is $K$ such that the eigenvalues of $[sE - A - BK]$ are the elements of $\Lambda$, while both conditions $\det[sE - A - BK] \not\equiv 0$ and $\det[sE - H - BK] \not\equiv 0$ hold.

To give the proof of Theorem 2, we need some preliminary results. As proved in [2] the matrices $E$, $A$, $B$, may be reduced via unitary transformations to the form

$$B = \begin{bmatrix} b_1 & \cdot & \times & \times \\ & \cdot & \cdot & \cdot \\ & & b_s & \times \\ & & & 0 \end{bmatrix} \quad E = \begin{bmatrix} E_1 & \cdot & \cdot & \times & \times \\ & E_2 & \cdot & \times & \times \\ & & & E_s & \times \\ & & & & E_{nc} \end{bmatrix} \quad A = \begin{bmatrix} A_1 & \cdot & \cdot & \times & \times \\ & A_2 & \cdot & \times & \times \\ & & & A_s & \times \\ & & & & A_{nc} \end{bmatrix} \tag{7}$$

for a certain $s \leqq q$, where $\times$ are generic matrices and the subsystems $(b_j, E_j, A_j)$, $j = 1, \dots, s$ of dimension $n_j$ have the following structure

$$b_j = \begin{bmatrix} \otimes \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad E_j = \begin{bmatrix} \times & \times & \times & \times & \times \\ & \otimes & \times & \times & \times \\ & & \cdot & \cdot & \cdot \\ & & & \otimes & \times \\ & & & & \otimes \end{bmatrix} \quad A_j = \begin{bmatrix} \times & \times & \times & \times & \times \\ \otimes & \times & \times & \times & \times \\ & \cdot & \cdot & \cdot & \cdot \\ & & \otimes & \times & \times \\ & & & \otimes & \times \end{bmatrix}. \tag{8}$$

(we denote by $\times$ a generic entry, by blank the zero elements and by $\otimes$ the nonzero ones). The variables of the subsystem $(E_{nc}, A_{nc})$ are all uncontrollable.

If the system is SC, then the eigenvalues of (7) are those of the CC system $[B_r, E_r, A_r]$ obtained by neglecting the rows and the columns related to $(E_{nc}, A_{nc})$. Then we consider a matrix $K$ of the form

$$K = \begin{bmatrix} k_1 & \times & \cdot & \times & \times \\ & k_2 & \cdot & \times & \times \\ & & \cdot & \cdot & \cdot \\ & & & k_s & \times \\ & & & & 0 \end{bmatrix}, \tag{9}$$

in which $k_j$, $j = 1, \dots, q$, are row vectors of the same dimensions $n_j$ of $(b_j, E_j, A_j)$. The entries of $k_j$, $j = 1, \dots, s$ are to be determined in order to assign the eigenvalues of the $j$th subsystem $(b_j, E_j, A_j)$. In [2] it is pointed out that we cannot in general assign directly all the eigenvalues of $[sE - A - BK]$ in this way because, denoting by $m_j = \text{rank}\{E_j\}$, $m^* = m_1 + m_2 + \dots + m_s$ is not necessarily equal to $m = \text{rank}\{E\}$. In fact in general there may be $m - m^*$ infinite eigenvalues that can be removed in a second step. However, in this second step a system of the form

$$[B, A, E] = \left[\begin{array}{c|cc|cc} B_1 & A_{11} & A_{12} & E_{11} & E_{22} \\ B_2 & 0 & A_{22} & 0 & E_{22} \end{array}\right] \tag{10}$$

is considered, where $E_1$ is invertible while $[B_2, A_2, E_2]$ is in the form (7) and it contains the system infinite eigenvalues, that may be assigned by a matrix of the form $K = [0, K_2]$. So without restriction, we may consider the single input problems of choosing $k_j$ such that $[sE_j - A_j - b_jk_j]$ has a set of desired eigenvalues. This problem may be solved as suggested in [2] where it is shown that, if $E_j$ is singular and if only finite eigenvalues are desired, the first element $k_j^{(1)}$ of $k_j$, $j = 1, \ldots, s$, may be arbitrarily chosen with the condition (assuring $\det[sE_j - A_j - b_jk_j] \not\equiv 0$) $k_j^{(1)} \neq \kappa_j$, where $\kappa_j$ is a critical value that depends on $E_j$, $A_j$, $b_j$.

Proof of the Theorem. Let (1) be reduced as in (7), and let consider a matrix **K** of the form (9). We shall refer to the first diagonal entries $e_{r_jr_j}$, $j = 1, \ldots, s$, of each block $E_j$, as the pivot elements. A pivot is zero if and only if the corresponding subsystem is singular [2]. We have to assure the condition $\det(sE - H - BK) \not\equiv \not\equiv 0$. The condition $\det(sE - H - BK) \equiv 0$ is equivalent to the existence of a constant vector $w \neq 0$ such that $(sE - H - BK)^T w \equiv 0$. This is equivalent to the conditions $E^T w = 0$ and $(H + BK)^T w = 0$, that may be rewritten as

$$w^T[E, H + BK] = 0$$

This equality holds for $w \neq 0$ if and only if the matrix $[E, H + BK]$ has not full rank. Consider the following matrix:

$$M = [g_1, F_1^*, g_2, F_2^*, \ldots, g_s, F_s^*]$$

where $g_j$, $j = 1, \ldots, s$, is the $r_j$th column of $E$ (i.e. that containing the $j$th pivot) if $e_{r_jr_j} \neq 0$, and it is $r_j$th column of $[H + BK]$ if $e_{r_jr_j} = 0$, while the columns in $M$ that form $F_i^*$ are the corresponding columns of $E$ (which do not contain a pivot).

It is immediate to see that $M$ has the following form ($y$ denotes a generic entry):

$$M = \left|\begin{array}{cccccccccccc}
\xi_1 & \times & \cdots & \times & \times & \cdots & \times & \times & \cdots & \times \\
y & \otimes & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
y & & \ddots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
y & & & \xi_2 & \times & \cdots & \times & \times & \cdots & \times \\
\vdots & & & y & \otimes & \cdots & \times & \times & \cdots & \times \\
\vdots & & & \vdots & & \ddots & \vdots & \vdots & \cdots & \vdots \\
\vdots & & & \vdots & & & \xi_s & \times & \cdots & \times \\
\vdots & & & \vdots & & & \vdots & \otimes & \cdots & \times \\
y & & & y & & & y & & \ddots & \times \\
y & & & y & & & y & & & \otimes
\end{array}\right| \tag{11}$$

where the elements denoted by $\xi_j$ are given by $\xi_j = h_{r_jr_j} + b_i^{(1)}k_i^{(1)}$, if the $j$th pivot $e_{r_jr_j} = 0$, where $k_j^{(1)}$ and $b_i^{(1)}$ denote the first entries of $k_j$ and $b_i$ in (7), otherwise, if $e_{r_j r_j} \neq 0$ we have $\xi_j = e_{r_j r_j}$. In the second case, the elements denoted by $y$, are all zero, say $m_{h,r_j} = 0$, $h = r_j + 1, \ldots, n$.

Since $M$ is built by a selection of columns of $E$ and $[H + BK]$, if we prove that it is a nonsingular matrix, we have that rank $[E, H + BK] = n$ and so the regularity pencil $[sE - H + BK]$ is assured.

Without restriction, we assume that all the pivot elements are zero.

Since we may arbitrarily choose $k_j^{(1)} \neq \kappa_j$, we set $k_j^{(1)} = \omega$, $j = 1, 2, \ldots, s$. Then consider the set $\Omega^* = \{\omega \in \mathbb{R}, \omega \neq \kappa_j, j = 1, \ldots, s\}$. We have to assure that $M$ is nonsingular for some value in $\Omega$. Note that $M$ may be rewritten as $M = [\omega N^* - M^*]$, where $M^*$ is the matrix obtained by replacing $\xi_j$ with $h_{r_j r_j}$ in (11) and $N^*$ is the matrix having entries $b_i^{(1)} \neq 0$ correspondingly to the position of the $j$th pivot and zero elsewhere. It is immediate that $[\omega N^* - M^*]$ is a regular pencil. To prove this one may think to apply Gaussian elimination in order to annihilate each one of the $y$-entries of $M^*$ say $m_{h,r_i}$, $h = n, n - 1, \ldots, r_j + 1$, using the $h$th column. In this way a nonsingular triangular pencil is derived, since, as it can be immediately seen, this elimination does not modify $N^*$. Let $\Omega = \{\omega \in \Omega^* \colon \det[\omega N^* - M^*] \neq 0\}$. Then for all $\omega \in \Omega$ both conditions $\det[sE - A - BK] \not\equiv 0$ and $\det[sE - H - BK] \not\equiv 0$ are fulfilled and the proof is complete. □

We see that the values of $\omega$ that must be avoided are the eigenvalues of the regular pencil $[\omega N^* - M^*]$. However, as is known, Gauss elimination may be numerically unsatisfactory and other more efficient tools to handle this eigenvalue problem may be more conveniently used.

## 4. CONCLUSIONS

The problem of compensator design via state observer for generalized state space systems has been considered.

A state observer of order $m = \text{rank}\{E\}$ is proposed, which is especially useful when applied to systems having many nondynamic variables and few input ones. Under the SC and SO conditions, the eigenvalues of the resulting closed-loop system may be placed to arbitrary assigned positions. However, in the singular case, attention must be put on the choice of the feedback matrix assigning the eigenvalues because it may happen that the resulting compensator is not solvable.

To assure solvability, we may perform two steps. With the first one we assign the observer eigenvalues, with the second one we assign the closed loop eigenvalues while assuring regularity for the pencil of the resulting compensator.

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