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SYNTHESIS OF OPTIMAL TRAJECTORY OF INDUSTRIAL ROBOTS

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The paper contains a description of a quite new algorithm for the synthesis of energetically optimal trajectory of industrial robots. Each iteration of this algorithm satisfies all constraints, thus each iteration is completely applicable. There are distinguished two classes of problems — positional and path tracking control. The efficiency of the alfgorithm is demonstrated on examples.

1. INTRODUCTION

Industrial robots driven electrically with a feedback control loop (and even then hydraulically) create highly nonlinear dynamic systems, the control of which in real time is a difficult problem of the control theory. From the theoretical point of view it is a classical problem of optimal control. A system is to be controlled from the initial state to the terminal state so as to minimize some performance index (e.g. energy consumption, time etc.). But the synthesis of the optimal control as a feedback control has not been till now successful because of high nonlinearity with couplings among the joints and of strict constraint conditions at the terminal state [5], [6]. Otherwise only the computation of the time dependence of optimal control is possible only by numeric iterations the convergence of which has been questionable [7], [3], [13], [2], [11]. So the conventional control of today's industrial robots [5], [6], [8], [1] is universally realized by the programme CNC control, i.e. the controller generates the desired corner points of the path and a conventional positional feedback control (linear servo) for each joint stabilizes the system state according to the varying desired system state. The trajectory of motion is the input to the control system which may be numerically fed into the system (e.g. from a simple programme), or furnished through so-called "teaching by doing", i.e. the corner points are recorded while the hand of the robot is led through these points manually by an operator.

The mass use of robots for more difficult application (i.e. the surroundings of the robot is dynamic, the accuracy and complexity of robot's positioning exceeds the human possibilities in "teaching by doing" and/or a quick change of robot's job is necessary according to the production change by many robots in one moment), needs the synthesis of suitable trajectories of industrial robots by a computer without direct interaction with the robot.

This necessity of the synthesis of robot's trajectories by a computer enables us to use an alternative approach towards optimal control, i.e. the computation of the optimal control and the optimal trajectory for the desired robot's movement and the realization of this optimal trajectory by the conventional positional linear CNC control. Then, it is a programme optimal control of an industrial robot.

We regard a suitable trajectory as the trajectory which is convenient from the technological requirements and which is admissible for the dynamics of joint servos and which minimizes the performance index of optimality. The energy consumption is a very natural and advantageous performance index of robot's motion.

A very efficient method of suboptimal approximations for solving this problem was developed in [12]. This method computes from the admissible control and admissible trajectory of robot (i.e. one satisfying constraint conditions, restrictions of servo's dynamics, obstacles in space etc.) the next admissible control and admissible trajectory on which the performance index of optimality is lower. The convergence of the method has been proved. Its main advantage is the complete applicability of each approximation.

Further the use of the method of suboptimal approximation for positional and path optimal control of industrial robots is described. The problem of the synthesis of initial admissible trajectory and the problem of control structure of the programme CNC control of industrial robots which realizes dynamically admissible trajectory dynamically accurately are important and have been solved in [11], but they are not mentioned here.

2. SYNTHESIS OF THE TRAJECTORY OF AN INDUSTRIAL ROBOT

Before one differentiates among particular problems of the synthesis of trajectory of an industrial robot, one must answer the following questions:

1. Must the hand follow a specified path?
2. Are all degrees of freedom of an industrial robot determined by the constraints at its terminal position or along the specified path (the initial position is given and so fixed)?
3. Is the work space free from obstacles?
4. Are there any further requirements on the movement of robot such as minimum-time movement, minimum energy consumption, minimum impacts in drives?
5. Whether and which parts of particular problems of synthesis must be accomplished in real time of the robot's movement or may be prepared in the time of planning before the proper motion?

The answer to each of these questions could be essentially either yes or no. They combine to form several classes of problems. Further the following problems will be analyzed in details.

A. *Positional control.* The initial and terminal positions of an indistrial robot is given and the trajectory between them is not specified. In this way the problem has an infinite number of solutions. The ambiguity of solution can be used for solving the 4th question, in our case for the minimization of energy consumption. Among many performance indexes of optimality the most advantageous one from several points of view is the energy consumption.

Other possible performance indexes of optimality [5] are the minimization of motion time, of average performance effort, of values of reaction forces etc. The minimization of motion time is a natural performance index, but it has a lot of disadvantages. At the numerical solution there can appear degenerated cases with infinite number of solutions, which are eliminated only by the more or less strong nonlinear couplings between robot's joints. During the movement according to this index all actuators have their maximum output. Their energy consumption is also maximum. And at the same time the robot must wait at the final position for the time between the minimum one necessary for the movement and the time resulting from the technological requirements on the robot's movement. During this movement the actuator torque is being switched from the maximum driven value in one direction to the opposite one and this causes the impacts in the robot's gears and the efforts of the robot's actuators and robot's construction.

By the others performance indexes these disadvantages are decreased in a different degree. But only the performance index of energy consumption fluently removes all these disadvantages. The degenerated cases cannot arise because among the robot's joints there is always a stable coupling through the performance index. By this movement there is obviously the minimum energy consumption and the time of the movement which is chosen can completely conform to the technological requirements. From the temporal point of view we don't lose anything because the chosen time of the robot's movement can be shortened to the minimum time. Then the control for the minimum energy consumption is equal to the control for the minimum motion time. The impacts at the robot's actuators and the extermal efforts of the robot's construction don't occur because the time performances of the robot's actuator torques are continuous.

We speak about positional optimal control of industrial robots.

B. *Path tracking control or programme control.* The trajectory along which the industrial robot is to move is specified. The trajectory is usually given only by pure geometric shape of the path and so the problem has even here an infinite number of solutions determined by a possible choice of the velocity profile along the specified path. The ambiguity can be used again for the minimization of energy consumption. We speak about path tracking optimal control of industrial robots.

Both problems are solved in [12] with redundant degrees of freedom, too. Only the solutions without redundant degrees of freedom will be described here. Collision-free path planning and the use of ambiguity of solution of inverse kinematics without or with redundant degrees of freedom for initial path synthesis are rather combinatorial problems of artificial intelligence and together with dynamic properties of motion will not be treated here.

3. POSITIONAL ENERGETICALLY OPTIMAL CONTROL OF INDUSTRIAL ROBOTS

The dynamic equations of an industrial robot with electric servos can be compactly written [1]

$$\mathbf{n}_h = \mathbf{I}(\mathbf{s})\,\ddot{\mathbf{s}} + \dot{\mathbf{s}}^T\,\mathbf{C}(\mathbf{s})\,\dot{\mathbf{s}} + \mathbf{V}(\mathbf{s})\,\dot{\mathbf{s}} + \mathbf{F}(\mathbf{s})\,\dot{\mathbf{s}} + \mathbf{g}(\mathbf{s}), \quad \mathbf{n}^- \leqq \mathbf{n}_h \leqq \mathbf{n}^+ \tag{1}$$

where

$\mathbf{n}_h$ is the n-dimensional vector of constant joint torques,

$\mathbf{I}(\mathbf{s})$ is the $n \times n$ generalized inertia tensor,

$\mathbf{C}(\mathbf{s})$ is the $n \times n \times n$ generalized tensor in the formulation of the Coriolis and centrifugal forces,

$\mathbf{V}(\mathbf{s})$ is the $n \times n$ generalized tensor in the formulation of the viscous friction forces,

$\mathbf{F}(\mathbf{s})$ is the $n \times n$ generalized tensor in the formulation of velocity actuator torque dependency such as for the back EMF of electric motors,

$\mathbf{g}(\mathbf{s})$ is the n-dimensional vector of gravity forces,

$\mathbf{s} = (s_1, \ldots, s_n)^T$ is the n-dimensional vector of robot's joint coordinates,

$\mathbf{n}^+ = (n_1^+, \ldots, n_n^+)^T$ and $\mathbf{n}^- = (n_1^-, \ldots, n_n^-)^T$ are maximum and minimum constant torque limits.

This description can be easily transferred to the state description with the variables

$$\begin{aligned} x_{1i} &= s_i \quad i = 1, \ldots, n \\ x_{2i} &= \dot{s}_i \\ u_i &= \frac{n_{hi} - \dfrac{n_i^+ + n_i^-}{2}}{\dfrac{n_i^+ - n_i^-}{2}} \end{aligned} \tag{2}$$

and then

$$\begin{aligned} \dot{x}_{1i} &= x_{2i} \quad i = 1, \ldots, n\,; \quad j = 1, \ldots, n\,; \quad k = 1, 2\,; \\ \dot{x}_{2i} &= f_i(u_m, \ldots, x_{kj}, \ldots) = \sum_{m=1}^{n} H_{im}(x_{kj})\, u_m + L_i(x_{kj}) \end{aligned} \tag{3}$$

with the control variable restriction

$$|u_i| \leqq 1 \quad i = 1, \ldots, n \tag{4}$$

The initial x_{ki0} and terminal state x_{kif} of robot motion are specified. There is given a time T during which the motion is to be realized so as to minimize the energy consumption given by the functional

(5) $$I = \int_0^T \left(\sum_{i=1}^{n} e_i u_i^2 \right) \mathrm{d}t$$

where the weighting coefficients e_i express contributions of particular control variables on the total energy consumption.

Now we derive a different description of robot dynamics which enables us to construct the control exactly satisfying the constraints. We use the method of the change of dynamic properties of a system by the transformation of input variables [10]. To the given system (3), (4) we choose another dynamic systems with desirable properties

(6) $$\dot{y}_{1i} = y_{2i} \quad i = 1, \ldots, n$$
$$\dot{y}_{2i} = \sum_{m=1}^{n} K_{im} w_m$$

with inputs w_m and outputs y_{ki}. We require the agreement of outputs of both systems

(7) $$y_{ki} = x_{ki} \quad i = 1, \ldots, n \,; \quad k = 1, 2 \,.$$

Comparing the equations (7) with (3) we get

(8) $$\sum_{m=1}^{n} H_{im}(x_{kj})\, u_m + L_i(x_{kj}) = \sum_{m=1}^{n} K_{im} w_m \,.$$

After designating the matrices of values $H_{im}(x_{kj})$, u_m, $L_i(x_{kj})$, K_{im}, w_m as $\mathbf{H}(x_{kj})$, $\mathbf{u}$, $\mathbf{L}(x_{kj})$, $\mathbf{K}$, $\mathbf{w}$ we can derive from the equation (8)

(9) $$\mathbf{u} = \mathbf{H}^{-1}(x_{kj}) \left(\mathbf{K}\mathbf{w} - \mathbf{L}(x_{kj}) \right)$$

and express the equation (9) in its elements

(10) $$u_i = \left[\mathbf{H}^{-1}(x_{kj}) \left(\mathbf{K}\mathbf{w} - \mathbf{L}(x_{kj}) \right) \right]_i = Q_i(w_m, x_{kj}) =$$
$$= \sum_{m=1}^{n} M_{im}(x_{kj})\, w_m - N_i(x_{kj})$$

Thus according to [10] we derived the transformation equations between the input variables u_i and w_m. If these equations (10) are realized in real time, the original dynamic system would have new dynamic properties given by the equations

(11) $$\dot{x}_{1i} = x_{2i} \quad i = 1, \ldots, n$$
$$\dot{x}_{2i} = \sum_{m=1}^{n} K_{im} w_m$$

with new inputs w_m. This method can be applied to the derivation of robot control structure [11] which is capable to realize a dynamic admissible trajectory dynamically accurately. We use equations (10), (11) to satisfy the constraints. If the robot dynamics is described by (11), the equations (4), (5) change to the form

$$|Q_i(w_m, x_{kj})| \leqq 1 \tag{12}$$

$$I = \int_0^T \left(\sum_{i=1}^n e_i Q_i^2(w_m, x_{kj}) \right) dt \tag{13}$$

It is obvious that the solution of the optimal problem described either by (3), (4), (5) or by (11), (12), (13) must be indentical because there is the same robot dynamic system. This is shown in [12]. But it is necessary to use the generalized optimum principle of V. F. Krotov [4] instead of Pontryagin's maximum principle because the original Pontryagin's maximum principle does not hold in the case the set of admissible control inputs (12) is a function of system states and time. But we don't need it for the derivation of suboptimal approximations.

We will not initially consider the control constraints (12) and further constraints (phase etc.) if any. We use Pontryagin's maximum principle (it is now equal to Krotov's optimum principle).

We assemble the hamiltonian

$$\mathscr{H} = -\sum_{i=1}^n e_i Q_i^2(w_m, x_{kj}) + \sum_{i=1}^n \left(\psi_{1i} x_{2i} + \psi_{2i}\left(\sum_{m=1}^n K_{im} w_m\right)\right) \tag{14}$$

and the corresponding conjugate system of differential equations

$$\dot\psi_{1j} = -\frac{\partial \mathscr{H}}{\partial x_{1j}} = \sum_{i=1}^n 2e_i \left(\sum_{m=1}^n M_{im}(x_{kj})\, w_m - N_i(x_{kj})\right) \left(\sum_{m=1}^n \frac{\partial M_{im}}{\partial x_{1j}} w_m - \frac{\partial N_i}{\partial x_{1j}}\right) \tag{15}$$

$$j = 1, \ldots, n$$

$$\dot\psi_{2j} = -\frac{\partial \mathscr{H}}{\partial x_{2j}} = \psi_{1j} + \sum_{i=1}^n 2e_i \left(\sum_{m=1}^n M_{im}(x_{kj})\, w_m - N_i(x_{kj})\right) \left(\sum_{m=1}^n \frac{\partial M_{im}}{\partial x_{2j}} w_m - \frac{\partial N_i}{\partial x_{2j}}\right)$$

and the condition of ham:tonian maximum

$$\frac{\partial \mathscr{H}}{\partial w_m} = 0 \quad m = 1, \ldots, n \tag{16}$$

$$\sum_{i=1}^n e_i \left(\sum_{m=1}^n M_{im}(x_{kj})\, M_{il}(x_{kj})\, w_m\right) = \tfrac{1}{2} \sum_{i=1}^n \psi_{2i} K_{il} + \sum_{i=1}^n e_i N_i(x_{kj})\, M_{il}(x_{kj}) \quad l = 1, \ldots, n\,.$$

This special form of equations (15), (16) is used for the method of suboptimal approximations. Let us have an admissible control of the robot as a J-approximation of the optimal control $w_m^J(t)$, $x_{1i}^J(t)$, $x_{2i}^J(t)$ which satisfies the boundary conditions of motion

$$x_{1i}^J(0) = x_{1i0} \quad x_{1i}^J(T) = x_{1if} \tag{17}$$

$$x_{2i}^J(0) = x_{2i0} \quad x_{2i}^J(T) = x_{2if}$$

and later even further constraints. We substitute it in (15) and we obtain a system of linear differential equations with the right side

$$\dot{\psi}^J_{1j} = P^J_{1j}(t) \tag{18}$$
$$\dot{\psi}^J_{2j} = -\psi^J_{1j} + P^J_{2j}(t)\,.$$

We also substitute in (16)

$$\sum_{m=1}^{n} P^J_{3lm}(t)\, w_m = \sum_{i=1}^{n} K_{il}\psi^J_{2i} + P^J_{4l}(t)\,. \tag{19}$$

Equations (18) can be simply integrated with respect to the time

$$\psi^J_{1j}(t) = \psi^J_{1j}(0) + \int_0^t P^J_{1j}(t)\,\mathrm{d}t \tag{20}$$
$$\psi^J_{2j}(t) = \psi^J_{2j}(0) - \psi^J_{1j}(0)\, t + \int_0^t \left(P^J_{2j}(t) - \int_0^t P^J_{1j}(\tau)\,\mathrm{d}\tau \right) \mathrm{d}t$$

and by substituting (20) in (19) we calculate

$$w_m = \sum_{i=1}^{n} \left(P^J_{5mi}(t)\, \psi^J_{1i}(0) + P^J_{6mi}(t)\, \psi^J_{2i}(0) \right) + P^J_{7m}(t)\,. \tag{21}$$

We substitute (21) in (11) and we integrate with respect to the time from 0 to T and we use (17)

$$x_{2jf} = x_{2j0} + \sum_{i=1}^{n} \left(\psi^J_{1i}(0) \sum_{m=1}^{n} K_{jm} \int_0^T P^J_{5mi}(t)\,\mathrm{d}t + \psi^J_{2i}(0) \sum_{m=1}^{n} K_{jm} \int_0^T P^J_{6mi}(t)\,\mathrm{d}t \right) +$$
$$+ \sum_{m=1}^{n} K_{jm} \int_0^T P^J_{7m}(t)\,\mathrm{d}t \quad j = 1, \ldots, n \tag{22}$$
$$x_{1jf} = x_{1j0} + x_{2j0}T + \sum_{i=1}^{n} \left(\psi^J_{1i}(0) \sum_{m=1}^{n} K_{jm} \int_0^T \int_0^t P^J_{5mi}(\tau)\,\mathrm{d}\tau\,\mathrm{d}t \, + \right.$$
$$\left. + \, \psi^J_{2i}(0) \sum_{m=1}^{n} K_{jm} \int_0^T \int_0^t P^J_{6mi}(\tau)\,\mathrm{d}\tau\,\mathrm{d}t \right) + \sum_{m=1}^{n} K_{jm} \int_0^T \int_0^t P^J_{7m}(\tau)\,\mathrm{d}\tau\,\mathrm{d}t$$

Finally the equations (22) are linear algebraic equations for unknown initial values $\psi^J_{ki}(0)$. We substitute them in (21) and we obtain the time dependence of control variables $w^{J*}_m(t)$, which from (11) gives the time dependence of $x^{J*}_{1i}(t)$ and $x^{J*}_{2i}(t)$. This J^*-approximation satisfies the boundary conditions (17). Now we define

$$w^{J+1}_m = (1 - \alpha)\, w^J_m + \alpha w^{J*}_m \tag{23}$$
$$x^{J+1}_{2i} = (1 - \alpha)\, x^J_{2i} + \alpha x^{J*}_{2i}$$
$$x^{J+1}_{1i} = (1 - \alpha)\, x^J_{1i} + \alpha x^{J*}_{1i}$$

If α is constant, equations (23) define a new approximation satifying (11) which exactly satisfies the boundary conditions (17) for every α. The unknown α is determined from one-dimensional minimization of energy consumption

$$I^{J+1} = \int_0^T (\sum_{i=1}^n e_i Q_i^2(w_m^{J+1}(t), x_{kj}^{J+1}(t)))\,dt = \tag{24}$$

$$= \int_0^T (\sum_{i=1}^n e_i (\sum_{m=1}^n M_{im}(x_{kj}^{J+1})\, w_m^{J+1} - N_i(x_{kj}^{J+1}))^2)\,dt$$

because (24) is just a function of α and for $\alpha \in [0; 1]$ the $(J + 1)$-approximation is varying from J to J^*-approximation.

We derive the change of energy consumption in the neighbourhood of the J-approximation. According to [9] we derive from (11), (13), (14)

$$I = \int_0^T (\sum_{i=1}^n (\psi_{1i}\dot{x}_{1i} + \psi_{2i}\dot{x}_{2i}) - \mathscr{H})\,dt = \tag{25}$$

$$= \sum_{i=1}^n (\psi_{1i}x_{1i} + \psi_{2i}x_{2i})\Big|_{t=0}^{t=T} - \int_0^T (\sum_{i=1}^n (\dot{\psi}_{1i}x_{1i} + \dot{\psi}_{2i}x_{2i}) + \mathscr{H})\,dt$$

The variation (25) along the approximation is then given from (13), (17) by

$$\delta I = \sum_{i=1}^n (\psi_{1i}\delta x_{1i} + \psi_{2i}\delta x_{2i})\Big|_{t=0}^{t=T} - \int_0^T \left(\sum_{i=1}^n \left(\dot{\psi}_{1i} + \frac{\partial \mathscr{H}}{\partial x_{1i}}\right)\delta x_{1i} + \right. \tag{26}$$

$$\left. + \left(\dot{\psi}_{2i} + \frac{\partial \mathscr{H}}{\partial x_{2i}}\right)\delta x_{2i} + \sum_{l=1}^n \frac{\partial \mathscr{H}}{\partial w_l}\delta w_l\right) dt = - \int_0^T \left(\sum_{l=1}^n \frac{\partial \mathscr{H}}{\partial w_l}\delta w_l\right) dt$$

The constructed variation along the J-approximation is from (23)

$$\begin{aligned} \delta x_{1i} &= x_{1i}^{J+1} - x_{1i}^J = \alpha(x_{1i}^{J*} - x_{1i}^J) \\ \delta x_{2i} &= x_{2i}^{J+1} - x_{2i}^J = \alpha(x_{2i}^{J*} - x_{2i}^J) \\ \delta w_l &= w_l^{J+1} - w_l^J = \alpha(w_l^{J*} - w_l^J) \end{aligned} \tag{27}$$

By substituting from (16), (27) to (26) we obtain

(28)

$$\delta I = - \int_0^T \sum_{l=1}^n (- \sum_{i=1}^n 2e_i(\sum_{m=1}^n Mi_m(x_{kj}^J)\, w_m^J - N_i(x_{kj}^J))\, M_{il}(x_{kj}^J) + \sum_{i=1}^n \psi_{2i}^J K_{il})\,\delta w_l\,dt$$

From (16) we derived w_l^{J*} from J-approximation. By substituting in (28) instead of ψ_{2i}^J

$$\delta I = - \int_0^T \sum_{l=1}^n (- \sum_{i=1}^n 2e_i(\sum_{m=1}^n M_{im}(x_{kj}^J)\, w_m^J - N_i(x_{kj}^J))\, M_{il}(x_{kj}^J) + \tag{29}$$

$$+ \sum_{i=1}^n (2e_i(\sum_{m=1}^n M_{im}(x_{kj}^J)\, M_{il}(x_{kj}^J)\, w_m^{J*}) -$$

$$- 2e_i N_i(x_{kj}^J)\, M_{il}(x_{kj}^J)))\,\delta w_l\,dt =$$

$$= -2 \int_0^T \alpha \sum_{i=1}^n e_i(\sum_{m=1}^n M_{im}(x_{kj}^J)\,(w_m^{J*} - w_m^J))^2\,dt$$

Equation (29) proves that if the linear theory is valid these approximations leads to the decrease of energy consumption. The validity of linear theory guarantees that the change of energy consumption $I^{J+1} - I^J$ is given by the first differential of optimum functional. Its validity is guaranteed by a small variation from J-approximation to $(J + 1)$-approximation which can be ensured by the choice of sufficiently small α in (24). On the basis of (29) we can prove the following theorem.

Theorem. The sequence of approximations (23) is either finite and then its last point satisfies the necessary conditions of Pontryagin's maximum principle, or is infinite and then its limit point satisfies the necessary conditions of Pontryagin's maximum principle.

Proof. According to Theorem 1.3.3 of [14] it is sufficient to prove that the performace index (13), (24) is a continuous functional of the functions w_m, x_{kj} and that for every w_m, x_{kj} which don't fulfil (16), there exist such $\varepsilon > 0$ and $\delta < 0$, that $I^{J+1}(w_m^{J+1}, x_{kj}^{J+1}) - I^J(w_m^J, x_{kj}^J) \leqq \delta < 0$ for all w_m^J, x_{kj}^J from the ε-neighbourhood of w_m, x_{kj}. The functional (13), (24) is obviously continuous. Let us suppose that the condition (16) is not satisfied for some w_m, x_{kj}. From w_m, x_{kj} we calculate w_m^{J*}, x_{kj}^{J*} which differ from w_m, x_{kj} because the condition (16) doesn't fulfil for it. As the functional is continuous, in the neighbourhood of w_m, x_{kj} there exist w_m^{J+1}, x_{kj}^{J+1} determined from (23) by the choice of such sufficiently small α that the change of the functional (13), (24) is given by its first differential. Then according to (29) it is $I^{J+1}(w_m^{J+1}, x_{kj}^{J+1}) - I^J(w_m, x_{kj}) = -D < 0$. As all operations by the calculation of a new approximation are continuous, there exists the ε-neighbourhood of w_m, x_{kj} for which it is valid $I^{J+1}(w_m^{J+1}, x_{kj}^{J+1}) - I^J(w_m^J, x_{kj}^J) \leqq -(D/2) = \delta < 0$. □

The above described approximations can be further modified for consideration of constraints (4), (12) and further state constraints. Let the J-approximation satisfy the constraints. If the whole J-approximation is on the boundary of constraints and the $J*$-approximation leads into the region where some constraint is not fulfilled, then the value of performance index cannot be decreased. On the contrary let the $(J + 1)$-approximation satisfy the constraints for some α_1 but not for α_2 and let both α decrease the performance index. We construct a new $(J + 1)_\circ$-approximation. Equation (29) is also valid for α as a time function and for every positive $\alpha(t)$ it leads to the decrease of the performance index.

Instead of (23) we set

$$
\begin{aligned}
(30) \qquad w_m^{(J+1)\circ}(t) &= (1 - \alpha(t))\, w_m^J(t) + \alpha(t)\, w_m^{J*}(t) \\
x_{2i}^{(J+1)\circ}(t) &= (1 - \alpha(t))\, x_{2i}^J(t) + \alpha(t)\, x_{2i}^{J*}(t) + \\
&\quad - \int_0^t \frac{\mathrm{d}\alpha(t)}{\mathrm{d}t} \left(x_{2i}^{J*}(t) - x_{2i}^J(t)\right) \mathrm{d}t \\
x_{1i}^{(J+1)\circ}(t) &= (1 - \alpha(t))\, x_{1i}^J(t) + \alpha(t)\, x_{1i}^{J*}(t) +
\end{aligned}
$$

$$- 2\int_0^t \frac{\mathrm{d}\alpha(t)}{\mathrm{d}t}\,(x_{1i}^{J*}(t) - x_{1i}^J(t))\,\mathrm{d}t + \int_0^t \int_0^t \frac{\mathrm{d}^2\alpha(\tau)}{\mathrm{d}\tau^2}\,(x_{1i}^{J*}(\tau) - x_{1i}^J(\tau))\,\mathrm{d}\tau\,\mathrm{d}t\,.$$

We substitute from (21), (11), (17) and for given $\alpha(t)$ we obtain a system of linear algebraic equations for $\psi_{ki}^J(0)$, which together with (21) determine $(J + 1)_\circ$-approximation by (30). By the choice of $\alpha(t)$ between $\alpha(t) = \alpha_1$ and $\alpha(t) = \alpha_2$ (e.g. in the form of a spline function) we easily find a $(J + 1)_\circ$-approximation which satisfies the constraints and decreases the performance index below $\alpha(t) = \alpha_1$. The other possibility is to set $w_m^{(J+1)\circ} = w_m^{Jb}$, where w_m^{Jb} is the control of the movement along the boundary of the region of constraints, on the time interval where the $(J + 1)$-approximation fails to satisfy the constraints. If we set

$$\alpha(t) = \frac{w_m^{Jb} - w_m^J}{w_m^{J*} - w_m^J} \tag{31}$$

then for $\alpha(t) > 0$ this construction decreases the performance index.

The described problem of positional energetically optimal control was solved for several industrial robots. An interesting illustrative example is the motion of the

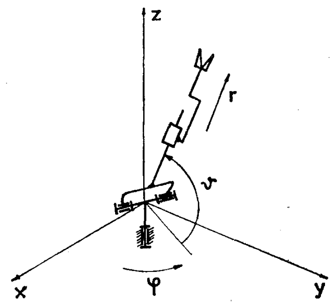

Fig. 1. The kinematic structure of an industrial robot.

robot with kinematic structure shown in Fig. 1 from the position $r = 0{\cdot}5$ m, $\varphi = 0$, $\vartheta = 1$ to the position $r = 2$ m, $\varphi = 3$, $\vartheta = -1$ in time $T = 2$ s. The dynamic properties are very simplified by considering a mass point, but nevertheless this model keeps significant nonlinear dynamic properties of motion. The equations of motion are

$$\begin{aligned}
\dot{x}_{11} &= x_{21}\\
\dot{x}_{21} &= \frac{K_1 u_1}{m} - g \sin x_{13} + x_{11} x_{23}^2 + x_{11} \cos^2 x_{13} x_{22}^2\\
\dot{x}_{12} &= x_{22}\\
\dot{x}_{22} &= \frac{K_2 u_2}{m x_{11}^2 \cos^2 x_{13}} - 2 x_{22} \left(\frac{x_{21}}{x_{11}} - x_{23} \operatorname{tg} x_{13}\right)
\end{aligned} \tag{32}$$

$$\dot{x}_{13} = x_{23}$$

$$\dot{x}_{23} = \frac{K_3 u_3}{m x_{11}^2} - \frac{2 x_{21} x_{23}}{x_{11}} - x_{22}^2 \sin x_{13} \cos x_{13} - \frac{g \cos x_{13}}{x_{11}}$$

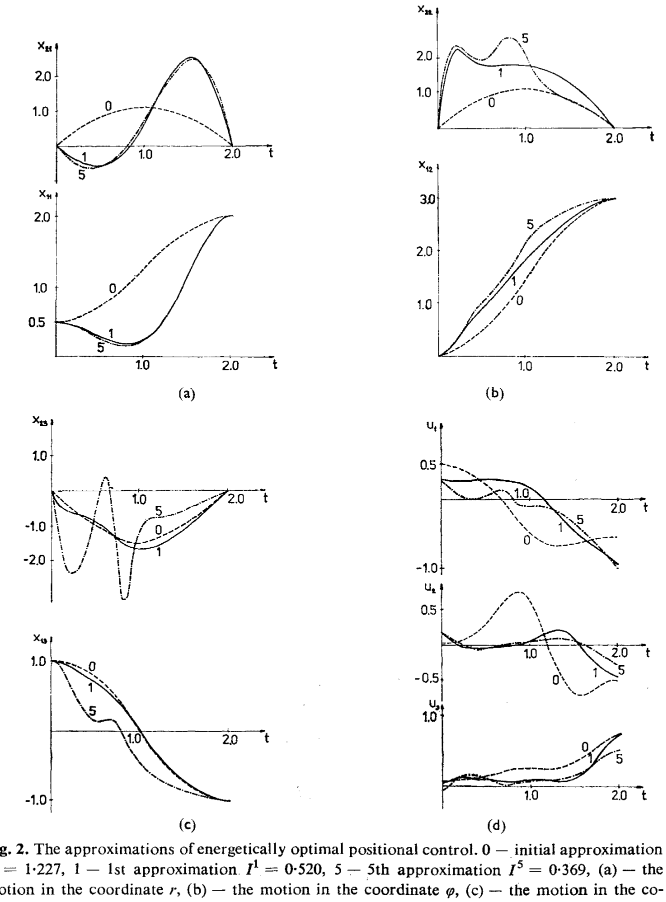

Fig. 2. The approximations of energetically optimal positional control. 0 — initial approximation $I^0 = 1{\cdot}227$, 1 — 1st approximation $I^1 = 0{\cdot}520$, 5 — 5th approximation $I^5 = 0{\cdot}369$, (a) — the motion in the coordinate r, (b) — the motion in the coordinate φ, (c) — the motion in the coordinate ϑ, (d) — the control variables.

where $x_{11} = r$, $x_{12} = \varphi$, $x_{13} = \vartheta$, u_i are control variables of servos, the mass $m = 1$ kg, the gains $K_1 = 20$, $K_2 = 10$, $K_3 = 30$, the gravity $g = 9{\cdot}80665\ \mathrm{ms}^{-2}$, the coefficients in (5), (13) $e_i = 1$. The results are in Fig. 2. The approximations were stopped when the decrease of energy consumption was less than 2% of the previous value of energy consumption.

4. PATH TRACKING ENERGETICALLY OPTIMAL CONTROL OF INDUSTRIAL ROBOTS

The industrial robot is required to follow the requested geometric path

$$\mathbf{R} = \mathbf{R}(p) \tag{33}$$

where generally $\mathbf{R} = [R_i, O_i]^{\mathrm{T}}$, $[R_i]$ is the radius vector of points on the path in cartesian coordinates, $[O_i]$ is the orientation of the robot gripper in cartesian coordinates and p is a geometric parameter of the path

$$0 \leqq p \leqq p_{\max} \,. \tag{34}$$

We distinguish the geometric trajectory (path) which is just the shape of geometric path in the space, and the trajectory, which is the complete time behaviour of movement on the path in the space including velocities and accelerations. The geometric trajectory variables (33) and the manipulator joint variables $\mathbf{s} = (s_1, \ldots, s_n)^{\mathrm{T}}$ are related by the kinematic equations

$$\mathbf{R} = \mathbf{G}(\mathbf{s}) \,. \tag{35}$$

We suppose that there are no redundant degrees of freedom for the motion (33) (i.e. the number of conditions in (33) including orientation is equal to the number of degrees of freedom of robot) and therefore there exist an inverse kinematic solution

$$\mathbf{s} = \mathbf{G}^{-1}(\mathbf{R}) \,. \tag{36}$$

The general case of redundant degrees of freedom is solved in [12]. The equations (30), (33) can be, however, replaced by a trajectory plan

$$\mathbf{s} = \mathbf{s}(p) \,. \tag{37}$$

We will synthetise dynamically realizable movement along the requested geometric trajectory (33) as the time parametrization of the geometric parameter p

$$p = p(t) \,. \tag{38}$$

We designate

$$\frac{\mathrm{d}p}{\mathrm{d}t} = d_1 \,, \quad \frac{\mathrm{d}d_1}{\mathrm{d}t} = d_2 \,. \tag{39}$$

It follows from (36), (39)

$$\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{G}^{-1}(\mathbf{R})}{\mathrm{d}p} \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{G}^{-1}(\mathbf{R})}{\mathrm{d}p} d_1 \tag{40}$$

$$\frac{d^2\mathbf{s}}{dt^2} = \frac{d^2\mathbf{G}^{-1}(\mathbf{R})}{dp^2}\left(\frac{dp}{dt}\right)^2 + \frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}\frac{d^2p}{dt^2} = \frac{d^2\mathbf{G}^{-1}(\mathbf{R})}{dp^2}d_1^2 + \frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}d_2$$

We substitute (40) in (1)

$$\text{(41)} \qquad \mathbf{n}^- \leqq \mathbf{n}_h = \left(\mathbf{I}(\mathbf{s})\frac{d^2\mathbf{G}^{-1}(\mathbf{R})}{dp^2} + \left(\frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}\right)^{\mathrm{T}}\mathbf{C}(\mathbf{s})\frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}\right)d_1^2 +$$

$$+ (\mathbf{V}(\mathbf{s}) + \mathbf{F}(\mathbf{s}))\frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}d_1 + \left(\mathbf{I}(\mathbf{s})\frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}\right)d_2 + \mathbf{g}(\mathbf{s}) \leqq \mathbf{n}^+$$

where equation (36) is still valid. After designating

$$[A_i] = \mathbf{A} = \mathbf{I}(\mathbf{s})\frac{d^2\mathbf{G}^{-1}(\mathbf{R})}{dp^2} + \left(\frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}\right)^{\mathrm{T}}\mathbf{C}(\mathbf{s})\frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}$$

$$\text{(42)} \qquad [B_i] = \mathbf{B} = (\mathbf{V}(\mathbf{s}) + \mathbf{F}(\mathbf{s}))\frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}$$

$$[D_i] = \mathbf{D} = \mathbf{I}(\mathbf{s})\frac{d\mathbf{G}^{-1}(\mathbf{R})}{dp}$$

$$[E_i] = \mathbf{E} = \mathbf{g}(\mathbf{s})$$

we can write from (41)

$$\text{(43)} \qquad \mathbf{n}^- \leqq \mathbf{n}_h = \mathbf{A}d_1^2 + \mathbf{B}d_1 + \mathbf{D}d_2 + \mathbf{E} \leqq \mathbf{n}^+ \,.$$

According to (1) (2,) (5) we can express the performance index-energy consumption in the form

$$\text{(44)} \qquad I = \int_0^T \left(\sum_{i=1}^n e_i n_{hi}^2\right) dt = \int_0^T \left(\sum_{i=1}^n e_i(A_i d_1^2 + B_i d_1 + D_i d_2 + E_i)^2\right) dt \,.$$

Let us be given a time interval T for the movement (33), (34). It follows from (34)

$$\text{(45)} \qquad p(0) = 0\,, \quad p(T) = p_{\max}\,.$$

At the start and at the end of the movement we have a requested velocity and from (33) we calculate after the differentiation

$$\text{(46)} \qquad d_1(0) = d_{10}\,, \quad d_1(T) = d_{1f}\,.$$

Thus the problem of path tracking optimal control is transformed to the problem of positional optimal control of the system (39) with state variables p, d_1 and control variable d_2 for the movement between positions (45), (46) within the constraints (43) so as to minimize (44).

This problem will be solved by the method of suboptimal approximations. We assemble the hamiltonian

$$\text{(47)} \qquad \mathscr{H} = -\sum_{i=1}^n e_i(A_i d_1^2 + B_i d_1 + D_i d_2 + E_i)^2 + \psi_1 d_1 + \psi_2 d_2\,,$$

the corresponding conjugate system of differential equations

$$(48)\quad \dot{\psi}_1 = \sum_{i=1}^{n} 2e_i(A_i d_1^2 + B_i d_1 + D_i d_2 + E_i)\left(\frac{\partial A_i}{\partial p} d_1^2 + \frac{\partial B_i}{\partial p} d_1 + \frac{\partial D_i}{\partial p} d_2 + \frac{\partial E_i}{\partial p}\right)$$

$$\dot{\psi}_2 = -\psi_1 + \sum_{i=1}^{n} 2e_i(A_i d_1^2 + B_i d_1 + D_i d_2 + E_i)(2A_i d_1 + B_i)$$

and from the maximum principle without consideration of the constraints (43)

$$(49)\qquad d_2 = \frac{\psi_2 - \sum_{i=1}^{n} 2e_i(A_i d_1^2 + B_i d_1 + E_i) D_i}{2\sum_{i=1}^{n} e_i D_i^2}$$

From the J-approximation $p^J(t)$, $d_1^J(t)$, $d_2^J(t)$ we substitute in the right side of (48) and (49) and we obtain a system of linear differential equations with right side as a time function. By their integration, substituting in (49), (39), further integration and substituting (45), (46) we obtain a system of linear algebraic equations for $\psi_1^J(0)$, $\psi_2^J(0)$. From them we calculate $J*$-approximation $p^{J*}(t)$, $d_1^{J*}(t)$, $d_2^{J*}(t)$, which satisfies the equations (45), (46). Now we define

$$(50)\qquad \begin{aligned} p^{J+1}(t) &= (1-\alpha)\, p^J(t) + \alpha p^{J*}(t) \\ d_1^{J+1}(t) &= (1-\alpha)\, d_1^J(t) + \alpha d_1^{J*}(t) \\ d_2^{J+1}(t) &= (1-\alpha)\, d_2^J(t) + \alpha d_2^{J*}(t) \end{aligned}$$

and we determine the unknown $\alpha \in [0; 1]$ by one-dimensional minimization (44) with respect to α. We obtain the $(J+1)$-approximation. The linear part of the change of (44) is

$$(51)\qquad \delta I = -2\int_0^T \alpha \sum_{i=1}^{n} e_i D_i^2 (d_2^{J*} - d_2^J)^2\, dt$$

which proves the convergence of the method. The constraints (43) can be incorporated by varying α in time as in the previous chapter (eqs. (30), (31)).

The above described problem was solved for the movement of robot (32) between the same positions and at the same time as in the previous chapter, but along a straight line path. The initial admissible trajectory was

$$(52)\qquad \begin{aligned} x &= 0{\cdot}2701511 - 1{\cdot}3399415p \\ y &= 0 + 0{\cdot}1524949p \\ z &= 0{\cdot}4207354 - 2{\cdot}1036773p \\ p &= \tfrac{3}{4}(t^2 - \tfrac{1}{3}t^3) \\ t &\in [0; 2] \end{aligned}$$

The inverse kinematic solution (36) is

$$
\begin{aligned}
r &= \sqrt{(x^2 + y^2 + z^2)} \\
\varphi &= \operatorname{atan} 2(y, x) \\
\vartheta &= \operatorname{atan} 2(z, \sqrt{(x^2 + y^2)})
\end{aligned} \tag{53}
$$

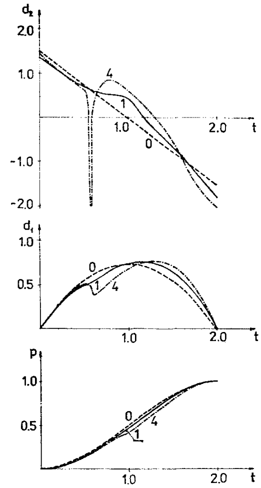

Fig. 3. The approximations of energetically optimal path tracking control. 0 — initial approximation $I^0 = 0{\cdot}439$, 1 — 1st approximation $I^1 = 0{\cdot}434$, 4 — 4th approximation $I^4 = 0{\cdot}418$.

The results are shown in Fig. 3. The approximations were stopped when the decrease of energy consumption was less than 1% of the previous value of energy consumption.

5. CONCLUSION

The developed method of suboptimal approximations is a very efficient method of the synthesis of manipulator's optimal trajectory, especially advantageous is the property of complete applicability of each approximation. There are solved both positional and path tracking optimal control problems. The obtained results enable us to construct the CAM programming system for the synthesis of dynamically admissible and energetically optimal trajectories of industrial robots.

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