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OPTIMAL CONTROL OF A CLASS OF THE DISCRETE-TIME DISTRIBUTED-PARAMETER SYSTEMS

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The optimal feedback control for discrete-time distributed-parameter systems is discussed. Sufficient conditions for stability of steady state are derived through Lyapunov's direct method for a class of discrete-time distributed-parameter systems. The technique, which uses Lyapunov function to optimization problem, is applied to a number of feedback optimal control problems of discrete-time distributed-parameter systems typical of those encountered in practice.

1. INTRODUCTION

The optimal control theory for the continuous-time distributed-parameter systems is relatively advanced. In the continuous-time distributed-parameter systems, a linear feedback law can be found for linear distributed-parameter systems with quadratic performance criterion. The optimal control theory of distributed-parameter systems has been developed for many cases with complete measurement and for many cases with incomplete measurement, too [1], [2], [3], [4]. Theoretical results can be used for a very large number of practical problems of control in real time control of tubular chemical reactors, control of absorption and distillation columns, etc. only with difficultes. Riccati-like equations for the optimal gain can be solved iteratively only (see e.g. [3]).

In this paper we present sufficient conditions for stability for discrete-time distributed-parameter systems through Lyapunov functional techniques. Other applications of model that is dicsrete in time and continuous in space are shown in [5], [6], [7] , [8]. We derive the optimal feedback control for discrete-time distributed-parameter systems. Equations for the feedback optimal gain that has been derived with the help of Lyapunov theory need not be solved iteratively. Theoretical results can be used for optimal control of tubular heat exchangers, tubular chemical reactors, etc. Illustrative calculations are given for a tubular plug flow heat exchanger.

2. SYSTEM EQUATIONS

Let us consider a linear distributed-parameter system which is described by the equation

$$\mathbf{x}(z, i + 1) = \mathbf{A}(z)\, \mathbf{x}(z, i) + \mathbf{A}_z(z) \frac{\partial \mathbf{x}(z, i)}{\partial z} + \mathbf{B}(z)\, \mathbf{w}(z, i) \tag{1}$$

where z is the dimensionless spatial coordinate, $0 \leqq z \leqq 1$, $i = 0, 1, 2, \ldots$, $t_i = iT$ is discrete instant of time, T is dimensionless sampling period, $\mathbf{x}(z, i) = [x_1(z, i), x_2(z, i), \ldots, x_n(z, i)]^{\mathrm{T}}$ is the state vector of n components, $\mathbf{w}(z, i) = [w_1(z, i), w_2(z, i), \ldots, w_h(z, i)]^T$ is the distributed control vector of h components, $h \leqq n$, $\mathbf{A}(z)$ is an $n \times n$ matrix, $\mathbf{A}_z(z)$ is an $n \times n$ matrix, $\mathbf{B}(z)$ is an $n \times h$ matrix.

Elements of $\mathbf{A}(z)$ and $\mathbf{B}(z)$ matrix are continuous. Elements of $\mathbf{A}_z(z)$ matrix are continuously differentiable on z.

The initial and boundary conditions are

$$\mathbf{x}(z, 0) = \mathbf{x}_0(z) \tag{2}$$

$$\mathbf{x}(0, i) = 0 \tag{3}$$

3. LYAPUNOV ANALYSIS

Consider the distributed parameter system

$$\mathbf{x}(z, i + 1) = \mathbf{A}(z)\, \mathbf{x}(z, i) + \mathbf{A}_z(z) \frac{\partial \mathbf{x}(z, i)}{\partial z} \tag{4}$$

Let the Lyapunov functional be given by the positive-definite

$$V = \tfrac{1}{2} \int_0^1 \int_0^1 \mathbf{x}^{\mathrm{T}}(z, i)\, \mathbf{N}(z, \xi)\, \mathbf{x}(\xi, i)\, \mathrm{d}z\, \mathrm{d}\xi \tag{5}$$

where $\mathbf{N}(z, \xi)$ is a positive-definite matrix and

$$\mathbf{N}(z, \xi) = \mathbf{N}^{\mathrm{T}}(\xi, z) \tag{6}$$

ξ is the dimensionless spatial coordinate, $0 \leqq \xi \leqq 1$.

We compute

$$\Delta V = \tfrac{1}{2} \int_0^1 \int_0^1 \mathbf{x}^{\mathrm{T}}(z, i + 1)\, \mathbf{N}(z, \xi)\, \mathbf{x}(\xi, i + 1)\, \mathrm{d}z\, \mathrm{d}\xi - - \tfrac{1}{2} \int_0^1 \int_0^1 \mathbf{x}^{\mathrm{T}}(z, i)\, \mathbf{N}(z, \xi)\, \mathbf{x}(\xi, i)\, \mathrm{d}z\, \mathrm{d}\xi \tag{7}$$

Using relation in Eqn. (4) and relations

$$(8)\qquad \tfrac{1}{2}\int_0^1\int_0^1 \mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)\,\frac{\partial \mathbf{x}(\xi,i)}{\partial \xi}\,\mathrm{d}z\,\mathrm{d}\xi =$$

$$= \tfrac{1}{2}\int_0^1 \mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}^{\mathrm{T}}(z)\left\{[\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)\,\mathbf{x}(\xi,i)]_{\xi=0}^{\xi=1} - \int_0^1 \frac{\partial[\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)]}{\partial \xi}\,\mathbf{x}(\xi,i)\right\}\mathrm{d}z$$

$$(9)\qquad \tfrac{1}{2}\int_0^1\int_0^1 \left(\frac{\partial \mathbf{x}(z,i)}{\partial z}\right)^{\mathrm{T}} \mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}(\xi)\,\mathbf{x}(\xi,i)\,\mathrm{d}z\,\mathrm{d}\xi =$$

$$= \tfrac{1}{2}\int_0^1 \left\{[\mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)]_{z=0}^{z=1} - \int_0^1 \mathbf{x}^{\mathrm{T}}(z,i)\,\frac{\partial[\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)]}{\partial z}\,\mathrm{d}z\right\}\mathbf{A}(\xi)\,\mathbf{x}(\xi,i)\,\mathrm{d}\xi$$

$$(10)\qquad \tfrac{1}{2}\int_0^1\int_0^1 \left(\frac{\partial \mathbf{x}(z,i)}{\partial z}\right)^{\mathrm{T}} \mathbf{A}_z(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)\,\frac{\partial \mathbf{x}(\xi,i)}{\partial \xi}\,\mathrm{d}z\,\mathrm{d}\xi =$$

$$= \tfrac{1}{2}\int_0^1 [\mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)]_{z=0}^{z=1}\,\mathbf{A}_z(\xi)\,\frac{\partial \mathbf{x}(\xi,i)}{\partial \xi}\,\mathrm{d}\xi -$$

$$- \tfrac{1}{2}\int_0^1 \mathbf{x}^{\mathrm{T}}(z,i)\left\{\frac{\partial[\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)]}{\partial z}\,\mathbf{x}(\xi,i)\right\}_{\xi=0}^{\xi=1}\mathrm{d}z +$$

$$+ \tfrac{1}{2}\int_0^1\int_0^1 \mathbf{x}^{\mathrm{T}}(z,i)\,\frac{\partial^2[\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)]}{\partial z\,\partial \xi}\,\mathbf{x}(\xi,i)\,\mathrm{d}\xi\,\mathrm{d}z$$

we obtain

$$(11)\qquad \Delta V = \tfrac{1}{2}\int_0^1\int_0^1 \left\{-\mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}^{\mathrm{T}}(z)\,\frac{\partial[\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)]}{\partial \xi}\,\mathbf{x}(\xi,t) -\right.$$

$$- \mathbf{x}^{\mathrm{T}}(z,i)\,\frac{\partial[\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)]}{\partial z}\,\mathbf{A}(\xi)\,\mathbf{x}(\xi,i) + \mathbf{x}^{\mathrm{T}}(z,i)\,\frac{\partial^2[\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)]}{\partial z\,\partial \xi}\,\mathbf{x}(\xi,i) -$$

$$\left. - \mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{N}(z,\xi)\,\mathbf{x}(\xi,i) + \mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}(\xi)\,\mathbf{x}(\xi,i)\right\}\mathrm{d}z\,\mathrm{d}\xi +$$

$$+ \tfrac{1}{2}\int_0^1 \mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}^{\mathrm{T}}(z)\,[\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)\,\mathbf{x}(\xi,i)]_{\xi=0}^{\xi=1}\,\mathrm{d}z +$$

$$+ \tfrac{1}{2}\int_0^1 [\mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)]_{z=0}^{z=1}\,\mathbf{A}(\xi)\,\mathbf{x}(\xi,i)\,\mathrm{d}\xi +$$

$$+ \tfrac{1}{2}\int_0^1 [\mathbf{x}^{\mathrm{T}}(z,i)\,\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)]_{z=0}^{z=1}\,\mathbf{A}_z(\xi)\,\frac{\partial \mathbf{x}(\xi,i)}{\partial \xi}\,\mathrm{d}\xi -$$

$$- \tfrac{1}{2}\int_0^1 \mathbf{x}^{\mathrm{T}}(z,i)\left\{\frac{\partial[\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)]}{\partial z}\,\mathbf{x}(\xi,i)\right\}_{\xi=0}^{\xi=1}\mathrm{d}z$$

or

$$\Delta V = -\tfrac{1}{2}\int_0^1\int_0^1 \mathbf{x}^{\mathrm{T}}(z, i)\, \boldsymbol{\mu}(z, \xi)\, \mathbf{x}(\xi, i)\, \mathrm{d}z\, \mathrm{d}\xi \tag{12}$$

since $\boldsymbol{\mu}(z, \xi)$, $0 \leqq z \leqq 1$, $0 \leqq \xi \leqq 1$ is an $n \times n$ positive-definite matrix. The matrix $\mathbf{N}(z, \xi)$ is the unique solution of

$$-\mathbf{A}^{\mathrm{T}}(z)\frac{\partial[\mathbf{N}(z, \xi)\,\mathbf{A}_z(\xi)]}{\partial \xi} - \frac{\partial[\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{A}(z, \xi)]}{\partial z}\,\mathbf{A}(\xi) + \frac{\partial^2[\mathbf{A}_z^{\mathrm{T}}(z)\,\mathbf{N}(z, \xi)\,\mathbf{A}_z(\xi)]}{\partial z\, \partial \xi} - \tag{13}$$

$$- \mathbf{N}(z, \xi) + \mathbf{A}^{\mathrm{T}}(z)\,\mathbf{N}(z, \xi)\,\mathbf{A}(\xi) = -\boldsymbol{\mu}(z, \xi)$$

$$\mathbf{N}(z, 1) = 0 \tag{14}$$

$$\mathbf{N}(1, \xi) = 0 \tag{15}$$

$$\boldsymbol{\mu}(z, \xi) = \boldsymbol{\mu}^{\mathrm{T}}(\xi, z) \tag{16}$$

Since $\Delta V < 0$ implies that the system is asymptotically stable, we can state the following.

Theorem 1. The distributed-parameter system, Eqn. (4) is asymptotically stable in-the-large at the origin if and only if, given any positive-definite matrix $\boldsymbol{\mu}(z, \xi) = \boldsymbol{\mu}^{\mathrm{T}}(\xi, z)$, $0 \leqq z \leqq 1$, $0 \leqq \xi \leqq 1$, there exists a positive-definite matrix $\mathbf{N}(z, \xi) = \mathbf{N}^{\mathrm{T}}(\xi, z)$, $0 \leqq z \leqq 1$, $0 \leqq \xi \leqq 1$, which is the unique solution of Eqn. (13) with boundary condition (14), (15).

The "only if" part (necessity) of the theorem is more difficult to establish, as the "if" part (sufficiency) of the theorem. A proof for the lumped system is given by Kalman and Bertram [9]. Since the distributed system in Eqn. (4) may be considered as the limit of a large approximating lumped system in which spatial derivatives are replaced by differences, it follows that an equivalent result must hold [10] for the "only if" part (necessity) of the theorem in the distributed system.

4. CONTROLLER DESIGN

Consider the linear system, Eqn. (1) and restrict attention to the case where the free system, Eqn. (4), is asymptotically stable. We arbitrarily choose a positive-definite matrix $\boldsymbol{\mu}(z, \xi) = \boldsymbol{\mu}^{\mathrm{T}}(\xi, z)$ and calculate $\mathbf{N}(z, \xi)$ from Eqns. (13), (14), (15). Let V be given by Eqn. (5).

Then

(17)

$$\Delta V = \tfrac{1}{2}\int_0^1\int_0^1 \left\{\mathbf{A}(z)\,\mathbf{x}(z, i) + \mathbf{A}_z(z)\frac{\partial \mathbf{x}(z, i)}{\partial z} + \mathbf{B}(z)\,\mathbf{w}(z, i)]^{\mathrm{T}}\, \mathbf{A}(z, \xi)\, [\mathbf{A}(\xi)\,\mathbf{x}(\xi, i) + \right.$$

$$\left. + \mathbf{A}_z(\xi)\frac{\partial \mathbf{x}(\xi, i)}{\partial \xi} + \mathbf{B}(\xi)\,\mathbf{w}(\xi, i) - \mathbf{x}^{\mathrm{T}}(z, i)\,\mathbf{N}(z, \xi)\,\mathbf{x}(\xi, i)\right\} \mathrm{d}z\, \mathrm{d}\xi$$

(18)

$$\Delta V = \int_0^1 \int_0^1 [\mathbf{w}^T(z,i)\,\mathbf{B}^T(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}(\xi)\,\mathbf{x}(\xi,i) + \mathbf{w}^T(z,i)\,\mathbf{B}^T(z)\,\mathbf{N}(z,\xi)\,\mathbf{A}_z(\xi)\frac{\partial \mathbf{x}(\xi,i)}{\partial \xi} +$$

$$+ \tfrac{1}{2}\mathbf{w}^T(z,i)\,\mathbf{B}^T(z)\,\mathbf{N}(z,\xi)\,\mathbf{B}(\xi)\,\mathbf{w}(\xi,i)]\,\mathrm{d}z\,\mathrm{d}\xi - \tfrac{1}{2}\int_0^1\int_0^1 \mathbf{x}^T(z,i)\,\boldsymbol{\mu}(z,\xi)\,\mathbf{x}(\xi,i)\,\mathrm{d}z\,\mathrm{d}\xi$$

If we choose $\mathbf{w}(z, i)$ to minimize $(-\Delta V)$, we are assumed that $(-\Delta V)$ will be at least $\frac{1}{2}\int_0^1\int_0^1 \mathbf{x}^T(z,i)\,\boldsymbol{\mu}(z,\xi)\,\mathbf{x}(\xi,i)\,\mathrm{d}z\,\mathrm{d}\xi$ (we may always choose $\mathbf{w}(z,i) = 0$) and hence, the system will be stable. Also maximizing $(-\Delta V)$ corresponds to a local maximization of the rate of return to the origin. Optimal control can be directly obtained from $\partial(\Delta V)/\partial \mathbf{w}(z,i) = 0$ which has the solution

$$\text{(19)}\quad \mathbf{w}(\xi,i) = -[\mathbf{N}^B(\xi)\,\mathbf{B}(\xi)]^{-1}\left[\mathbf{N}^B(\xi)\,\mathbf{A}(\xi)\,\mathbf{x}(\xi,i) + \mathbf{N}^B(\xi)\,\mathbf{A}_z(\xi)\frac{\partial \mathbf{x}(\xi,i)}{\partial \xi}\right]$$

where

$$\text{(20)}\qquad \mathbf{N}^B(\xi) = \int_0^1 \mathbf{B}^T(z)\,\mathbf{N}(z,\xi)\,\mathrm{d}z$$

As long as there are no constraints on the components of the control vector $\mathbf{w}$. Eqn. (19) gives the correct choice of control, provided the matrix $\mathbf{B}$ satisfies the property that $\mathbf{Bw} = 0$ which implies that $\mathbf{w} = 0$. In addition, this condition will guarantee that, in case of constraints, $\mathbf{w}(z, i)$ should be set as close to the value given in Eqn. (19) as allowed by the constraints.

Each choice of $\boldsymbol{\mu}(z, \xi)$ results in a different $\mathbf{N}(z, \xi)$ and, hence, a different control. However, in the special case where the number, h, of control variables in $\mathbf{w}$ is the same as the number, n, of state variables in $\mathbf{x}$, the matrix $\mathbf{B}$ has the dimension $x \times n$. Provided $\mathbf{B}$ nonsingular, Eqn. (19) reduces to

$$\text{(21)}\qquad \mathbf{w}(\xi,i) = -[\mathbf{B}(\xi)]^{-1}\left[\mathbf{A}(\xi)\,\mathbf{x}(\xi,i) + \mathbf{A}_z(\xi)\frac{\partial \mathbf{x}(\xi,i)}{\partial \xi}\right]$$

and Eqn. (1) reduces to

$$\text{(22)}\qquad \mathbf{x}(z,i) = 0\,, \quad i > 0$$

for all $\mathbf{x}(z, 0)$. Therefore, when there are as many control variables as state variables, and when constraints on the control do not preclude Eqn. (21), the state of the system can be returned to the origin in one step, and the arbitrary choice of $\boldsymbol{\mu}(z, \xi)$ does not affect our ability to reach this result. This is a satisfying aspect of the method.

The space-independent control is

(23)

$$\mathbf{u}(i) = -\left[\int_0^1 \mathbf{N}^B(\xi)\,\mathbf{B}(\xi)\,\mathrm{d}\xi\right]^{-1}\left[\int_0^1 \mathbf{N}^B(\xi)\,\mathbf{A}(\xi)\,\mathbf{x}(\xi,i)\,\mathrm{d}\xi + \int_0^1 \mathbf{N}^B(\xi)\,\mathbf{A}_z(\xi)\frac{\partial \mathbf{x}(\xi,i)}{\partial \xi}\mathrm{d}\xi\right]$$

5. SCALAR EXAMPLE

As an example of the control theory which we have developed, we now apply the theory to obtain optimal control of the tubular plug flow heat exchanger by the manipulation of the wall temperature. The control is to drive the exchanger from an initial undersired steady state to a new steady maximizing $(-\Delta V)$. The dynamics of the heat exchanger can be represented by the following equation [3], [4]

(24) $$x_1(z, i + 1) = A_{11}\, x_1(z, i) + A_{z11} \frac{\partial x_1(z, i)}{\partial z} + B_{11}\, w_1(z, i)$$

where

z is the dimensionless spatial coordinate, $0 \leqq z \leqq 1$,
$t = iT$ is the dimensionless time-like variable, $0 \leqq t < \infty$,
$i = 0, 1, 2, \ldots,$
T is the dimensionless sampling period,
$x_1(z, i)$ is the dimensionless state variable,
$w_1(z, i)$ is the dimensionless control variable,
$A_{11} = 1 - TP$,
$A_{z11} = -T$,
$B_{11} = TP$,
P is the ratio of the heat exchanger to heat capacity,
$x_1(z, i) = x_1(z, iT)$,
$w_1(z, i) = w_1(z, iT)$.

The boundary condition is

(25) $$x_1(0, t) = 0$$

The initial condition is

(26) $$x_1(z, 0) = 1 - e^{-Pz}$$

The system is completely controllable [4].

Lyapunov functional is given by

(27) $$V = \tfrac{1}{2} \int_0^1 \int_0^1 x_1(z, i)\, N_{11}(z, \xi)\, x_1(\xi, i)\, dz\, d\xi$$

Using Eqn. (21) the optimal feedback distributed control is

(28) $$w_1(\xi, i) = -\frac{1}{B_{11}} \left[A_{11}\, x_1(\xi, i) + A_{z11} \frac{\partial x_1(\xi, i)}{\partial \xi} \right]$$

Using Eqn. (23) the optimal feedback space-independent control is

(29)
$$u_1(i) = -\frac{1}{\int_0^1 N_{11}^B(\xi)\, B_{11}\, d\xi} \left[\int_0^1 N_{11}^B(\xi)\, A_{11}\, x_1(\xi, i)\, d\xi + \int_0^1 N_{11}^B(\xi)\, A_{z11} \frac{\partial x_1(\xi, i)}{\partial \xi}\, d\xi \right]$$

where

$$N_{11}^{B}(\xi) = \int_{0}^{1} B_{11}\, N_{11}(z, \xi)\, \mathrm{d}z \tag{30}$$

$N_{11}(z, \xi)$ will satisfy the equation

$$-A_{11} \frac{\partial N_{11}(z, \xi)}{\partial \xi} A_{z11} - A_{z11} \frac{\partial N_{11}(z, \xi)}{\partial z} A_{11} + A_{z11} \frac{\partial^2 N_{11}(z, \xi)}{\partial z\, \partial \xi} A_{z11} - -N_{11}(z, \xi) + A_{11} N_{11}(z, \xi) A_{11} = -\mu_{11}(z, \xi) \tag{31}$$

with boundary conditions

$$N_{11}(z, 1) = 0 \tag{32}$$

$$N_{11}(1, \xi) = 0 \tag{33}$$

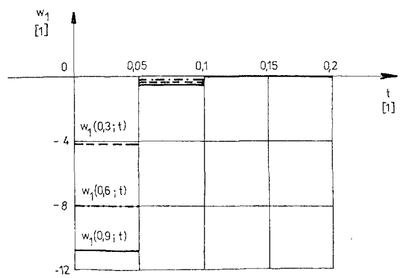

Fig. 1. Control variable profiles.

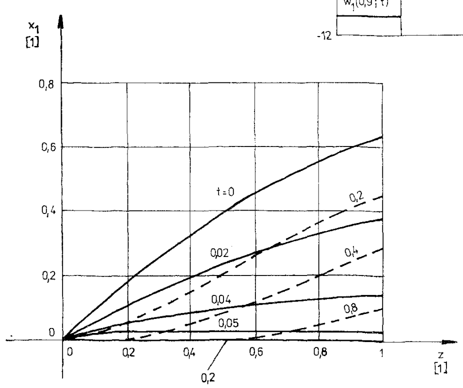

Fig. 2. State variable profiles.

Computational results for distributed control and for $P = 1$, $T = 0{\cdot}05$ are shown in Figures 1 and 2. The dashed profiles in Figure 2 are for $w_1 = 0$.

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