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The Quadratic Criterion of Control Process Quality Using the Exponential Weighting Function

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The paper deals with the quadratic criterion of control process quality, which is defined as the minimum value of the sum of the squares of error the discrete values of which are multiplied by the exponential weighting function. The paper follows with a preceding one [2]. The expressions are derived for coefficients important for determination of squares of discrete values.

The quadratic criterion using linear weighting function is introduced in the paper [2]. This criterion gives very good results with respect to classical quadratic criterion, but it has one disadvantage: increases the degree of determinant. Therefore the quadratic criterion was improved by using of exponential weighting function. The criterion, described in this paper, gives two advantages with respect to the classical one described in [2]: it decreares the overshut, and not increases the degree of determinant.

In this paper the notations introduced in the book by Prof. Strejc [1] and in paper [2] are used.

Presupposing the DL-transform of an error in the form of a rational function

$$E^*(q) = \frac{\sum_{i=0}^{l} b_i \, e^{qi}}{\sum_{i=0}^{l} a_i \, e^{qi}} . \tag{1}$$

The exponential function

$$f(n) = e^{\lambda n} \tag{2}$$

will be used as weighting function, where λ is a constant value.

The criterion is defined as

(3)
$$I_\lambda = \sum_{n=0}^{\infty} [e^{\lambda n} e(n)]^2 .$$

Using a new notation for the expression in square brackets

(4)
$$g_\lambda(n) = e^{\lambda n} e(n) .$$

According to the shifting theorem [1] the following expression

(5)
$$E^*(q - \lambda) = \sum_{n=0}^{\infty} e^{-(q-\lambda)n} e(n)$$

is valid. This one can be written in other form

(6)
$$E^*(q - \lambda) = \sum_{n=0}^{\infty} e^{-qn} e^{\lambda n} e(n) ,$$

or with respect to (4)

(7)
$$E^* (q - \lambda) = \sum_{n=0}^{\infty} e^{-qn} g_\lambda(n) .$$

According to basic definition of DL-transform, the expression (7) is DL-transform of (4). DL-transform of the function $g_\lambda(n)$ is determined by:

(8)
$$G_\lambda^*(q) = E^*(q - \lambda) .$$

Performing the shifting in DL-transform of error (1) we get

(9)
$$E^*(q - \lambda) = \frac{\sum_{i=0}^{l} b_i e^{(q-\lambda)i}}{\sum_{i=0}^{l} a_i e^{(q-\lambda)i}} = G_\lambda^*(q) .$$

It is necessary to arrange the numerator and denominator of (9) by dividing by powers of e

(10)
$$G_\lambda^*(q) = \frac{\sum_{i=0}^{l} b_i e^{-\lambda i} e^{qi}}{\sum_{i=0}^{l} a_i e^{-\lambda i} e^{qi}} .$$

By means of simple substitution

(11)
$$b_i\,e^{-\lambda i} = {}^\lambda b_i\,,$$
$$a_i\,e^{-\lambda i} = {}^\lambda a_i\,,$$

it is possible to simplify the expression (10) to the form

(12)
$$G_\lambda^*(q) = \frac{\sum_{i=0}^{l} {}^\lambda b_i\,e^{qi}}{\sum_{i=0}^{l} {}^\lambda a_i\,e^{qi}}\,.$$

The determinants can be compilled from the coefficients of (12)

(13)
$${}^\lambda\Delta_a = \begin{bmatrix} {}^\lambda a_0 & {}^\lambda a_1 & {}^\lambda a_2 & \dots & {}^\lambda a_{l-2} & {}^\lambda a_{l-1} & {}^\lambda a_l \\ {}^\lambda a_1 & {}^\lambda a_2 + {}^\lambda a_0 & {}^\lambda a_3 & \dots & {}^\lambda a_{l-1} & {}^\lambda a_l & 0 \\ {}^\lambda a_2 & {}^\lambda a_3 + {}^\lambda a_1 & {}^\lambda a_4 + {}^\lambda a_0 & \dots & {}^\lambda a_l & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ {}^\lambda a_l & {}^\lambda a_{l-1} & {}^\lambda a_{l-2} & \dots & {}^\lambda a_2 & {}^\lambda a_1 & {}^\lambda a_0 \end{bmatrix},$$

(14)
$${}^\lambda\Delta_b = \begin{bmatrix} {}^\lambda \gamma_0 & {}^\lambda a_1 & {}^\lambda a_2 & \dots & {}^\lambda a_{l-2} & {}^\lambda a_{l-1} & {}^\lambda a_l \\ {}^\lambda \gamma_1 & {}^\lambda a_2 + {}^\lambda a_0 & {}^\lambda a_3 & \dots & {}^\lambda a_{l-1} & {}^\lambda a_l & 0 \\ {}^\lambda \gamma_2 & {}^\lambda a_3 + {}^\lambda a_1 & {}^\lambda a_4 + {}^\lambda a_0 & \dots & {}^\lambda a_l & 0 & 0 \\ \vdots & \vdots & \vdots & & & & \\ {}^\lambda \gamma_l & {}^\lambda a_{l-1} & {}^\lambda a_{l-2} & \dots & {}^\lambda a_2 & {}^\lambda a_1 & {}^\lambda a_0 \end{bmatrix}.$$

The first column terms of determinant (14) are determined by

(15)
$${}^\lambda\gamma_h = \frac{1}{h!}\lim_{q\to-\infty}\frac{d^h}{d(e^q)^h}\,{}^\lambda\beta(q)\,,$$

where

(16)
$${}^\lambda\beta(q) = \frac{\sum_{i=0}^{l} {}^\lambda b_i\,e^{qi}\sum_{i=0}^{l} {}^\lambda b_i\,e^{q(l-i)}}{\sum_{i=0}^{l} {}^\lambda a_i\,e^{q(l-i)}}\,.$$

The ratio of determinants (13) and (14) gives the final expression for the sum of the squares of discrete values multiplied by the exponential weighting function

(17)
$$I_\lambda = \frac{{}^\lambda\Delta_b}{{}^\lambda\Delta_a}\,.$$

The further process of calculation is similar to the process introduced in paper [2]. It is perceptible from expressions (13) and (14) that by introducing of exponential weighting function, the order of determinant not increases. It is very important advantage of described criterion with respect to the criterion, using linear weighting function.

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