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ON TOTAL MATCHING NUMBERS AND TOTAL COVERING NUMBERS FOR k-UNIFORM HYPERGRAPHS

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In [3] P. Erdős and A. Meir investigate upper and lower bounds for $\alpha_2(G)+\alpha_2(\bar{G})$ and $\beta_2(G)+\beta_2(\bar{G})$, where G is an undirected graph without loops and multiple edges and $\bar{G}$ is the complement of G. $\alpha_2(G)$ or $\alpha_2(\bar{G})$ is the total covering number of G or $\bar{G}$ respectively and $\beta_2(G)$ or $\beta_2(\bar{G})$ is the total matching number of G or $\bar{G}$ respectively. In this paper these results are generalized for k-uniform hypergraphs. First let us introduce the necessary notions.

(Cf. Berge [1].) By a hypergraph H we mean a couple $\langle X, \mathscr{E}\rangle$, where X is a finite set of elements called vertices and $\mathscr{E}=\{E_1, \dots, E_m\}$ is a finite system of non-empty subsets of X called edges, where $E_i \neq E_j$ for $i, j \in \{1, \dots, m\}$, $i \neq j$.

A hypergraph is said to be k-uniform, $k>1$, if all its edges have cardinality k. A k-uniform hypergraph with $n \geq k$ vertices is called complete if its set of edges has the cardinality $\binom{n}{k}$.

The complement of a k-uniform hypergraph $H=\langle X, \mathscr{E}\rangle$ is the hypergraph $\bar{H}=\langle X, \bar{\mathscr{E}}\rangle$ if $|\mathscr{E}\cup\bar{\mathscr{E}}|=\binom{n}{k}$ and $\mathscr{E}\cap\bar{\mathscr{E}}=\emptyset$. ($|\mathscr{E}\cup\bar{\mathscr{E}}|$ denotes the cardinality of the set $\mathscr{E}\cup\bar{\mathscr{E}}$.)

A hypergraph $H\langle N\rangle=\langle X, \mathscr{E}_N\rangle$ is said to be a k-uniform subhypergraph of a k-uniform hypergraph $H=\langle X, \mathscr{E}\rangle$ induced by a set N if $N\subseteq X$ and $\mathscr{E}_N$ is the system of all edges $E_i \in \mathscr{E}$ such that $E_i \subseteq N$.

A vertex x of a k-uniform hypergraph H is said to cover itself, all edges incident with x and all vertices adjacent to x. An edge E_i of a k-uniform hypergraph H covers itself, the vertices incident with E_i and all edges adjacent to E_i.

A subset P of elements of $X\cup\mathscr{E}$ is called a total covering of $H=\langle X, \mathscr{E}\rangle$ if the elements of P cover H and P is a minimal set with this property.

Two elements of the set $X\cup\mathscr{E}$ are called strongly independent if they do not cover each other. A subset F of $X\cup\mathscr{E}$ is called a strong total matching if elements of F are pairwise strongly independent and F is maximal.

A subset N of X is called stable if for each edge $E_i \in \mathscr{E}$, $|E_i \cap N| \leqslant k-1$. A subset S of X is called strongly stable if for each edge E_i, $|E_i \cap S| \leqslant 1$.

A subset T of $X \cup \mathscr{E}$ is said to be a weak total matching if T is maximal and has the following properties:

1° The elements of $T \cap \mathscr{E}$ are pairwise independent (disjoint)

2° No element of $T \cap \mathscr{E}$ covers an element of $T \cap X$

3° The elements of $T \cap X$ form a stable set of H.

The cardinality of a minimum set which is a total covering of H is called the total covering number $\alpha_2(H)$ of H.

The cardinality of a maximum strong total matching of H is called the strong total matching number $\beta_2(H)$ of H.

The cardinality of a maximum weak total matching of H is called the weak total matching number $\gamma_2(H)$ of H.

The cardinality of a maximum stable set of H is called the stability number $\alpha(H)$ of H.

The cardinality of a maximum strong stable set of H is called the strong stability number $\alpha_0(H)$ of H.

In the sequel we supose that $n \geqslant k \geqslant 3$.

Theorem 1. *For a k-uniform hypergraph $H = \langle X, \mathscr{E} \rangle$ with n vertices and its complement $\bar{H}$*

$$\left]\frac{n}{k}\right[+ 2 \leqslant \beta_2(H) + \beta_2(\bar{H}) \leqslant \left]\frac{(k+1)n}{k}\right[\tag{1}$$

holds.

Proof. Let F or $\bar{F}$ be a strong total matching of H or $\bar{H}$ with cardinality $\beta_2(H)$ or $\beta_2(\bar{H})$ respectively. Let $F = F_x \cup F_y$ and $\bar{F} = \bar{F}_x \cup \bar{F}_y$, where F_x or $\bar{F}_x$ is a set of vertices of F or $\bar{F}$ respectively and F_y or $\bar{F}_y$ is a set edges of F or $\bar{F}$ respectively. Thus $\beta_2(H) = |F_x| + |F_y|$ and $\beta_2(\bar{H}) = |\bar{F}_x| + |\bar{F}_y|$ holds.

Let $V(F_y)$ or $V(\bar{F}_y)$ be the set of vertices incident with edges of F_y or $\bar{F}_y$ respectively. Without loss of generality we can suppose that the sets F_y or $\bar{F}_y$ are maximal independent sets of H or $\bar{H}$ respectively, so that the subhypergraphs $H\langle X - V(F_y)\rangle$ and $\bar{H}\langle X - V(\bar{F}_y)\rangle$ have no edges.

A. We prove the upper bound from Theorem 1.

$$\beta_2(H) = |F_x| + |F_y|$$

holds, thus

$$\beta_2(\bar{H}) \leqslant \left]\frac{|X - V(F_y)|}{k}\right[+ k|F_y|.$$

Then

$$\beta_2(H) + \beta_2(\bar{H}) \leqslant |F_x| + |F_y| + \left]\frac{n - k|F_y|}{k}\right[+ k|F_y|.$$

Since

$$|F_x| + k|F_y| \leqslant n,$$

$$\beta_2(H) + \beta_2(\bar{H}) \leqslant n + \left]\frac{n}{k}\right[= \left]\frac{(k+1)n}{k}\right[$$

holds.

B. We prove the lower bound from Theorem 1.

For $k \leqslant n \leqslant 2k$ the theorem holds.

Let $n > 2k$.

Since $H\langle X - V(F_y)\rangle$ or $\bar{H}\langle X - V(\bar{F}_y)\rangle$ are empty subhypergraphs of H or $\bar{H}$ respectively,

$$|F_y| + |\bar{F}_y| \geqslant \left[\frac{n}{k}\right] \tag{2}$$

holds.

Let us analyse five possibilities:

I. If $|F_y| = \left[\frac{n}{k}\right]$ and $n \equiv 0 \pmod{k}$, then $\beta_2(\bar{H}) \geqslant 2$, thus the assertion of the theorem holds.

II. If $|F_y| = \left[\frac{n}{k}\right]$ and $n \not\equiv 0 \pmod{k}$, then $|F_x| \geqslant 1$ and $\beta_2(\bar{H}) \geqslant 2$, thus the assertion of the theorem holds.

III. If $0 < |F_y| < \left[\frac{n}{k}\right]$ and $n \equiv 0 (\mathrm{mod}\ k)$, then $|F_x| \geqslant 1$ and $|\bar{F}_x| + |\bar{F}_y| \geqslant \left[\frac{n}{k}\right] - |F_y| + 1$, thus the assertion of the theorem holds.

IV. If $0 < |F_y| < \left[\frac{n}{k}\right]$ and $n \not\equiv 0 \pmod{k}$ and $|F_y| + |\bar{F}_y| > \left[\frac{n}{k}\right]$, then $|F_x| \geqslant 1$, $|\bar{F}_x| \geqslant 1$, thus the assertion of the theorem holds.

V. Let $0 < |F_y| < \left[\frac{n}{k}\right]$ and $n \not\equiv 0 \pmod{k}$ and

$$|F_y| + |\bar{F}_y| = \left[\frac{n}{k}\right], \tag{3}$$

then

$$|F_x| + |\bar{F}_x| > 2.$$

Suppose in fact the assertion does not hold.
Then

$$|F_x| + |\bar{F}_x| = 2, \quad (\text{i.e. } |F_x| = 1, \ |\bar{F}_x| = 1). \tag{4}$$

We will show that the hypergraph satisfying both the hypotheses of V and (4) does not exist. We can suppose that the sets $V(F_y)$ and $V(\bar{F}_y)$ are disjoint, because $H\langle X-V(F_y)\rangle$ has no edges, hence as a maximal set of disjoint edges of $\bar{H}\langle X-V(F_y)\rangle$ we can consider $\bar{F}_y$.

Let $N=X-V(F_y)-V(\bar{F}_y)$.

The hypergraph satisfying both the hypotheses of V and (4) must have the following properties:

(a) $0<|N|\leq k-1$, because $|V(F_y)|+|V(\bar{F}_y)|=k\left[\frac{n}{k}\right]$ and $|N|=|X-V(F_y)-V(\bar{F}_y)|=n-k\left[\frac{n}{k}\right]$.

(b) $H\langle V(F_y)\cup N\rangle$ or $\bar{H}\langle V(\bar{F}_y)\cup N\rangle$ is a complete subhypergraph of H or $\bar{H}$ respectively.
If $H\langle V(F_y)\cup N\rangle$ is not complete, then in $\bar{H}\langle V(F_y)\cup N\rangle$ there exists at least one edge, which is a contradiction to (3).

(c) Each vertex of X covers all vertices of both H and $\bar{H}$. Let there exist vertices x_1, x_2, which are not incident in H. From (b) it follows that in the set N all vertices are incident, i.e.

(i) $x_1\in V(F_y)$ and $x_2\in V(\bar{F}_y)$, or

(ii) $x_1, x_2\in V(\bar{F}_y)$

(in the case $x_1\in N$ and $x_2\in V(\bar{F}_y)$ there would be a contradiction to $|F_x|=1$).
In case (i) all edges containing the vertices x_1, x_2 are in $\bar{H}$. Let us take such an edge E from $\bar{H}$, which has $(k-1)$ vertices in the set $V(F_y)$. From (b) it follows that in $\bar{H}\langle V(\bar{F}_y)\cup N-\{x_2\}\rangle$ there exists an independent set of edges $\bar{F}_{1y}$, for which $|\bar{F}_{1y}|=|\bar{F}_y|$. But in $\bar{H}$ we can add an edge E to $\bar{F}_{1y}$ and obtain an independent set $\bar{F}_{2y}$ whose cardinality is $|\bar{F}_{2y}|=|\bar{F}_y|+1$. Then $|F_y|+|\bar{F}_{2y}|>\left[\frac{n}{k}\right]$, which is a contradiction to (3). In case (ii) we take $F_x=\{x_1, x_2\}$, which is a contradiction to $|F_x|=1$.

(d) Each vertex of $V(F_y)\cup N$ forms an edge with arbitrary $(k-1)$ vertices of $V(\bar{F}_y)$ in $\bar{H}$. Otherwise there exist $(k-1)$ vertices $x_2, \ldots, x_k$ in $V(\bar{F}_y)$ and $x_0\in V(F_y)\cup N$, that $\{x_0, x_2, \ldots, x_k\}$ forms an edge in H. But in $H\langle V(F_y)\cup N-\{x_0\}\rangle$ there exists an independent set of edges of cardinality $|F_y|$ and thus in H there exist a set of disjoint edges of cardinality $|F_y|+1$, which is a contradiction to (3).

(e) Each vertex of $V(\bar{F}_y)\cup N$ forms an edge with arbitrary $(k-1)$ vertices of $V(F_y)$ in the hypergraph H, which follows from an analogous consideration to that in (d).

For $k=3$, (c), (d), (e) and (3) can not hold the same time, thus for a 3-uniform hypergraph satisfying the condition from V the Theorem 1 holds.

Let $k\geq 4$. By induction we will prove an assertion (A):

(A) In a hypergraph H which satisfies (3) and (4), there does not exist an edge which has exactly i vertices in $V(F_y)$, for $i = 2, 3, \ldots, k-1$. This will be a contradiction to (e), because according to (e) each edge exactly $(k-1)$ of whose vertices are in $V(F_y)$ must belong to H.

Proof of (A):

1. Let $i = 2$. Let there exist an edge E_1 in H such that $|E_1 \cap V(F_y)| = 2$. According to (d), in $\bar{H}$ there exists an edge E_2 such that $|E_1 \cap E_2 \cap V(F_y)| = 1$ and $|E_1 \cap E_2| = k-1$. Let us consider a set of vertices $R \subseteq V(F_y) \cup V(\bar{F}_y)$ such that $|R \cap V(F_y)| = k-2$, $|R \cap V(\bar{F}_y)| = 2$, $R \cap E_1 = \emptyset$ and $|R \cap E_2| = 1$. Subhypergraph $H\langle R \cup N \rangle$ does not contain any edge (otherwise in $H\langle R \cup V(F_y) \cup E_1 \rangle$ there exists an independent set of edges of cardinality $|F_y| + 1$ which is a contradiction with (3)), and so $\bar{H}\langle R \cup N \rangle$ is a complete subhypergraph of $\bar{H}$. But in this case $H\langle R \cup N \cup V(F_y) \cup E_2 \rangle$ contains an independent set of edges of cardinality at least $|F_y| + 1$, which is a contradiction to (3). Let $v \in N$. Then the set of vertices $E_3 = (R - E_2) \cup \{v\}$ forms an edge in $\bar{H}$ and $E_3 \cap E_2 = \emptyset$, which is a contradiction to (3), thus for $i = 2$ the assertion (A) holds.

2. Suppose that for $i = r$, $2 < r \leqslant k-2$, the assertion (A) holds and for $i = r+1$ it does not hold, then in H there exists an edge E_1 such that $|E_1 \cap V(F_y)| = r+1$. According to the induction assumption there exists in $\bar{H}$ an edge E_2 such that $|E_1 \cap E_2| = k-1$ and $|E_1 \cap E_2 \cap V(F_y)| = r$. Let us consider a set of vertices $R \subseteq V(F_y) \cup V(\bar{F}_y)$ for vhich $|R \cap V(F_y)| = k-r$, $|R \cap V(\bar{F}_y)| = r$, $R \cap E_1 = \emptyset$ and $|R \cap E_2| = 1$. Then $\bar{H}\langle R \cup N \rangle$ is a complete subhypergraph of $\bar{H}$, otherwise we have a contradiction to (3). But in this case $H\langle R \cup N \cup V(F_y) \cup E_2 \rangle$ contains an independent set of edges of cardinality at least $|F_y| + 1$, which is a contradiction to (3). Thus the auxiliary assertion is proved.

From (A) it follows for $i = k-1$ that in H there does not exist any edge E for which $|E \cap V(F_y)| = k-1$, which is a contradiction to (e). This completes the proof of the assertion for case V and therefore also of Theorem 1.

Remark. The equality in the upper bound (1) holds for an arbitrary complete k-uniform hypergraph.

The equality in the lower bound (1) holds, e.g., for $H = \langle X, \mathscr{E} \rangle$ with the following structure:

1° There exists a vertex $x \in X$ such that $H\langle X - \{x\} \rangle$ is a complete subhypergraph of H.

2° In H there exist exactly $\left]\frac{n-1}{k-1}\right[$ edges containing a vertex x, among which there exist $\left[\frac{n-1}{k-1}\right]$ edges such that any two edges have in common exactly the vertex x.

3° The vertex x is adjacent to all vertices of H.

For such a hypergraph H

$$\beta_2(H) = \left]\frac{n}{k}\right[\quad \text{and} \quad \beta_2(\bar{H}) = 2 \quad \text{holds.}$$

This means that the upper and lower bounds (1) are the best possible.

Theorem 2. *For a k-uniform hypergraph $H = \langle X, \mathscr{E} \rangle$ and its complement $\bar{H} = \langle X, \bar{\mathscr{E}} \rangle$*

$$\left]\frac{n-1}{k}\right[+ 1 \leqslant \alpha_2(H) + \alpha_2(\bar{H}) \leqslant \left]\frac{(k+1)n}{k}\right[\tag{5}$$

holds.

Proof. The upper bound in (5) follows form the inequality

$$\alpha_2(H) \leqslant \beta_2(H), \quad \alpha_2(\bar{H}) \leqslant \beta_2(\bar{H})$$

and from Theorem 1.

Let $P = P_x \cup P_y$ be a total covering of H, where P_x is a set of vertices and P_y is a set of edges.

If $|P_x| = 0$, then $n \leqslant k|P_y|$, thus

$$\alpha_2(H) = |P_y| \geqslant \left]\frac{n}{k}\right[.$$

As $\alpha_2(\bar{H}) \geqslant 1$, the lower bound in (5) is satisfied.

Let $|P_x| \geqslant 1$. Let us denote $N = X - P_x - V(P_y)$. If $|N| \leqslant k-1$, then

$$\alpha_2(H) = |P_x| + |P_y| \geqslant |P_x| + \frac{|V(P_y)|}{k} \geqslant \frac{k|P_x|}{k} + \frac{|V(P_y)|}{k} =$$

$$= \left]\frac{k|P_x| + |V(P_y)|}{k}\right[\geqslant \left]\frac{n}{k}\right[$$

holds and $\alpha_2(\bar{H}) \geqslant 1$, thus the lower bound in (5) is satisfied.

If $|N| \geqslant k$, then $\bar{H}\langle N \rangle$ is a complete k-uniform subhypergraph of $\bar{H}$, thus

$$\alpha_2(\bar{H}) \geqslant \left]\frac{|N|}{k}\right[.$$

It follows that

$$\alpha_2(H) + \alpha_2(\bar{H}) \geqslant |P_x| + |P_y| + \left]\frac{|N|}{k}\right[= \left]\frac{1}{k}\left(|P_x| + k|P_y| + |N| + (k-1)|P_x|\right)\right[.$$

$$|P_x| + k|P_y| + |N| \geqslant n,$$

thus

$$\alpha_2(H)+\alpha_2(\bar{H})\geqslant \left]\frac{1}{k}(n+(k-1)|P_x|)\right[=\left]\frac{n-|P_x|}{k}\right[+|P_x|\geqslant \left]\frac{n-1}{k}\right[+1.$$

The proof of Theorem 2 is now complete.

Remark. The equality in the upper bound (5) holds for an arbitrary complete k-uniform hypergraph.

The equality in the lower bound (5) holds, e.g., for $H=\langle X, \mathscr{E}\rangle$ with the following structure:

1° There exists a vertex $x \in X$ such that the subhypergraph $H\langle X-\{x\}\rangle$ is complete.

2° The vertex x is incident with exactly one edge of H.

For such a hypergraph H

$$\alpha_2(H)=\left]\frac{n-1}{k}\right[\quad \text{and} \quad \alpha_2(\bar{H})=1$$

holds. This shows that the upper and lower bounds in (5) are the best possible ones.

Lemma 1. *For a k-uniform hypergraph $H=\langle X, \mathscr{E}\rangle$ and its complement $\bar{H}=\langle X, \bar{\mathscr{E}}\rangle$*

$$\alpha(H)+\alpha(\bar{H})\leqslant n+k-1 \tag{6}$$

holds.

Proof. Let $\alpha(H)=r$. Then in $\bar{H}$ there exists a complete subhypergraph with r vertices, thus $\alpha(\bar{H})\leqslant n-r+k-1$. From this, the assertion of the lemma follows.

Theorem 3. *For a k-uniform hypergraph $H=\langle X, \mathscr{E}\rangle$ and its complement $\bar{H}=\langle X, \bar{\mathscr{E}}\rangle$*

$$\gamma_2(H)+\gamma_2(\bar{H})\leqslant\left[\frac{(k+1)n+1}{k}\right]+k-2 \tag{7}$$

holds.

Proof. Let $\alpha(H)$ be the cardinality of the greatest stable set of vertices in H. Then

$$\gamma_2(H)\leqslant\alpha(H)+\left[\frac{n-\alpha(H)}{k}\right]$$

holds. Also

$$\gamma_2(\bar{H})\leqslant\alpha(\bar{H})+\left[\frac{n-\alpha(\bar{H})}{k}\right]$$

holds. After the addition of these inequalities we get

$$\gamma_2(H)+\gamma_2(\bar{H})\leqslant\alpha(H)+\alpha(\bar{H})+\left[\frac{2n-(\alpha(H)+\alpha(\bar{H}))}{k}\right].$$

By using Lemma 1 we get

$$\gamma_2(H)+\gamma_2(\bar{H})\leq n+k-1+\left[\frac{n-k+1}{k}\right],$$

after appropriate modifications we get the assertion of Theorem 3.

Remark. The equality in (7) holds for an arbitrary complete k-uniform hypergraph H.

A k-uniform hypergraph $H=\langle X, \mathscr{E}\rangle$ is connected if for each non-empty set of vertices $S\subset X$ the following holds: $\mathscr{E}_1\cup\mathscr{E}_2\neq\mathscr{E}$, where $\mathscr{E}_1$ or $\mathscr{E}_2$ is a set of edges of the subhypergraph $H\langle S\rangle$ or $H\langle X-S\rangle$, respectively.

Lemma 2. *For a connected k-uniform hypergraph $H=\langle X, \mathscr{E}\rangle$*

$$\alpha_2(H)\leq \left]\frac{n}{2}\right[\tag{8}$$

holds.

Proof. From a hypergraph $H=\langle X, \mathscr{E}\rangle$ we construct an undirected graph $G=\langle X, E\rangle$ without loops or multiple edges, by which the vertices x_i, $x_j\in X$ form the edge in G, if in H there exists at least one edge which contains them. G is connected and $\alpha_2(H)\leq\alpha_2(G)$. For a connected graph with n vertices, the inequality

$$\alpha_2(G)\leq \left]\frac{n}{2}\right[$$

holds [2]. From this, the assertion of Lemma 2 follows.

Lemma 3. *For a connected k-uniform hypergraph $H=\langle X, \mathscr{E}\rangle$*

$$\alpha_2(H)\leq n-\alpha_0(H)+2-k \tag{9}$$

$$\beta_2(H)\leq \alpha_0(H)+\frac{n-\alpha_0(H)}{k} \tag{10}$$

$$\alpha_2(H)\leq n-\alpha(H) \tag{11}$$

$$\gamma_2(H)\leq \alpha(H)+\frac{n-\alpha(H)}{k} \tag{12}$$

holds.

Proof. The above follows directly from the definition of the characteristic numbers treated and from the connectivity of H.

Theorem 4. *For a connected k-uniform hypergraph $H=\langle X, \mathscr{E}\rangle$*

$$\alpha_2(H)+\beta_2(H)\leq n+\left[\frac{1}{k}\left(\left]\frac{n}{2}\right[-2\right)\right]+3-k \tag{13}$$

$$\alpha_2(H)+\gamma_2(H)\leqslant n+\left[\frac{1}{k}\right]\frac{n}{2}\left[\;\right] \tag{14}$$

$$\beta_2(H)+\gamma_2(H)\leqslant 2n-k \tag{15}$$

holds.

Proof. From (10) it follows that

$$\alpha_0(H)\geqslant\frac{k\beta_2(H)-n}{k-1}$$

and after substitution into (9) we get

$$\alpha_2(H)\leqslant n-\frac{k\beta_2(H)-n}{k-1}+2-k,$$

and further

$$\alpha_2(H)+\beta_2(H)\leqslant n-\frac{\alpha_2(H)}{k}+3-k+\frac{2}{k}.$$

After substitution for $\alpha_2(H)$ from (8) we get the assertion (13).

From (11) and (12) it follows that

$$\alpha_2(H)\leqslant n-\frac{k\gamma_2(H)-n}{k-1}$$

and after a modification we get

$$\alpha_2(H)+\gamma_2(H)\leqslant n+\frac{\alpha_2(H)}{k}.$$

From this and (8) we get the assertion (14).

For the connected hypergraph H

$$\beta_2(H)\leqslant n-k+1$$
$$\gamma_2(H)\leqslant n-1.$$

After addition we get the assertion (15).

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О ЧИСЛЕ ТОТАЛЬНОЙ НЕЗАВИСИМОСТИ И ТОТАЛЬНОГО ПОКРЫТИЯ ДЛЯ k-УНИФОРМНЫХ ГИПЕРГРАФОВ

František Olejník

Резюме

В этой работе приведены верхние и нижние оценки для суммы числа сильной тотальной независимости, (числа слабой тотальной независимости, числа тотального покрытия) для k-униформного гиперграфа H и его дополнения $\bar{H}$.