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ON FUZZY INPUT DATA AND THE WORST SCENARIO METHOD*

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Abstract. In practice, input data entering a state problem are almost always uncertain to some extent. Thus it is natural to consider a set $\mathcal{U}_{\rm ad}$ of admissible input data instead of a fixed and unique input. The worst scenario method takes into account all states generated by $\mathcal{U}_{\rm ad}$ and maximizes a functional criterion reflecting a particular feature of the state solution, as local stress, displacement, or temperature, for instance. An increase in the criterion value indicates a deterioration in the featured quantity. The method takes all the elements of $\mathcal{U}_{\rm ad}$ as equally important though this can be unrealistic and can lead to too pessimistic conclusions. Often, however, additional information expressed through a membership function of $\mathcal{U}_{\rm ad}$ is available, i.e., $\mathcal{U}_{\rm ad}$ becomes a fuzzy set. In the article, infinite-dimensional $\mathcal{U}_{\rm ad}$ are considered, two ways of introducing fuzziness into $\mathcal{U}_{\rm ad}$ are suggested, and the worst scenario method operating on fuzzy admissible sets is proposed to obtain a fuzzy set of outputs.

Keywords: fuzzy sets, uncertainty, worst scenario method

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1. Introduction

We can distinguish three basic ingredients of a modeling process: an input parameter (or parameters) a, an a-dependent state problem $S(a;u)$ the solution of which is $u = u(a)$, and a criterion Φ evaluating a feature of $u(a)$. The criterion can also directly depend on a, i.e., $\Phi = \Phi(a;u(a))$. Problem $S(a;u)$ represents a mathematical model of the observed phenomenon, engineering structure, etc. Local (mean) values of temperature, velocity, mechanical displacement or stress are examples of Φ. Parameter(s) a can take the form of coefficients of an equation underlying the problem $S(a;u)$, for instance.

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In modeling real-life problems, parameters a often originate from measurements, identification procedures or even hypotheses and so they are burdened with uncertainty. As a consequence, it is natural and responsible to consider $a \in \mathcal{U}_{\rm ad}$, where $\mathcal{U}_{\rm ad}$ is a set of admissible parameters. The admissible set $\mathcal{U}_{\rm ad}$ reflects the uncertainty of inputs. Having the set $\mathcal{U}_{\rm ad}$ of admissible inputs, we arrive at the set of possible outputs $\{u(a) \mid a \in \mathcal{U}_{\rm ad}\}$ and, consequently, at the set $\mathcal{C} = \{\Phi(a; u(a)) \mid a \in \mathcal{U}_{\rm ad}\}$ comprising achievable values of the criterion functional. An analysis of $\mathcal{C}$ is the primary goal in many applications.

If additional information is known about $\mathcal{U}_{\rm ad}$, then it is highly desirable to utilize it in determining the features of $\mathcal{C}$. Take the probabilistic character of inputs transformed into the probabilistic character of outputs, for instance. Different approaches are presented in [3].

The scheme described in the previous paragraphs has been at least partly applied to many models and corresponds to well established engineering rules of design safety analysis, though not all implementations of this scheme have been mathematically rigorous. However, we can observe growing interest in uncertainty in inputs and its propagation into outputs during the last decade, see, e.g., [1], [2], [8], [9], [12].

Investigating inputs a and states $u(a)$ indicated by high values of $\Psi(a) = \Phi(a; u(a))$, one wishes to know $\sup_{a \in \mathcal{U}_{\rm ad}} \Psi(a)$. Though general, the supremum concept leads to difficulties both in theory and numerical approximation. This is why the *worst scenario method* aims at a modified problem:

$$\text{Find } a^0 \in \mathcal{U}_{\rm ad}, \quad a^0 = \mathop{\arg\max}_{a \in \mathcal{U}_{\rm ad}} \Psi(a). \tag{1}$$

Conditions sufficient for the existence of a^0 are presented in [11]: $\mathcal{U}_{\rm ad}$ is a compact subset of a Banach space, $u(a)$ belongs to a reflexive Banach space V and is unique, the strong convergence of $\{a_n\} \subset \mathcal{U}_{\rm ad}$ implies the weak convergence of $\{u(a_n)\} \subset V$, and $\Phi\colon \mathcal{U}_{\rm ad} \times V \to \mathbb{R}$ is upper semi-continuous with respect to the strong convergence in $\mathcal{U}_{\rm ad}$ and the weak convergence in V. Further details and modifications can be found in [11].

Knowing a^0, we are sure that the criterion value $\Psi(a^0)$ cannot be exceeded, i.e., that "nothing worse can happen" if a is taken from $\mathcal{U}_{\rm ad}$. This is related to the classical engineering safe-side rule, which asks safe design even if the worst possible circumstances occur. In terms of (1), $\Psi(a^0) \leqslant \psi_{\rm saf}$, where $\psi_{\rm saf}$ is a lower bound of the safety measure.

A sound mathematical formulation of the worst scenario method was allegedly first suggested in [4], [5]. In [1], where convex $\mathcal{U}_{\rm ad}$ are considered, it appears as *convex modeling of uncertainty*. The worst scenario method is also known as *anti-optimization* in [7].

Problem (1) gives only an upper bound of the achievable values. However, it is also desirable to know the whole range of the criterion values generated by the admissible uncertain input parameters. To this end, we suppose that Ψ is continuous and define the *maximum* (or *range*) *span problem*:

$$\text{(2)} \qquad \text{Besides } a^0, \text{ find also } a_0 = \operatorname*{arg\,min}_{a \in \mathcal{U}_{\mathrm{ad}}} \Psi(a).$$

The range of $\Psi(a)$ on $\mathcal{U}_{\mathrm{ad}}$ is then equal to $[\Psi(a_0), \Psi(a^0)]$.

The worst scenario method takes all the elements of $\mathcal{U}_{\mathrm{ad}}$ as equally important though this can be unrealistic and can lead to too pessimistic conclusions. This can happen if more information on the admissible set $\mathcal{U}_{\mathrm{ad}}$ is known. Some input values are quite possible, some are possible but with a shred of doubt, others seem to be almost excluded. As a consequence, we wish to use this additional information and to transform it into a stratification of the possibility of outputs. We are led to a coupling of the worst scenario method with fuzzy sets.

2. Fuzzy admissible sets

A set U of a universe set X is uniquely related to the characteristic function $\chi_U\colon X \to \{0,1\}$, $x \in U \Rightarrow \chi_U(x) = 1$, $x \notin U \Rightarrow \chi_U(x) = 0$. A fuzzy set U is characterized by a membership function $\mu_U\colon X \to [0,1]$ expressing the degree of truth of the statement "$x \in X$ belongs to U." This characterization suggests $\mu_U(x) = 0 \Rightarrow x \notin U$. As U will be closed in our setting, $\mu_U(x) = 0$ might lead to an ambiguous interpretation, i.e., $x \notin U$ or $x \in \partial U$ if μ_U is continuous, for instance.

This is why we interpret μ_U in a slightly different way. We consider U given in the classical (crisp) sense and use $\mu_U(x)$ to express the degree of possibility that $x \in U$ represents a really feasible input parameter. It means that $\mu_U(x) = 0$ and $x \in U$ are no longer in contradiction, x is used in computation but, from the view point of possibility, it plays the role of a dummy value. Moreover, $x \in X \setminus U \Rightarrow \mu_U(x) = 0$ can be in effect. The range of μ_U need not be the whole set $[0,1]$.

We will exploit another fuzzy sets theory notion, namely the *α-cut of a fuzzy set*:

$${}^{\alpha}U = \{x \in U \mid \mu_U(x) \geqslant \alpha\}, \quad \alpha \in [0,1].$$

Let us note that ${}^{0}U = U$.

Lemma 1. *Let U be a compact subset of a Banach space X and let μ_U restricted to U be a continuous map $U \to [0,1]$. Then for any $\alpha \in [0,1]$ the set ${}^{\alpha}U$ is a compact subset of X.*

P r o o f. Due to the continuity of μ_U, the α-cut ${}^{\alpha}U$ is a closed subset of the compact set U. $\square$

Let us suppose that the admissible set $\mathcal{U}_{\rm ad}$ is endowed with a membership function $\mu_{\mathcal{U}_{\rm ad}}$. If J is, for example, a finite set of m equidistant values α_j, $0 = \alpha_1 < \alpha_2 < \ldots < \alpha_m = 1$, then we can define a sequence of α-cuts $\{{}^{\alpha}\mathcal{U}_{\rm ad}\}_{\alpha\in J}$ and solve the maximum span problem (2) on every ${}^{\alpha}\mathcal{U}_{\rm ad}$ to get a sequence of intervals ${}^{\alpha}I = [\Psi(a_{0,\alpha}), \Psi(a^{0,\alpha})]$. Here, the compactness of α-cuts is substantial. The intervals ${}^{\alpha}I$ can be interpreted as α-cuts derived from the fuzzy set $M_{\Phi} = \{\Phi(a, u(a)) \mid a \in \mathcal{U}_{\rm ad}\}$ through a (not necessarily continuous) membership function defined by

$$\mu_{M_{\Phi}}(y) = \max_{\alpha\in J} \min(\alpha, \chi_{{}^{\alpha}I}(y));$$

see [3], where also examples based on finite-dimensional sets of fuzzy parameters are given.

However, admissible sets are infinite-dimensional in advanced worst scenario problems, see [6], [10], for example. To apply the formula for $\mu_{M_{\Phi}}$, i.e., to extend the fuzziness from inputs to outputs, we need membership functions suitable for admissible sets common for complex worst scenario problems.

We will deal with a fairly typical example of an infinite-dimensional admissible set $\mathcal{U}_{\rm ad}$ in the next paragraphs. To this end, we introduce the set $C_B^{(0),1}(\Omega)$ of Lipschitz functions defined on $\Omega \equiv [0, l] \subset \mathbb{R}$ and such that $|a'(x)| \leqslant B$ a.e. in $(0, l)$, $B > 0$, the prime standing for the derivative. We define

$$\mathcal{U}_{\rm ad} = \bigl\{a \in C_B^{(0),1}(\Omega) \mid c_0(x) \leqslant a(x) \leqslant c^0(x) \;\; \forall\, x \in \Omega\bigr\}, \tag{3}$$

where $c_0, c^0 \in C_B^{(0),1}(\Omega)$ are functions given and positive on $[0, l]$. In virtue of the Ascoli-Arzelà theorem, the admissible set $\mathcal{U}_{\rm ad}$ is compact in the space of continuous functions, i.e., in the $C(\Omega)$-norm.

It may happen that elements of $\mathcal{U}_{\rm ad}$ are not equally possible. Then we need a tool for introducing fuzziness into $\mathcal{U}_{\rm ad}$. In this article, we present two ways of making $\mathcal{U}_{\rm ad}$ fuzzy.

The first method uses fuzzy bounds instead of crisp functions c_0 and c^0. Let us explain it in detail.

We assume that besides c_0 and c^0, two other functions $\bar{c}_0, \bar{c}^0 \in C_B^{(0),1}(\Omega)$ are given such that

$$c_0(x) < \bar{c}_0(x) \leqslant \bar{c}^0(x) < c^0(x) \quad \forall\, x \in \Omega.$$

Then, avoiding the direct presentation of a fuzzy admissible set, we define its (*compact*) α-cuts for $\alpha \in [0,1]$:

$$ {}^{\alpha}\mathcal{U}_{\rm ad} = \bigl\{a \in C_B^{(0),1}(\Omega) \mid (1-\alpha)c_0(x) + \alpha\bar{c}_0(x) \leqslant a(x) \leqslant (1-\alpha)c^0(x) + \alpha\bar{c}^0(x) \ \ \forall\, x \in \Omega\bigr\}. \tag{4} $$

The corresponding membership function $\bar{\mu}_{\mathcal{U}_{\rm ad}}$ reads

$$ \bar{\mu}_{\mathcal{U}_{\rm ad}}(a) = \max_{\{\alpha\in[0,1] \mid a \in {}^{\alpha}\mathcal{U}_{\rm ad}\}} \alpha. \tag{5} $$

Let us discuss (4) and (5). Definition (4) is advantageous for its simplicity, and ${}^{\alpha}\mathcal{U}_{\rm ad}$ are easy to approximate as we will see later. However, the membership function (5) does not distinguish between a local and global behavior of a function $a \in \mathcal{U}_{\rm ad}$. We illustrate this by an example.

Let $a_1 = (1-\alpha)c_0 + \alpha\bar{c}_0$ for a fixed $\alpha \in [0,1/2]$, and let a_2 also belong to ${}^{\alpha}\mathcal{U}_{\rm ad}$, $a_1(0) = a_2(0)$. If B is sufficiently large, then we can find a_2 such that $a_2|_K$ falls between $\bar{c}_0$ and $\bar{c}^0$, where $K \subset \Omega$ and $\operatorname{meas}(K)$ is not small, i.e., a substantial part of a_2 is constrained by bounds defining the α-cut with $\alpha = 1$. Despite of this, $\bar{\mu}_{\mathcal{U}_{\rm ad}}(a_1) = \bar{\mu}_{\mathcal{U}_{\rm ad}(a_2)}$.

If functions a come from measurements, for instance, the equality does not seem to properly reflect the phenomena which we observe there. Indeed, roughly speaking, we expect that fluctuations only rarely and locally exceeding $\bar{c}_0$ or $\bar{c}^0$ are more possible than those exceeding $\bar{c}_0$ or $\bar{c}^0$ often or almost globally.

The other method introducing fuzziness into $\mathcal{U}_{\rm ad}$ is designed to avoid the just mentioned drawback of (4) and (5).

First, we define a set

$$ Q = \{(x,y) \in \mathbb{R}^2 \mid x \in \Omega,\ \ c_0(x) \leqslant y \leqslant c^0(x)\}. $$

Next, we consider a continuous function

$$ \varrho\colon\ Q \to [0,1] \tag{6} $$

such that

$$ \forall\, x \in \Omega \ \ \varrho(x,\cdot) \ \text{ is concave in } \ [c_0(x), c^0(x)]. \tag{7} $$

We are ready to define a new membership function

$$ \hat{\mu}_{\mathcal{U}_{\rm ad}}(a) = l^{-1}\int_0^l \varrho(x, a(x))\,{\rm d}x. \tag{8} $$

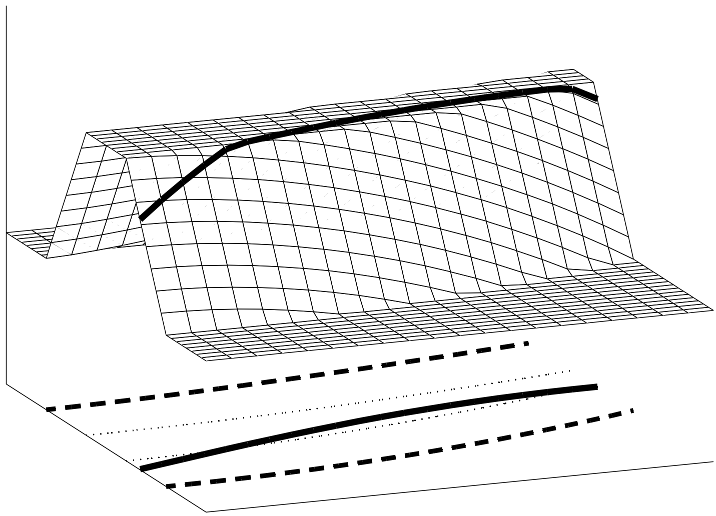

Figure 1.

Fig. 1 depicts c_0 and c^0 (dashed lines), a function a (solid line) as well as the graph of ϱ and $\varrho(x, a(x))$ (solid line located on the surface determined by ϱ). The function ϱ is continuously extended by zero outside Q, i.e., $\varrho(x, c_0(x)) = 0 = \varrho(x, c^0(x))$, $x \in \Omega$, in this example.

Going back to the above-mentioned example of functions a_1, a_2 indistinguishable by $\overline{\mu}_{\mathcal{U}_{\rm ad}}$, we see that $\hat{\mu}_{\mathcal{U}_{\rm ad}}(a_1) < \hat{\mu}_{\mathcal{U}_{\rm ad}}(a_2)$.

Lemma 2. *Let the admissible set and the membership function be given by* (3) *and* (8), *respectively. Then the corresponding α-cuts are compact with respect to the $C(\Omega)$-norm.*

P r o o f. The mapping $\hat{\mu}_{\mathcal{U}_{\rm ad}}\colon \mathcal{U}_{\rm ad} \to [0,1]$ is continuous with respect to the $C(\Omega)$-norm, therefore Lemma 1 can be applied. □

The function ϱ can be designed on the basis of measurements, for instance. Lemma 2 guarantees that we can construct the intervals ${}^{\alpha}I$ and investigate the fuzziness of the range of Ψ.

The parameter B (see (3)) can also be endowed with a sort of fuzziness.

Following the idea leading to (5), we start with a constant $\overline{B}$, $0 < \overline{B} < B$, and define

$$\overline{\overline{\mu}}_{\mathcal{U}_{\rm ad}}(a) = \begin{cases} 1 & \text{if } \|a'\|_\infty \leqslant \overline{B}, \\ (B - \|a'\|_\infty)/(B - \overline{B}) & \text{if } \|a'\|_\infty \geqslant \overline{B}, \end{cases} \tag{9}$$

where $\|a'\|_\infty$ stands for the $L^\infty(\Omega)$-norm of a', $a \in \mathcal{U}_{\rm ad}$.

The fuzzy sets intersection rule takes the minimum membership value at points belonging to the intersection to establish the intersection membership function. We do the same defining

$$\check{\mu}_{\mathcal{U}_{\rm ad}}(a) = \min(\overline{\mu}_{\mathcal{U}_{\rm ad}}(a), \overline{\overline{\mu}}_{\mathcal{U}_{\rm ad}}(a)), \quad a \in \mathcal{U}_{\rm ad}. \tag{10}$$

A parallel to (6)–(8) is constructed through a continuous function π,

$$\pi\colon\ \Omega \times [-B, B] \to [0, 1], \quad \forall\, x \in \Omega\ \ \pi(x, \cdot)\ \text{ is concave in }\ [-B, B],$$
$$\tilde{\mu}_{\mathcal{U}_{\rm ad}}(a) = l^{-1} \int_0^l \pi(x, a'(x))\,\mathrm{d}x.$$

Just as in (10), we set

$$\mu_{\mathcal{U}_{\rm ad}}(a) = \min(\hat{\mu}_{\mathcal{U}_{\rm ad}}(a), \tilde{\mu}_{\mathcal{U}_{\rm ad}}(a)), \quad a \in \mathcal{U}_{\rm ad}. \tag{11}$$

Lemma 3. *Let ${}^{\alpha}\mathcal{U}_{\rm ad}$ be an α-cut of $\mathcal{U}_{\rm ad}$ determined by $\alpha \in [0, 1]$ and by the membership function* (10) *or* (11)*. Then ${}^{\alpha}\mathcal{U}_{\rm ad}$ is compact in the $C(\Omega)$-norm.*

P r o o f. Since the membership functions are continuous, ${}^{\alpha}\mathcal{U}_{\rm ad}$ is a closed subset of a compact set. □

3. Approximation of $\mathcal{U}_{\rm ad}$

To approximate $\mathcal{U}_{\rm ad}$, we fix a mesh $\{x_i\}_{i=1}^N$ in Ω, $0 = x_1 < x_2 < \ldots < x_N = l$, and set $h = \max\limits_{i \in \{2,\ldots,N\}} (x_i - x_{i-1})$.

By means of

$$P_1^N = \{g \in C(\Omega) \mid g|_{[x_{i-1}, x_i]} \ \text{ is linear,}\ \ i = 2, \ldots, N\},$$

we define

$$\mathcal{U}_{\rm ad}^h = \bigl\{a \in P_1^N \cap C_B^{(0),1}(\Omega) \mid c_0(x_i) \leqslant a(x_i) \leqslant c^0(x_i),\ \ i = 1, \ldots, N\bigr\}. \tag{12}$$

R e m a r k 1. In general, $\mathcal{U}_{\rm ad}^h \not\subset \mathcal{U}_{\rm ad}$ (take c_0 concave or c^0 convex, for example). To guarantee $\mathcal{U}_{\rm ad}^h \subset \mathcal{U}_{\rm ad}$, we could start with a sequence $\{\mathcal{U}_{\rm ad}^h\}_{h \to 0}$ and define $\mathcal{U}_{\rm ad}$ as the closure of $\bigcup\limits_{h \to 0} \mathcal{U}_{\rm ad}^h$, see [6].

It can be proved that for any $\varepsilon > 0$ there exists a parameter $h > 0$ such that $\mathcal{U}_{\rm ad}$ is uniformly approximated by $\mathcal{U}_{\rm ad}^h$ with the error less than or equal to ε, i.e., $\forall\, a \in \mathcal{U}_{\rm ad}\ \ \exists a_h \in \mathcal{U}_{\rm ad}^h\ \ \|a - a_h\|_{C(\Omega)} \leqslant \varepsilon$.

The fact that $\{\mathcal{U}_{\rm ad}^h\}_{h\to 0}$ converges to $\mathcal{U}_{\rm ad}$ is substantial for convergence relationships between the original state and the worst scenario problems and their approximations, see [10], [11]. We do not focus on these subjects here, however.

We take an interest in computational aspects of the worst scenario problems stemming from fuzzy admissible sets.

To supply fuzziness to $\mathcal{U}_{\rm ad}^h$, we can use approaches already described in Section 2.

Let us suppose that, after a full discretization of the worst scenario problem (1), the criterion Ψ becomes a continuous mapping $\Psi_h\colon\ \mathcal{U}_{\rm ad}^h \to \mathbb{R}$. Then the pivotal task is to solve the maximum span problem on α-cuts, i.e., to find

$$a_{0,h} = \operatorname*{arg\,min}_{a_h \in {}^{\alpha}\mathcal{U}_{\rm ad}^h} \Psi_h(a_h), \quad a_h^0 = \operatorname*{arg\,max}_{a_h \in {}^{\alpha}\mathcal{U}_{\rm ad}^h} \Psi_h(a_h). \tag{13}$$

Each function a_h can be identified with the N-tuple $\boldsymbol{y}_h = (a_h(x_1), \ldots, a_h(x_N))$. As a consequence, each ${}^{\alpha}\mathcal{U}_{\rm ad}^h$ is identified with ${}^{\alpha}\mathbb{U}_{\rm ad}^h \subset \mathbb{R}^N$ and $\Psi_h(a_h)$ defines a continuous function

$$\psi_h\colon\ {}^{\alpha}\mathbb{U}_{\rm ad}^h \to \mathbb{R}, \quad \psi_h(\boldsymbol{y}_h) = \Psi_h(a_h).$$

Problem (13) is then interpreted as the search for extremes over a constrained subset of $\mathbb{R}^N$. Let us further analyze the nature of those constraints.

If ${}^{\alpha}\mathbb{U}_{\rm ad}^h$ originates from a fuzzy set given through α-dependent bounds as in (4), then the constraints are simple:

$$(1-\alpha)c_0(x_i) + \alpha\bar{c}_0(x_i) \leqslant a_h(x_i) \leqslant (1-\alpha)c^0(x_i) + \alpha\bar{c}^0(x_i), \tag{14}$$

$$-B \leqslant (a_h(x_j) - a_h(x_{j-1}))/(x_j - x_{j-1}) \leqslant B, \tag{15}$$

where $i = 1, \ldots, N$ and $j = 2, \ldots, N$. Box (see (14)) and linear (see (15)) constraints are implemented in many software routines used in advanced optimization. Therefore, we can profit from standard and ready-to-use procedures in solving (13).

We omitted the fuzziness of B (cf. (9), (10)) but it could have been implemented in a way similar to (14).

We face more difficult situation if fuzzy admissible sets are given via (6)–(8). Though (15) applies to this case too and some constraints copy (14),

$$c_0(x_i) \leqslant a_h(x_i) \leqslant c^0(x_i), \quad i = 1, \ldots, N,$$

the others are more complicated as we will see below.

Since the goal is to express $\hat{\mu}_{\mathcal{U}_{\rm ad}}(a_h)$ as a function λ of $\boldsymbol{y}_h$, $\lambda(\boldsymbol{y}_h) = \hat{\mu}_{\mathcal{U}_{\rm ad}}(a_h)$, the complexity (simplicity) of ϱ is crucial. The function λ is nonlinear in general. Computer algebra systems can assist in deriving λ.

We can make the problem somewhat easier if we resort to a numerical integration formula, i.e., if we approximate $\hat{\mu}_{\mathcal{U}_{\rm ad}}(a_h)$ by $\sum_{i=1}^{N} w_i\theta_i$, where $\theta_i = \varrho(x_i, a_h(x_i))$ and w_i is the respective weight parameter. Generally, the function $\varrho(x_i,\cdot)$ is nonlinear.

We can go further in simplification. It is reasonable to have ϱ such that the graph of $\varrho(x,\cdot)$ has a triangular or trapezoidal shape. Then each θ_i, viewed as $\theta_i(y) = \varrho(x_i, y)$, is a continuous piecewise linear function and this linearity is reflected in piecewise linear constraints put on $\boldsymbol{y}_h$. However, due to their piecewise character, the constraints depend on $\boldsymbol{y}_h$. In detail, the matrix A appearing in inequalities $A\boldsymbol{y}_h \leqslant \boldsymbol{b}$ comprises constant elements the value of which depends on $\boldsymbol{y}_h$, i.e., $A(\boldsymbol{y}_h)\boldsymbol{y}_h \leqslant \boldsymbol{b}$. The matrix A changes whenever $a_h(x_i)$ leaves one linear segment and enters the adjoint linear segment of $\varrho(x_i,\cdot)$. As a consequence, an algorithm for solving (13) is more complex than in the case of constraints (14) and (15).

4. Conclusion

Two sorts of fuzzy infinite-dimensional admissible sets have been proposed. As regards (4), it is easy to define and to calculate with but it does not properly reflect some features of uncertain input data. The way illustrated by (6)–(8) avoids this drawback but asks for more complicated numerical algorithms.

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