STABILITY AND THROUGHPUT IMPROVEMENT FOR MULTICHANNEL CSMA AND CSMA/CD PROTOCOLS WITH OPTIMAL BANDWIDTH ALLOCATION

Ioannis E. Pountourakis

This paper examines appropriate protocols for high speed multiple access communication systems where the bandwidth is divided into two separate asymmetric channels. Both channels operate using slotted non-persistent CSMA or CSMA/CD techniques. Free stations access the first channel while all retransmissions occur in the second channel. We define the stability regions and the rules for optimal bandwidth allocation among the two channels for improvement of the system performance in case of infinite population. Numerical results show that the optimal behaviour gives performance improvement as it compared with the single channel system with the same capacity.

1. INTRODUCTION

The exploitation of high speed networks depends on the type of communication medium, the system architecture and the suitable protocols that define the access rules for transmission. The most important factors for networks in baseband and broadband technology using the bus architecture are the propagation delay and the traffic load. These parameters set an upper bound on the performance depending on the medium access protocol. For example, slotted ALOHA and TDMA schemes give best performance at light and heavy loads respectively because these protocols are independent of propagation delay but inefficient for bursty traffic types. In contrast the CSMA scheme, which is very efficient for light and medium loads, is not the best choice when the propagation delay is large. In this protocol category the capacity utilization is limited by the ratio of the end-to-end propagation delay and the packet transmission time. Specifically the performance of CSMA degrades with increasing this ratio. Thus for a given topology, as the bandwidth of the channel increases, the packet transmission time decreases while the end-to-end propagation delay remains the same. So the above mentioned ratio increases and this results in inefficient utilization of the channel capacity.

Several approaches for overcoming this end-to-end propagation delay and the packet transmission time limitation have been proposed for the capacity utilization problem of a high speed channel. The most common used method is the division
of a given bandwidth of a high speed channel into bands in a system of parallel subchannels constructing a multichannel network. This architecture among others advantageous gives improvement in the capacity utilization because the low speed subchannels reduce the afore-mentioned ratio of the propagation delay to packet transmission time as compared with the high speed channel. On the other hand the multiplicity of the station interfaces increase the cost of transmitter/receiver mechanism which set practical limitations.

Alternatively, multichannel solution in single-mode optical fibre networks uses the wavelength division multiplexing access (WDMA) technique. The fibre optic WDMA technique utilises a system of parallel channels each one with different frequency. The main problem of WDMA is not the efficient capacity utilization of the communication medium, because of the plentiful supply of bandwidth, but the rate at which the channel can operate due to confined electronic processing capability of end stations [8, 9]. Contention-based schemes like CSMA protocols is not efficient in WDMA environment. The reason is that the packet transmission time becomes relative small to end-to-end propagation delay and the node processing time. Also the carrier sensing time, based on the immediate feedback, is not a practical feature in high speed systems.

A number of random access protocols have been introduced and analyzed using broadband multichannel networks and multiple access techniques. In [1, 6] a multichannel architecture was proposed. The performance and stability characteristics of the system were analyzed as an extension of the ALOHA protocol. In [2, 3, 4, 5, 7] an alternative architecture of broadcast multiple channel network was proposed. They use several parallel bus-structured channels for implementing a high bandwidth. Appropriate protocols have been described and analyzed as an extension of the non-persistent CSMA and CSMA/CD. It was shown that performance improvements with respect to the single channel protocols may be gained. In [7] two multichannel multiaccess slotted non-persistent CSMA protocols are proposed with asymmetric access rights of the stations among the channels. The asymmetry is related to the distinction of the channels as transmission and retransmission channels where the stations construct queues corresponding to each channel category. The protocols are examined under simplifying assumptions for the carrier sensing technique which saves much of the time wasted for scanning an idle channel and for (re)transmission, as well as much processing time which allows simplicity in the station interface functionality.

In this paper the proposed protocols use the asymmetric access rights idea [7] of the stations among the channels and extend the asymmetry to the bandwidth allocation. In the examined protocols, a given bandwidth is divided into two separate channels with different capacity and adopts the separate sensing policy of SSCSMA protocol [7]. One channel is dedicated for transmission of the new generated packets and the other for (re)transmission of backlogged stations. We examine the optimal bandwidth allocation among the two channels for throughput optimization in conjunction with stability conditions. The rest of the paper is organized as follows. In the next section, we give the basic concepts of computer communication systems and protocols. In Section 3 the basic assumptions about the examined protocols are
introduced. In Section 4, the analysis of the two asymmetric channels system of the slotted non-persistent CSMA and CSMA/CD protocols for an infinite population are presented. Also the stability and throughput optimization is analyzed and the problem of the optimal allocation of the bandwidth between the two asymmetric channels is provided. In Section 4 numerical results are presented. Comments on numerical results and explanation of the behaviour are discussed. Also some conclusions are made.

2. BASIC CONCEPTS

A communication system can be characterized as a centralized system in which all resources are located within a single unit, or a decentralized system in which no communication link or coordination exists among multiple independent systems. A distributed system is a hybrid between centralized and decentralized systems, with the choice of functions to be distributed depending on the specific tradeoffs involved. With proper design a distributed system may provide many of the advantages of both centralized and decentralized systems while avoiding most of their disadvantages. In packet communication systems, the problem of designing an efficient multiple access scheme is of prime importance. Brief definitions and characteristics of these schemes are given in the following subsections.

2.1. Fixed assignment schemes

In these schemes, a fixed portion of the total channel capacity is allocated to each station in the network. These schemes are simple to implement and use but suffer from inefficiency on two accounts: First, during the idle periods of a station, its portion of channel capacity cannot be used by other stations with traffic, and second, the response time is very poor due to the scaling channel capacity.

2.1.1. Frequency Division Multiple Access (FDMA)

The bandwidth of the channel is divided into different (usually) equal subchannels and each station is allocated a separate subchannel.

2.1.2. Time Division Multiple Access (TDMA)

Channel time is divided into slots and a station is assigned a number of slots (usually one slot) in each time frame in a round robin fashion. This scheme requires total cooperation among stations. With total cooperation, collisions can be avoided. Each station when its slot occurs, either transmits, if ready, or allows its assigned slot to go unused.

2.1.3. Code Division Multiple Access (CDMA)

The transmissions are allowed to overlap both in the frequency and time domains. Orthogonality is achieved by the use of different signalling codes in conjunction with
correlated detection at the intended receivers. Major features of attraction are message security from unauthorized access and immunity from natural and intentional noises. However, spectrum utilization is usually very poor.

2.2. Random access schemes

2.2.1. Pure ALOHA

The simplest form of a random access scheme is pure or unslotted ALOHA. In this scheme a station transmits a packet at the instant of its generation. If it overlaps in time (in part or full) with one or more transmissions from other stations, all packets involved in the collision are assumed destroyed and must be retransmitted later at randomly chosen times. Collisions can be detected either by the failure of an acknowledgement message to arrive (after suitable delay) from the intended receiver station or by equipping a station so that it receives its own transmissions. This scheme is simple but extracts a price from the system in the form of wasted channel capacity due to collisions. We assume that the total channel traffic \( G \), consisting of newly generated packets plus retransmitted packets, obeys to Poisson statistics with parameter \( G \). A retransmission occurs due to at least one collision. Since the packet transmission time is \( T \)-sec long, a collision with a given station’s packet will take place if any other station decides to transmit in a interval \( \pm T \) about the time of initiation of a given station. So the throughput \( S \) of the system is given by, \( S = Ge^{-2G} \). This the desired throughput equation for a pure ALOHA system operating under stable equilibrium conditions. If we differentiate the above equation with respect of \( G \), we find the \( S_{\text{max}} = \frac{1}{2e} = 0.18 \), that corresponds at \( G_{\text{opt}} = 1/2 \).

2.2.2. Slotted ALOHA

By the introduction of slotting, in which each station begins its transmission at the beginning of a fixed size time slot, the channel utilization can be improved. Consider a system in which all stations are synchronized to one common clock. Let the time scale now be divided into specified packet transmission time intervals or slots \( T \)-sec long. Stations are constrained to transmit at the beginning of these time slots. Since there can be no overlap of packets from adjacent intervals the throughput of this scheme is exactly twice that of the pure ALOHA system. So the throughput obeys to the equation, \( S = Ge^{-G} \). The \( S_{\text{max}} = \frac{1}{e} = 0.368 \), that corresponds at \( G_{\text{opt}} = 1 \) The static or uncontrolled slotted ALOHA (with a fixed distribution for the retransmission delays) performs very poorly (low channel utilization and large delays). The dynamic control procedures can improve the performance significantly. It has been shown that, for maximum throughput, the retransmission probabilities should equal the reciprocal of the number of busy stations. This scheme achieves the maximum channel utilization (under the class of symmetric access rights, where each station transmits with the same probability) and has been called Optimal ALOHA Scheme.
2.3. Carrier–Sense Multiple Access (CSMA)

CSMA refers to a family of protocols, with all members employing deference. The family members are differentiated by the action they take upon encounter a busy channel. The protocol strategy for a station to access the communication system, is based on listening to the channel before attempting transmission in order to determine whether another station's carrier is present or not. If such a carrier is detected, then the station refrains from transmitting, otherwise it transmits according to the following described protocols. The CSMA protocols are efficient when the ratio of propagation delay to the average packet transmission time is small.

2.3.1. Non-persistent CSMA

In this protocol, a ready station senses the channel and transmits a packet (or a message) if the channel is idle, that is if no carrier signal is present. If a carrier is present, the station waits \( \Delta t \) time units, with \( \Delta t \) drawn from a probability distribution called the retransmission delay distribution. After the \( \Delta t \) delay, the node repeats the sequence described. Nodes send acknowledgements messages. If, after a reasonable time, an acknowledgement is not returned, it is assumed that channel noise or collision destroyed the transmission, and the sequence is repeated. The length of the \( \Delta t \) delays is randomly drawn so as to serialize the transmission attempts of stations that become ready during a transmission, hence avoiding collision.

2.3.2. 1-persistent CSMA

In this protocol, a ready station senses the channel and transmits if it is sensed idle. If the channel is sensed busy, the station persists in sensing the channel until it goes idle, at which time the station transmits. Acknowledgements, noise, and collisions are handled in the same manner as in nonpersistent CSMA.

2.3.3. p-persistent CSMA

In this protocol, a ready node senses the channel and transmits with probability \( p \) if it is idle and delays a time unit with probability \( 1 - p \). If, after the a time-unit delay, the channel is again sensed idle, the decision process is repeated.

If, after a delay, the channel is sensed busy, a retransmission attempt is scheduled after a \( \Delta t \) delay (as in nonpersistent CSMA). If the ready node initially senses the channel busy, it persists in sensing the channel until it becomes idle, at which time it transmits with probability \( p \) and delays its point of decision by a time units with probability \( 1 - p \). Collisions, noise, and acknowledgements are handled as previously described.

2.3.4. Collision detection (CSMA-CD)

The CSMA-CD protocol, is also referred listen-while-transmit (LWT). If a station is ready to transmit, first senses the channel and transmits if it is idle. If it senses a collision (garbled message), transmission is immediately halted and transmits a brief
jamming signal to ensure that all stations know there was a collision. After transmitting the jamming signal, waits a random time $\Delta t$ as in the case of nonpersistent CSMA. The policy if the channel is sensed busy, is as in nonpersistent CSMA.

2.4. Optical networks

Optical networks using (Wavelength Division Multiplexing – WDM) techniques, enable the exploitation of the huge optical fiber bandwidth at low attenuation. Various (Wavelength Division Multiple Access – WDMA) protocols have been proposed for various network architectures. The fundamental problem of single channel high-speed networks is the rate at which the channel can operate which is confined by the electronics of end stations. By dividing a given optical fiber bandwidth, the WDM technique provides multiple parallel independent and noninterfering channels at lower data rates compatible with the existing station’s commercial devices. In WDM networks, the different channels correspond to different optical wavelengths. Stations may transmit packets on different channels using tuneable laser transmitter(s). Stations may receive packets from different channels using tuneable filter receiver(s). The data multichannel system that WDM technique provides, gives the solution to the problem of an electronic bottleneck because each station has to operate only on the data traffic which is intended for itself at the lower rate of that data channel.

In practice the performance features of protocols proposed in WDM network architecture and protocols depend on a number of key factors. Among the most important are 1) tuning times, 2) tuning range, 3) the processing requirements, 4) the propagation delay with respect to the packet transmission time, 5) the waiting time before packet transmission, 6) whether or not a scheme requires network synchronization, 7) channel collision and 8) receiver collision. These parameters set an upper bound on the maximum utilization of such protocols.

3. MODEL AND ASSUMPTIONS

A given channel of capacity $C$ (packets/sec) is divided into two separate channels the transmission and the retransmission channel for both CSMA and CSMA/CD protocol cases. In the transmission channel of capacity $C_f$, only free stations attempt transmissions as they generate new packets following the adopting access technique. The retransmission channel of capacity $C_b$, is dedicated to backlogged stations for collision resolution. So that the total capacity is divided into two channels $C_b/C_f = k$, where $k$ is a constant called the bandwidth allocation coefficient. An infinite population of stations each one connected by means of separate interfaces to both channels is assumed. The time axis for each channel is considered to be partitioned into minislots of length $\tau$, the end-to-end propagation delay which we use as time unit. It is supposed that each station can receive packets simultaneously from both channels. The two channels operate separately and each one applies the sensing procedure at different time instants. Thus we define two time interval cycles, one for each channel. The duration and the starting point of each cycle is different. The channels are error free and there are no capture phenomena. Thus, packets may be
corrupted only because of their concurrent transmission (collision). The set of rules that the slotted version of the non-persistent CSMA implies for the stations in the two channel system are as follows:

1. Every station has a buffer with capacity of one packet. If the buffer is empty, the station is said to be free, otherwise, it is backlogged. Free stations are obliged to transmit to transmission channel. As a free station generates a packet, it senses the transmission channel at the beginning of the minislot and if the channel is idle, it starts a packet transmission at the beginning of the next minislot. If the transmission is unsuccessful or the transmission channel sensed busy, the packet enters to station’s buffer, the station changes to backlogged and retransmission period starts. Packets of equal and constant length are collectively generated in a Poisson stream with mean rate of λ packets/minislot. If a station is backlogged and generates a new packet, the packet is lost and never returns.

2. Backlogged stations use the retransmission channel. A backlogged station waits for a random period of time according to the retransmission probability rule and then senses the retransmission channel and if the channel is idle, it starts transmitting at the beginning of the of the next minislot. If the retransmission channel is busy, it repeats the same procedure at a future random time until successful retransmission.

4. ANALYSIS

Throughput improvement is associated with the stability of the system for infinite population. We follow the work in [7, 10] assuming that the retransmission probabilities depend on the number n of backlogged stations that is.

\[ p = \min\{1, \alpha/n\}, \quad n > 0, \quad \text{and} \quad \alpha > 0 \quad \text{is a constant.} \quad (1) \]

A method to estimate the number of backlogged stations, n, is described in [11]. For a meaningful comparison of the bandwidth allocation with respect to a single channel system of the same total capacity, performance measures should refer to equal time units. Let T be the packet transmission of the single channel system with capacity C. For discussion on numerical results we consider T be the time unit. Let \( T_f \) be the packet transmission time on the transmission channel of capacity \( C_f \) and \( T_b \) be the packet transmission time of the retransmission channel of capacity \( C_b \), then we have that

\[ \frac{C_b}{C_f} = \frac{T_f}{T_b} = k, \quad T_f = (k + 1)T, \quad T_b = T(k + 1)/k. \quad (2) \]

For the transmission channel, the traffic is \( G_f = \lambda \), which is the mean rate of packets generated by free stations during a minislot and obeys to Poisson statistics. The probability that k free stations attempt transmission during a minislot time is given by

\[ C_k = \lambda^k e^{-\lambda} / k!. \quad (3) \]
For the retransmission channel, we assume that the retransmission probabilities are defined as in Eq. (1), and the traffic is given by $G_b = \alpha$, which is the mean rate of packets generated by backlogged stations during a minislot. We also assume that the traffic $G_b$ obeys the Poisson statistics. The conditional probability that $i$ out of $n$ backlogged stations attempt to retransmit with probability $p$ during a minislot time is given by

$$g_{in} = \frac{n!}{((n - i)!i!)} (1 - p)^{n-i} p^i. \tag{4}$$

The idle period is defined as the time interval, starting at the first point that no station (re)transmits and ending at the time instant that some stations begin to (re)transmit. The probability that the idle period ends is $d_f = 1 - C_0$ and the expected duration is $I_f = 1/d_f$ for the transmission channel. Also we take $d_b = 1 - g_{0n}$ and $I_b = 1/d_b$ for the retransmission channel. We have

$$g_{1n} = \alpha [1 - \alpha/n]^{n-1} \quad \text{and} \quad g_{0n} = [1 - \alpha/n]^n. \tag{5}$$

As $n \rightarrow \infty$, we take

$$\lim_{n \rightarrow \infty} g_{1n} = \alpha e^{-\alpha} \quad \text{and} \quad \lim_{n \rightarrow \infty} g_{0n} = e^{-\alpha} \tag{6}$$

also:

$$C_0 = e^{-\lambda} \quad \text{and} \quad C_1 = \lambda e^{-\lambda}. \tag{7}$$

According to the above equations the average length in minislots of the idle periods for transmission and retransmission channel is given by

$$I_f = 1/(1 - e^{-\lambda}) \tag{8}$$

$$I_b = 1/(1 - e^{-\alpha}). \tag{9}$$

The different length of the idle periods in the transmission and the retransmission channel is due to the independence of the two carrier sensing procedures. For slotted non persistent CSMA/CD protocol there are two types of busy periods (collision/abort) and (successful transmission) and their respective lengths are $R + 1$ and $X + 1$ minislots where $R$ is the required time to abort a collided packet from the network, and $X$ is the transmission time ($T_f$ or $T_b$) of a data packet. We define

$$P_{sf} = \frac{\lambda e^{-\lambda}}{1 - e^{-\lambda}} \tag{10}$$

as the conditional probability that a transmission is successful given that a transmission has occurred

$$P_{ef} = \frac{1 - e^{-\lambda} - \lambda e^{-\lambda}}{1 - e^{-\lambda}} \tag{11}$$

as the conditional probability that a transmission is unsuccessful given that a transmission has occurred and

$$P_{sb} = \frac{\alpha e^{-\alpha}}{1 - e^{-\alpha}} \tag{12}$$
as the conditional probability that a retransmission is successful given that a retransmission has occurred.
\[ P_{cb} = \frac{1 - e^{-\alpha} - \alpha e^{-\alpha}}{1 - e^{-\alpha}} \]  
(13)
as the conditional probability that a retransmission is unsuccessful given that a retransmission has occurred. The expected duration of busy periods are defined as follows:

\[ B_f = (T_f + 1) P_{sf} + (R + 1) P_{cf} \]  
(14)
\[ B_b = (T_b + 1) P_{sb} + (R + 1) P_{cb} \]  
(15)
A busy period followed by an idle period is called cycle. So the mean length of the cycle of the transmission channel is given by

\[ C_{tf} = B_f + I_f. \]  
(16)
For the retransmission channel the mean length of the cycle is given by

\[ C_{tb} = B_b + I_b. \]  
(17)
The expected throughput is defined as the ratio of the expected time during a cycle that the transmission/retransmission channel is used without conflicts to the mean length of the cycle. Thus for the transmission channel holds

\[ S_f = \frac{T_f P_{sf}}{C_{tf}} = \frac{T_f \lambda e^{-\lambda}}{1 + (1 + T_f) \lambda e^{-\lambda} + (1 + R)(1 - e^{-\lambda} - \alpha e^{-\alpha})} \]  
(18)
and for retransmission channel we take

\[ S_b = \frac{T_b P_{sb}}{C_{tb}} = \frac{T_b \alpha e^{-\alpha}}{1 + (1 + T_b) \alpha e^{-\alpha} + (1 + R)(1 - e^{-\alpha} - \alpha e^{-\alpha})}. \]  
(19)
The slotted non persistent CSMA protocol can be considered as a special case of CSMA/CD. Hence substituting \( R = T_f \) into (18) and \( R = T_b \) into (19) we have correspondingly

\[ S_f = \frac{T_f \lambda e^{-\lambda}}{1 + (1 + T_f)(1 - e^{-\lambda})} \]  
(20)
and

\[ S_b = \frac{T_b \alpha e^{-\alpha}}{1 + (1 + T_b)(1 - e^{-\alpha})}. \]  
(21)
Let \( S \) the total throughput of the system and \( \lambda T_f \) the mean input rate during the transmission period \( T_f \). In steady state we have

\[ S = \lambda T_f. \]  
(22)
We also define the normalized throughput as the average number of successfully transmitted packets during the time unit in steady state, that is,

\[ S_{nor} = \lambda T = S/(k + 1). \]  
(23)
4.1. Stability

In steady state, using the definition of stability in [7, 10] for infinitive population, the system is stable if the mean input rate during a minislot is less than the sum of the maximum mean output rate from the transmission channel (probability $P_{sf}$) normalized to the mean duration time $C_{tf}$, plus the mean output rate from retransmission channel (probability $P_{sb}$) normalized to the mean duration time $C_{tb}$. Thus we take:

$$\lambda < \frac{P_{sf}}{C_{tf}} + \frac{P_{sb}}{C_{tb}}. \quad (24)$$

We can rewrite (24) as follows:

$$\lambda T_f < \frac{T_f P_{sf}}{C_{tf}} + \frac{k T_b P_{sb}}{C_{tb}} \quad (25)$$

or

$$\lambda T_f < S_f + k S_b. \quad (26)$$

Substituting Eqs (18) and (19) into (26) for the non persistent CSMA/CD protocol, we take:

$$\lambda T_f < \frac{T_f \lambda e^{-\lambda}}{1 - (1 + T_f) \lambda e^{-\lambda} + (1 + R) (1 - e^{-\lambda} - \lambda e^{-\lambda})} + \frac{k T_b \alpha e^{-\alpha}}{1 + (1 + T_b) \alpha e^{-\alpha} + (1 + R) (1 - e^{-\alpha} - \alpha e^{-\alpha})}. \quad (27)$$

For the non persistent CSMA we have

$$\lambda T_f < \frac{T_f \lambda e^{-\lambda}}{1 + (1 + T_f) (1 - e^{-\lambda})} + \frac{k T_b \alpha e^{-\alpha}}{1 + (1 + T_b) (1 - e^{-\alpha})}. \quad (28)$$

The above inequalities denote that in steady state the input rate, $\lambda T_f$, should be less than the throughput, $S_f$, from the transmission channel plus $k$ times the throughput, $S_b$, from the retransmission channel. The retransmission channel throughput is multiplied by $k$, so that it refers to the time $T_f = k T_b$. The throughput $S_b$ from the retransmission channel, for a fixed value of $T$, can be maximized with respect to $\alpha$. For this purpose we set the first derivative of

$$S_b(k, \alpha) = \frac{T_b \alpha e^{-\alpha}}{1 + (1 + T_b) \alpha e^{-\alpha} + (1 + R) (1 - e^{-\alpha} - \alpha e^{-\alpha})} \quad (29)$$

with respect to $\alpha$ equal to zero and with the aid of numerical analysis techniques for each $k$ we calculate the optimal $\alpha_{opt}$ and then the corresponding optimal value $S_b(k, \alpha_{opt})$. We rewrite Eq. (26) assuming optimum performance from retransmission channel, as follows,

$$F = \lambda T_f - k S_b(k, \alpha_{opt}) < S_f. \quad (30)$$
Fig. 1. Average throughput from transmission channel and $F = \lambda T_f - kS_b(k, \alpha_{opt})$ versus input rate $\lambda T_f$ with infinite population for CSMA protocol with $T = 10$.

Fig. 2. Average throughput from transmission channel and $F = \lambda T_f - kS_b(k, \alpha_{opt})$ versus input rate $\lambda T_f$ with infinite population and $T = 10$ for CSMA/CD protocol with $R = 0.1T$. 
5. NUMERICAL RESULTS

For a stable system we evaluate $\alpha_{\text{opt}}$ and consequently $S_b(k, \alpha_{\text{opt}})$. Substituting this value into Eq. (30) and using numerical search techniques, for each $k$ we define an allowable maximum $\lambda_{\text{max}}$ which corresponds to a maximum $S_{\text{nor}}$ given by

$$ S_{\text{nor,max}} = \frac{S_f(k, \lambda_{\text{max}})}{k+1} + \frac{S_b(k, \alpha_{\text{opt}})k}{k+1} = T\lambda_{\text{max}}. \quad (31) $$

Figures 1 and 2 depict $S_f$ and $F$ respectively versus $\lambda T_f$ for $T = 10$. It is obvious that for each $k$ there is a maximum $\lambda$ value $\lambda_{\text{max}}$ that corresponds to the point where the $F$ curve intersects the $S_f$ curve.

Thus for fixed value of $T = 40$ and different values of $k$ we have:

Example 1. The CSMA protocol for $k = 1$, gives $S_{\text{nor,max}} = 0.72855$ with $\lambda_{\text{max}} = 0.01821$ and $\alpha_{\text{opt}} = 0.20377$. Also for $k = 2$, gives $S_{\text{nor,max}} = 0.7907$ with $\lambda_{\text{max}} = 0.01977$ and the same $\alpha_{\text{opt}}$.

Example 2. The CSMA/CD protocol($R = 0.5T$), for $k = 1$ gives $S_{\text{nor,max}} = 0.7438$ with $\lambda_{\text{max}} = 0.01859$ and $\alpha_{\text{opt}} = 0.27483$. Also for $k = 2$, gives $S_{\text{nor,max}} = 0.81343$ with $\lambda_{\text{max}} = 0.02034$ and the same $\alpha_{\text{opt}}$.

The stability regions for fixed values of $S_{\text{nor}}$, $T$ and different values of $k$ are defined from Eq. (26).

Example 3. The CSMA protocol for $T = 20$, $S_{\text{nor}} = 0.5$ we take for $k = 2$, $0.03 < \alpha < 1.629$ and for $k = 5$, $0.0375 < \alpha < 1.4124$. Also for $T = 30$, $S_{\text{nor}} = 0.6$ we take for $k = 1$, $0.0363 < \alpha < 1.1266$ and for $k = 3$, $0.0359 < \alpha < 1.136$.

Example 4. The CSMA/CD protocol for $T = 20$, $R = 0.1T$, $S_{\text{nor}} = 0.5$ we take for $k = 2$, $0.0297 < \alpha < 3.5289$ and for $k = 5$, $0.0370 < \alpha < 3.263$. Also for $T = 30$, $R = 0.2T$, $S_{\text{nor}} = 0.6$ we take for $k = 1$, $0.0356 < \alpha < 2.5035$ and for $k = 4$, $0.0373 < \alpha < 2.4481$.

The above examples show that for constant values of $(T, R, S_{\text{nor}})$ we can found a set of values of the control parameter $\alpha_{\text{min}} < \alpha < \alpha_{\text{max}}$ for which equation (26) holds. Figures 3 and 4 illustrate $S_{\text{nor}}$ versus $\alpha$ for $T = 20$ with $k = 0.6, 5$ and $T = 10, 20$ with $k = 5$ correspondingly for CSMA and CSMA/CD($R = 0.2T$) protocols. From these figures it is obvious that for a specific value of $S_{\text{nor}}$ the determination of stability is depicted by the two points where the curve is intersected with the dotted line as Examples 3 and 4 presents. The widening stable range for $\alpha$ is clear as we proceed from CSMA to CSMA/CD with $R = 2$. Also it can observed that for lower values of $S_{\text{nor}}$ or $k$ the stable range is increased which gives robustness of the stability control. An other advantage is the increasing flatness of $S_{\text{nor}}$ curve as $T$ or $k$ increase, which gives manoeuvrability for the choice of the control parameter.

Figures 5 and 6 illustrate $S_{\text{nor,max}}$ versus $k$ for a system with $T = 5, 10, 20, 30$ for CSMA and CSMA/CD ($R = 0.2T$) respectively. These figures show that for each fixed value of $T$ and at small values of $k$, $S_{\text{nor,max}}$ is an increasing function of $k$. As $k$
Fig. 3. Normalized throughput $S_{\text{nor}}$ versus control parameter $\alpha$ for $k = 0.6, 5$ with infinite population and $T = 20$ for CSMA and CSMA/CD($R = 2$) protocols.

Fig. 4. Normalized throughput $S_{\text{nor}}$ versus control parameter $\alpha$ for $k = 5$ with infinite population and $T = 10, 20$ for CSMA and CSMA/CD($R = 2$) protocols.
Fig. 5. The normalized throughput $S_{\text{nor,max}}$ versus bandwidth allocation coefficient $k$ with infinite population and $T = 5, 10, 20, 30$ for CSMA protocol.

Fig. 6. The normalized throughput $S_{\text{nor,max}}$ versus bandwidth allocation coefficient $k$ with infinite population and $T = 5, 10, 20, 30$ for CSMA/CD protocol with $R = 0.2T$. 
Fig. 7. Average throughput from transmission channel, average throughput from retransmission channel and average normalized throughput versus bandwidth allocation coefficient, with infinite population and $T = 10$ for CSMA protocol.

increases there is an optimal $k_{opt}$ that corresponds to maximum $\lambda_{max}$ and $S_{nor,max}$. But at higher values of $k$, $S_{nor,max}$ decreases slightly approaching saturation. The explanation comes from Eq. (31) which says that as $k$ tends to infinity,

$$S_{nor,max} \rightarrow S_{b}(k, \alpha_{opt})$$  \hspace{1cm} (32)$$

and

$$T_{b} \rightarrow T.$$  \hspace{1cm} (33)

From Figures 5 and 6 it is clear that there is small difference between $S_{nor,max}$ corresponding to $k_{opt}$ and $k > 5$ corresponding to saturation. For this reason we can accept any reasonable value of $k$ as optimum for the optimal bandwidth allocation.

Numerical results in Figures 7 and 8 illustrate the comparison of $S_{f}$, $S_{b}$ and $S_{nor}$ versus $k$ with $T = 10$ for CSMA and CSMA/CD protocol ($R = 0.2T$) correspondingly. It can be observed that as $k$ increases $S_{f}$ and $S_{nor}$ increases while $S_{b}$ decreases. There is value of $k$ in which the three curves coincides. This value corresponds to $k_{opt}$ that gives the $\lambda_{max}$. The reason is that at large values of $k$, from equations (22), (23) and (32) we get

$$\frac{T_{f}\lambda_{max}}{k + 1} \approx S_{b}(k, \alpha_{opt}) \Rightarrow T_{f}\lambda_{max} - kS_{b}(k, \alpha_{opt}) \approx S_{b}(k, \alpha_{opt}).$$  \hspace{1cm} (34)$$

Taking into account Eqs (27), (28) and (32) the above relation can be rewritten as

$$S_{f} \approx S_{b}(k, opt) \approx S_{nor}(max).$$  \hspace{1cm} (35)$$
Fig. 8. Average throughput from transmission channel, average throughput from retransmission channel and average normalized throughput versus bandwidth allocation coefficient, with infinite population and $T = 10$ for CSMA/CD protocol with $R = 0.2T$.

Fig. 9. Normalized throughput $S_{nor}$ versus bandwidth allocation coefficient $k$ for CSMA and CSMA/CD protocols for $T = 20$ and $R = 0.2T, 0.4T, 0.6T$ with infinite population.
The optimal behaviour of the examined system for various values of $T$ is summarised in the following Table 1 and Table 2, where the throughput $S_{\text{sin}}$ of corresponding single channel case is also listed. Numerical results from Table 1 shows the advantage of the proposed CSMA protocol as it compared with single channel system with the same capacity. It can be observed that normalised throughput performance is enhanced with 18.62% for $T = 5$ and 10.47% for $T = 10$. Comparison with SSCSMA protocol [7] shows the superiority of the proposed protocol. We take improvements of 21.6% in normalised throughput for $T=20$ in a two channel system and 59.20% for $T = 30$ in a three channel system with one transmission channel and two retransmission channels. Comparison of the numerical results from Table 2 presents the superiority of the proposed CSMA/CD protocol to the single channel system with the same capacity. Improvements in normalised throughput performance 17.98% for $T = 5$ and 8.96 % for $T = 10$ is achieved.

**Table 1. Maximum normalized throughput $S_{\text{nor, max}}$, $\lambda_{\text{max}}$, optimal control parameter $\alpha_{\text{opt}}$ and optimal bandwidth allocation coefficient $k_{\text{opt}}$ for a stable system and $S_{\text{sin}}$ of the single channel system in case of CSMA protocol.**

<table>
<thead>
<tr>
<th>$T$</th>
<th>$K_{\text{opt}}$</th>
<th>$\lambda_{\text{max}}$</th>
<th>$\alpha_{\text{opt}}$</th>
<th>$S_{\text{nor, max}}$</th>
<th>$S_{\text{sin}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.3</td>
<td>0.10720</td>
<td>0.4068</td>
<td>0.5362</td>
<td>0.4520</td>
</tr>
<tr>
<td>10</td>
<td>2.9</td>
<td>0.06451</td>
<td>0.3226</td>
<td>0.6415</td>
<td>0.5807</td>
</tr>
<tr>
<td>20</td>
<td>4.0</td>
<td>0.03648</td>
<td>0.2501</td>
<td>0.7295</td>
<td>0.6906</td>
</tr>
<tr>
<td>30</td>
<td>4.7</td>
<td>0.02571</td>
<td>0.2125</td>
<td>0.7712</td>
<td>0.7439</td>
</tr>
</tbody>
</table>

**Table 2. Maximum normalized throughput $S_{\text{nor, max}}$, $\lambda_{\text{max}}$, optimal control parameter $\alpha_{\text{opt}}$ and optimal bandwidth allocation coefficient $k_{\text{opt}}$ for a stable system and $S_{\text{sin}}$ of the single channel system, in case of CSMA/CD protocol with $R = 0.2T$.**

<table>
<thead>
<tr>
<th>$T$</th>
<th>$K_{\text{opt}}$</th>
<th>$\lambda_{\text{max}}$</th>
<th>$\alpha_{\text{opt}}$</th>
<th>$S_{\text{nor, max}}$</th>
<th>$S_{\text{sin}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.35</td>
<td>0.12084</td>
<td>0.65302</td>
<td>0.60419</td>
<td>0.51209</td>
</tr>
<tr>
<td>10</td>
<td>3.35</td>
<td>0.07195</td>
<td>0.58013</td>
<td>0.71950</td>
<td>0.66028</td>
</tr>
<tr>
<td>20</td>
<td>4.60</td>
<td>0.04039</td>
<td>0.48893</td>
<td>0.80777</td>
<td>0.77569</td>
</tr>
<tr>
<td>30</td>
<td>6.00</td>
<td>0.02824</td>
<td>0.43187</td>
<td>0.84722</td>
<td>0.82597</td>
</tr>
</tbody>
</table>

An alternative view is obtained from Figure 9 where $S_{\text{nor(max)}}$ is depicted as function of $R = 0.2T, 0.4T, 0.6T$ for CSMA/CD and for CSMA protocol with $T = 20$ minislots. Examination of the curves and comparison with Tables 1 and 2 show the improvement that CSMA/CD offers over CSMA in $S_{\text{nor(max)}}$.

6. CONCLUSIONS

The objective of this paper is the stability and optimization of the throughput performance of random multiple access high speed communication systems for non persistent CSMA and CSMA/CD protocols in conjunction with optimal bandwidth
allocation among two channels. The stability and optimization problem is considered for an infinite population with Poisson packet arrivals for a detailed study of the throughput behaviour. The stability and optimization are given as a function of the retransmission control parameter, \( \alpha \), and the bandwidth allocation coefficient, \( k \). The choice of retransmission probabilities as in equation (1) requires the estimation of the current backlogged station number. The proper choice of the above parameters optimizes the system behaviour in terms of the total capacity utilization. Numerical results show that the division of the total bandwidth into two channels results in a better performance than the single channel approach and that the corresponding optimal bandwidth allocation is rather insensitive to the bandwidth allocation coefficient \( k \) for \( k > 5 \). Comparison with the corresponding SSCSMA model in [7] demonstrates the throughput performance superiority of the proposed protocol. Also variation of \( \alpha \) in a wide range of values gives robustness in the stability and manoeuvrability to load demands.

(Received October 5, 1998.)

REFERENCES


Dr. Ioannis E. Pountourakis, Assistant Professor, National Technical University of Athens, Department of Electrical and Computer Engineering, Division of Computer Science, 157 73 Zographou, Athens, Greece.

e-mail: ipount@cs.ntua.gr