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FINITE-TIME COOPERATIVE TRACKING CONTROL FOR A CLASS OF SECOND-ORDER NONLINEAR MULTI-AGENT SYSTEMS

Haibo Du, Yigang He and Yingying Cheng

The problem of finite-time cooperative tracking control for a class of second-order nonlinear multi-agent systems is studied in this paper. The agent dynamic is described by a second-order nonlinear system with uncertain time-varying control coefficients and unknown nonlinear perturbations. Based on the finite-time control technique and graph theory, a class of distributed finite-time control laws are proposed which are only based on the neighbors' information. Under the proposed controller, it is shown that the states of all the agents can reach consensus in a finite time and the final consensus state is the desired signal. As an application of the proposed theoretic results, the problem of distributed finite-time attitude cooperative control for the roll channels of multiple bank-to-turn (BTT) missiles is solved. Simulation results are given to demonstrate the effectiveness of the proposed method.

Keywords: finite-time control, multi-agent systems, nonlinear system, bank-to-turn missiles

Classification: 93A14, 93C10,93D15, 93D21

1. INTRODUCTION

Distributed cooperative control of multi-agent systems has attracted more and more attention in recent years since it has broad applications in many areas, e. g., formation control [7, 9, 26], attitude alignment [4, 27], cooperative attack of multiple missiles [16], flocking [23], rendezvous [5, 30], data fusion [31], etc. As a basic issue in distributed control theory, the consensus problem of multi-agent systems requires that the states of all the agents converge to a common desired value by using an appropriate consensus protocol [22]. The common desired value is usually called the consensus state. Obviously, the consensus state and the convergence rate are crucial for the study of consensus problem.

Usually, the consensus state can be regarded as the state of a leader, such as the food source, rendezvous position, desired attitude, formation trajectory. The leader is usually independent of the followers, but has influence on the followers' behaviors. Then, the control objective of a group of agents is to design a distributed tracking control law for each follower to track the leader, i. e. consensus tracking for leader-follower multi-agent systems. Recently, many valuable results about this topic have been obtained. In [12], the consensus problem with a stationary leader under switching interconnection topologies was studied. When there exist multiple stationary leader spacecrafts, the co-operative attitude control algorithms were proposed in [4]. To guarantee all the follower can track a dynamic leader, the consensus control algorithms were designed for first-order and second-order multi-agent systems in [13, 14] respectively, where it is assumed that the leader's control input is available to each follower. For the case with communication delay, in [10] the consensus tracking problem for leader-follower multi-agent systems was also considered. Considering the fact only some of the followers can get the information of the dynamic leader, the consensus tracking problem were discussed in [28, 29]. In [7], distributed cooperative tracking control algorithms were proposed for multiple nonholonomic mobile robots, where the desired formation trajectory is regarded as the state of a virtual leader.

Another important topic in the study of consensus problem is the convergence rate. Nevertheless, most of the existing consensus algorithms for multi-agent systems are asymptotically stable, which means that the convergence rate is at best exponential with infinite settling time. In other words, the states can not reach consensus in a finite time. In order to enhance the convergence rate, recently the finite-time control technique [1, 21, 24] has been employed, which can guarantee the consensus is reached in a finite time, i. e. finite-time consensus. Meanwhile, the finite-time control can offer other advantages such as better precision, robustness to uncertainties and disturbances [1, 6, 18, 19, 20, 35, 37]. In [3, 15, 33, 34], for first-order linear multi-agent systems, several finite-time consensus algorithms were proposed. For the second-order case, some second-order finite-time consensus algorithms were also designed in [2, 17, 18, 36].

Note that the previous listed finite-time consensus algorithms are only applicable to linear multi-agent systems. This certainly limits the application of finite-time consensus algorithm in the control practice where exist many nonlinear systems with uncertain time-varying control coefficients and unknown nonlinear perturbations. For example, to design a distributed finite-time consensus algorithm for the roll angles of multiple (bank-to-turn) BTT missile (see Section 4), uncertain time-varying aerodynamic parameters will need to be addressed. To this end, this paper will aim to develop a finite-time consensus algorithm for this kind of second-order nonlinear multi-agent systems. The design procedure is divided into two steps. First, the technique of adding a power integrator [25] is employed to construct a finite-time consensus algorithm under the condition that the leader's velocity is available for all agents. Then, by using a distributed finite-time convergent observer, each agent will estimate the desired velocity in a finite time. Finally, substituting the estimate value into the previous proposed finite-time controller leads to the final distributed finite-time tracking controller. Rigorous theoretic analysis shows that under the proposed controller the consensus can be achieved in a finite time and the final consensus state is the desired signal.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Notations

Throughout of this paper, let $P > (<)\ 0$ denote a symmetric positive definite (negative definite) matrix. For any matrix P, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively.

2.2. Graph theory

Without loss of generality, assume that there exist n agents and the agent indexes belong to a finite index set $\Gamma = \{1, \ldots, n\}$. Each agent is regarded as a node and the information exchange among n agents is denoted by a directed graph $G(A) = \{V, E, A\}$. $V = \{v_i, i = 1, \ldots, n\}$ is the set of nodes, $E \subseteq V \times V$ is the set of edges and $A = [a_{ij}] \in R^{n \times n}$ is the weighted adjacency matrix of the graph $G(A)$ with non-negative adjacency elements a_{ij}. If there is an edge from agent j to agent i, i. e., $(v_j, v_i) \in E$, then $a_{ij} > 0$, which means there exists an available information channel from agent j to agent i. Moreover, we assume that $a_{ii} = 0$ for all $i \in \Gamma$. The set of neighbors of agent i is denoted by $N_i = \{j : (v_j, v_i) \in E\}$. The out-degree of node v_i is defined as $\deg_{\text{out}}(v_i) = d_i = \sum_{j=1}^{n} a_{ij} = \sum_{j \in N_i} a_{ij}$. Then the degree matrix of digraph G is $D = \text{diag}\{d_1, \ldots, d_n\}$ and the Laplacian matrix of digraph G is $L = D - A$.

A path in the directed graph $G(A)$ from node v_{i_1} to node v_{i_k} is a sequence of $v_{i_1}, v_{i_2}, \ldots, v_{i_k}$ of finite nodes starting with v_{i_1} and ending with v_{i_k} such that $(v_{i_l}, v_{i_{l+1}}) \in E$ for $l = 1, 2, \ldots, k-1$. The graph G is undirected means that $(v_i, v_j) \in E \Leftrightarrow (v_j, v_i) \in E$. The graph G is undirected connected if the graph G is undirected and there is a path between any two vertices.

2.3. Problem formulation

Without loss of generality, we consider n agents which are described by

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= a_i(t)u_i + f(x_i, v_i) + d_i(t), \quad i \in \Gamma, \end{aligned} \tag{1}$$

where x_i and v_i are the ith agent's position and velocity, u_i is the control law to be designed, $a_i(t)$ is uncertain time-varying parameter, $f(x_i, v_i)$ is unknown nonlinear perturbation, and $d_i(t)$ is time-varying external disturbance.

The control objective is to design a distributed control law which is only based on the neighbors' information such that all the agents' states reach consensus in a finite time and the consensus state is desired state. Let $x_d(t)$ and $v_d(t)$ denote the desired position and velocity and assume $\dot{v}_d(t)$ is bounded, i. e. $|\dot{v}_d(t)| \le L_1 < +\infty$, where L_1 is a known constant. As in [28], we assume that the desired state is represented by a virtual leader and only subset of the agents can obtain the leader's information. For the convenience of description of connection between the follower and the leader, define $b_i \ge 0, i \in \Gamma$ as follows:

$$b_i = \begin{cases} > 0, & \text{the } i\text{th agent has access to the leader's information;} \\ = 0, & \text{otherwise.} \end{cases}$$

Denote $B = \text{diag}\{b_1, \ldots, b_n\}$.

In addition, we need to impose the following assumption conditions for the considered system:

Assumption 2.1. For any $i \in \Gamma$, there are known positive constants $\underline{a}_i, \overline{a}_i, L_2$ and function $F_i(x_i, v_i)$ such that

$$\begin{aligned}&(i) \quad \underline{a}_i \le a_i(t) \le \overline{a}_i,\\ &(ii) \quad |f_i(x_i, v_i)| \le F(x_i, v_i)\\ &(iii) \quad |d_i(t)| \le L_2.\end{aligned}$$

Remark 2.2. It should be pointed out that the considered system (1) in this paper is more general than the double integrators model which is commonly studied in the literature, see for example [18]. First, in practice, the system parameters might not be precisely known due to the lack of detailed knowledge of system specifications. In some cases, the parameters are even varying caused by various reasons such as mechanical wears, model errors, environments changes, etc. A specific example is that the BTT missile model [8, 32] has time-varying aerodynamic parameters which is shown in Example part. Second, there usually are unknown nonlinear perturbations and external disturbance in the control channel, e. g. the model of BTT missile. Hence in this paper we consider the systems with unknown parameters and perturbations to encompass more practical systems included BTT missile in applications. In addition, although the problem of finite-time consensus tracking control for multi-agent systems with double-integrators has been solved in [18], the proposed method is not available here for system (1) due to the unknown parameters and perturbations.

About the communication topology, here as in [13, 18], we give the following assumptions.

Assumption 2.3. For the multi-agent system (1),

- the communication topology G is undirected connected;
- there is at least one agent that has access to the desired state, i. e. $B \neq 0$.

Moreover, a number of Lemmas are further used, which are placed in Appendix A.

3. MAIN RESULT

In this section, a distributed finite-time cooperative control law will be constructed for each agent to track the desired state in a finite time. To solve this problem, we divided the design procedure into two steps. First, we assume that the desired velocity v_d is available to each agent and then a finite-time control law is proposed. Second, to remove the previous assumption, a distributed finite-time convergent observer is designed to estimate the desired velocity v_d in a finite time. Now, we are in a position to present our first main result.

Theorem 3.1. For the nonlinear multi-agent systems (1) under Assumptions 2.1 − 2.3, if the desired velocity v_d is available to each agent and u_i is designed as

$$\begin{aligned}u_i &= -\frac{k_1}{\underline{a}_i} s_i^{2/p-1} - \frac{F(x_i, v_i) + L_1 + L_2}{\underline{a}_i} \operatorname{sign}(s_i),\\ s_i &= (v_i - v_d)^p + k_2^p \Big(\sum_{j \in N_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0) \Big), \qquad i \in \Gamma, \qquad (2)\end{aligned}$$

where k_3 is an arbitrary positive constant,

$$k_2 \geq \frac{p2^{1-1/p}}{1+p} + \frac{(\beta + n\gamma)}{1+p} + k_3,$$
$$k_1 \geq (2-1/p)2^{1-1/p}k_2^{1+p}\left(\frac{2^{1-1/p} + (\beta+n\gamma)p}{1+p} + \frac{(\beta+n\gamma)2^{1-1/p}}{k_2} + k_3\right),$$

$\beta = \max\limits_{\forall i \in \Gamma}\{b_i\} + \max\limits_{\forall i \in \Gamma}\{\sum\limits_{j \in N_i} a_{ij}\}$, $\gamma = \max\limits_{\forall i,j \in \Gamma}\{a_{ij}\}$, $1 < p = p_1/p_2 < 2$, p_1, p_2 are positive odd integers, then each agent can track the desired position in a finite time, i. e., $x_i(t) \to x_d$ in a finite time.

P r o o f. Define $\overline{x}_i = x_i - x_d, \overline{v}_i = v_i - v_d, i \in \Gamma$ as the tracking error, then it follows from system (1) that

$$\dot{\overline{x}}_i = \overline{v}_i, \dot{\overline{v}}_i = a_i(t)u_i + f(x_i, v_i) + d_i(t) - \dot{v}_d, \quad i \in \Gamma. \tag{3}$$

The proof procedure is divided into two steps. First, a virtual velocity $\overline{v}_i$ is designed such that each agent can track the desired position in a finite time. Then, the control law is designed such that the velocity can track the virtual velocity in a finite time.

Step 1: Virtual velocity design

Denote $\overline{x} = [\overline{x}_1, \ldots, \overline{x}_n]$. Based on Lemma A.5, construct a Lyapunov candidate

$$V_1 = \frac{1}{2}\overline{x}^T L \overline{x} + \frac{1}{2}\overline{x}^T B \overline{x} = \frac{1}{4}\sum_{i=1}^{n}\left(\sum_{j \in N_i} a_{ij}(\overline{x}_i - \overline{x}_j)^2 + 2b_i\overline{x}_i^2\right) \tag{4}$$

which yields

$$\dot{V}_1 = \sum_{i=1}^{n} \dot{\overline{x}}_i \left(\sum_{j \in N_i} a_{ij}(\overline{x}_i - \overline{x}_j) + b_i\overline{x}_i\right) = \sum_{i=1}^{n}\left(\sum_{j \in N_i} a_{ij}(\overline{x}_i - \overline{x}_j) + b_i\overline{x}_i\right)\overline{v}_i. \tag{5}$$

For the sake of brevity, let $\xi_i = \sum_{j \in N_i} a_{ij}(\overline{x}_i - \overline{x}_j) + b_i\overline{x}_i$. If take the velocity $\overline{v}_i^* = -k_2\xi_i^{1/p}$ as the virtual input, we obtain

$$\dot{V}_1 = \sum_{i=1}^{n}\xi_i\overline{v}_i = \sum_{i=1}^{n}\xi_i\overline{v}_i^* + \sum_{i=1}^{n}\xi_i(\overline{v}_i - \overline{v}_i^*) = -k_2\sum_{i=1}^{n}\xi_i^{1+1/p} + \sum_{i=1}^{n}\xi_i(\overline{v}_i - \overline{v}_i^*). \tag{6}$$

Step 2: Control law design

In this step, a control law is designed such that virtual velocity $\overline{v}_i^*$ is tracked by real velocity $\overline{v}_i$ in a finite time. The candidate Lyapunov function is chosen as

$$V = V_1 + \sum_{i=1}^{n} W_i, \tag{7}$$

where V_1 is the same as that in *Step 1*, and

$$W_i = \frac{1}{(2-1/p)2^{1-1/p}k_2^{1+p}}\int_{\overline{v}_i^*}^{\overline{v}_i}(\mu^p - \overline{v}_i^{*p})^{2-1/p}\mathrm{d}\mu, \quad \overline{v}_i^* = -k_2\xi_i^{1/p}, \quad i \in \Gamma. \tag{8}$$

From Propositions B1 and B2 in [25], we know that function W_i is differentiable, positive definite and proper.

First of all, we estimate the second term in (6). By Lemma A.2, we obtain

$$\xi_i(\overline{v}_i - \overline{v}_i^*) \leq |\xi_i| \cdot |(\overline{v}_i^p)^{1/p} - (\overline{v}_i^{*p})^{1/p}| \leq 2^{1-1/p}|\xi_i| \cdot |\overline{v}_i^p - \overline{v}_i^{*p}|^{1/p}. \tag{9}$$

For the convenience of statement, denote $s_i = \overline{v}_i^p - \overline{v}_i^{*p}, i \in \Gamma$, and $d = 1 + 1/p$. As a consequence, it follows from Lemma A.3 that

$$\xi_i(\overline{v}_i - \overline{v}_i^*) \leq 2^{1-1/p}|\xi_i| \cdot |s_i|^{1/p} \leq 2^{1-1/p}\left(\frac{p|\xi_i|^d}{1+p} + \frac{|s_i|^d}{1+p}\right). \tag{10}$$

Substituting (10) into (6) results in

$$\dot{V}_1 \leq -k_2\sum_{i=1}^n \xi_i^d + 2^{1-1/p}\frac{p}{1+p}\sum_{i=1}^n |\xi_i|^d + 2^{1-1/p}\frac{1}{1+p}\sum_{i=1}^n |s_i|^d. \tag{11}$$

The derivative of W_i along system (3) is

$$\begin{aligned}\dot{W}_i = &-\frac{1}{2^{1-1/p}k_2^{1+p}}\frac{\mathrm{d}\overline{v}_i^{*p}}{\mathrm{d}t}\int_{\overline{v}_i^*}^{\overline{v}_i}(\mu^p - \overline{v}_i^{*p})^{1-1/p}\mathrm{d}\mu \\ &+ \frac{1}{(2-1/p)2^{1-1/p}k_2^{1+p}}s_i^{2-1/p}\Big(a_i(t)u_i + f(x_i, v_i) + d_i(t) - \dot{v}_d\Big).\end{aligned} \tag{12}$$

Based on the definitions of β and γ, we obtain

$$\begin{aligned}\frac{\mathrm{d}v_i^{*p}}{\mathrm{d}t} &= -k_2^p\frac{\mathrm{d}\Big(\sum\limits_{j\in N_i} a_{ij}(\overline{x}_i - \overline{x}_j) + b_i\overline{x}_i\Big)}{\mathrm{d}t} = -k_2^p\left(\sum_{j\in N_i} a_{ij}(v_i - v_j) + b_iv_i\right) \\ &\leq k_2^p\left(\beta|v_i| + \gamma\sum_{m=1}^n |v_m|\right).\end{aligned} \tag{13}$$

With the help of this inequality, the estimate for the first term in (12) is given as

$$\begin{aligned}&-\frac{1}{2^{1-1/p}k_2^{1+p}}\frac{\mathrm{d}\overline{v}_i^{*p}}{\mathrm{d}t}\int_{\overline{v}_i^*}^{\overline{v}_i}(\mu^p - \overline{v}_i^{*p})^{1-1/p}\mathrm{d}\mu \\ &\leq \frac{1}{2^{1-1/p}k_2}\left(\beta|\overline{v}_i| + \gamma\sum_{m=1}^n|\overline{v}_m|\right)\left|\int_{\overline{v}_i^*}^{\overline{v}_i}(\mu^p - \overline{v}_i^{*p})^{1-1/p}\mathrm{d}\mu\right| \\ &\leq \frac{1}{2^{1-1/p}k_2}\left(\beta|\overline{v}_i| + \gamma\sum_{m=1}^n|\overline{v}_m|\right)|\overline{v}_i - \overline{v}_i^*||s_i|^{1-1/p}.\end{aligned} \tag{14}$$

Using Lemma A.2, we have for any $i, m \in \Gamma$

$$\begin{aligned}|\overline{v}_m||\overline{v}_i - \overline{v}_i^*||s_i|^{1-1/p} &= |\overline{v}_m||(\overline{v}_i^p)^{1/p} - (\overline{v}_i^{*p})^{1/p}||s_i|^{1-1/p} \\ &\leq 2^{1-1/p}(|s_i||\overline{v}_m - \overline{v}_m^*| + |s_i||\overline{v}_m^*|).\end{aligned} \tag{15}$$

Based on this inequality, it follows from Lemma A.2 and Lemma A.3 that

$$|s_i||\overline{v}_m - \overline{v}_m^*| \le 2^{1-1/p}|s_i||s_m|^{1/p} \le 2^{1-1/p}\left(\frac{p}{1+p}|s_i|^d + \frac{1}{1+p}|s_m|^d\right), \tag{16}$$

$$|s_i||\overline{v}_m^*| = k_2|s_i||\xi_m|^{1/p} \le k_2\left(\frac{p}{1+p}|s_i|^d + \frac{1}{1+p}|\xi_m|^d\right). \tag{17}$$

Substituting (16) and (17) into (15) results into

$$|\overline{v}_m||\overline{v}_i - \overline{v}_i^*||s_i|^{1-1/p} \le 2^{1-1/p}\left((2^{1-1/p}+k_2)\frac{p}{1+p}|s_i|^d + \frac{2^{1-1/p}}{1+p}|s_m|^d + \frac{k_2}{1+p}|\xi_m|^d\right). \tag{18}$$

With the help of (18), we have

$$\begin{aligned}
&\Big(\beta|\overline{v}_i| + \gamma\sum_{m=1}^{n}|\overline{v}_m|\Big)|\overline{v}_i - \overline{v}_i^*||s_i|^{1-1/p} \\
&= \beta|\overline{v}_i||\overline{v}_i - \overline{v}_i^*||s_i|^{1-1/p} + \gamma\sum_{m=1}^{n}|\overline{v}_m||\overline{v}_i - \overline{v}_i^*||s_i|^{1-1/p} \\
&\le 2^{1-1/p}\left(k_4|s_i|^d + \frac{\beta k_2}{1+p}|q_i|^d + \frac{\gamma 2^{1-1/p}}{1+p}\sum_{m=1}^{n}|s_m|^d + \frac{\gamma k_2}{1+p}\sum_{m=1}^{n}|\xi_m|^d\right),
\end{aligned} \tag{19}$$

where $k_4 = (\beta + n\gamma)(2^{1-1/p}+k_2)\frac{p}{1+p} + \frac{\beta 2^{1-1/p}}{1+p}$. Based on (12), (14) and (19), we get

$$\begin{aligned}
\dot{W}_i \le& \frac{k_4}{k_2}|s_i|^d + \frac{\beta}{1+p}|\xi_i|^d + \frac{\gamma 2^{1-1/p}}{k_2(1+p)}\sum_{m=1}^{n}|s_m|^d + \frac{\gamma}{1+p}\sum_{m=1}^{n}|\xi_m|^d \\
&+ \frac{1}{(2-1/p)2^{1-1/p}k_2^{1+p}}s_i^{2-1/p}\Big(a_i(t)u_i + f(x_i,v_i) + d_i(t) - \dot{v}_d\Big).
\end{aligned} \tag{20}$$

Putting together (11) and (20), and noticing that $|\xi_i|^d = |\xi_i|^{\frac{1+p}{p}} = |\xi_i|^{\frac{p_1+p_2}{p_1}} = \xi_i^{\frac{p_1+p_2}{p_1}} = \xi_i^d$, also $|s_i|^d = s_i^d$, we get

$$\begin{aligned}
\dot{V} =& \dot{V}_1 + \sum_{i=1}^{n}\dot{W}_i \\
\le& -\left(k_2 - \frac{p2^{1-1/p}}{1+p} - \frac{\beta+n\gamma}{1+p}\right)\sum_{i=1}^{n}\xi_i^d + \left(\frac{2^{1-1/p}}{1+p} + \frac{k_4}{k_2} + \frac{n\gamma 2^{1-1/p}}{k_2(1+p)}\right)\sum_{i=1}^{n}s_i^d \\
&+ \frac{1}{(2-1/p)2^{1-1/p}k_2^{1+p}}\sum_{i=1}^{n}s_i^{2-1/p}\Big(a_i(t)u_i + f(x_i,v_i) + d_i(t) - \dot{v}_d\Big).
\end{aligned} \tag{21}$$

Based on the definition of k_4, a straightforward calculation leads to

$$\frac{2^{1-1/p}}{1+p} + \frac{k_4}{k_2} + \frac{n\gamma 2^{1-1/p}}{k_2(1+p)} = \frac{2^{1-1/p} + (\beta+n\gamma)p}{1+p} + \frac{(\beta+n\gamma)2^{1-1/p}}{k_2}. \tag{22}$$

Select the control gains k_1, k_2 and k_3 as

$$k_3 > 0, k_2 \geq \frac{p2^{1-1/p}}{1+p} + \frac{(\beta + n\gamma)}{1+p} + k_3,$$
$$k_1 \geq (2 - 1/p)2^{1-1/p}k_2^{1+p}\left(\frac{2^{1-1/p} + (\beta + n\gamma)p}{1+p} + \frac{(\beta + n\gamma)2^{1-1/p}}{k_2} + k_3\right) \tag{23}$$

and u_i is designed as

$$u_i = -\frac{k_1}{\underline{a}_i}s_i^{2/p-1} - \frac{F(x_i, v_i) + L_1 + L_2}{\underline{a}_i}\text{sign}(s_i), \qquad i \in \Gamma. \tag{24}$$

Under Assumption 2.1, substituting the control law (24) into (21) results in

$$\begin{aligned}
\dot{V} \leq & - k_3 \sum_{i=1}^{n} \xi_i^d - k_3 \sum_{i=1}^{n} s_i^d \\
& + \frac{1}{(2 - 1/p)2^{1-1/p}k_2^{1+p}} \sum_{i=1}^{n} \left(|s_i|^{2-1/p}[|f(x_i, v_i)| + |d_i(t)| + |\dot{v}_d|]\right) \\
& - \frac{1}{(2 - 1/p)2^{1-1/p}k_2^{1+p}} \sum_{i=1}^{n} \left(|s_i|^{2-1/p}(F(x_i, v_i) + L_1 + L_2)\right) \\
\leq & - k_3 \sum_{i=1}^{n} \xi_i^d - k_3 \sum_{i=1}^{n} s_i^d.
\end{aligned} \tag{25}$$

Next, we will show that $V(t)$ will reach zero in finite time. First of all, by Lemma A.6, we know $L + B > 0$, which implies that $\lambda_{\min}(L + B) > 0$. Meanwhile, since

$$\begin{aligned}
(L + B)\overline{x} = & \begin{pmatrix}
\sum\limits_{j \in N_1} a_{1j} + b_1 & -a_{12} & \dots & -a_{1n} \\
-a_{21} & \sum\limits_{j \in N_2} a_{2j} + b_2 & \dots & -a_{2n} \\
\vdots & \vdots & & \vdots \\
-a_{n1} & -a_{n2} & \dots & \sum\limits_{j \in N_n} a_{nj} + b_n
\end{pmatrix} \overline{x} \\
= & \left[\sum_{j \in N_1} a_{1j}(\overline{x}_1 - \overline{x}_j) + b_1\overline{x}_1, \dots, \sum_{j \in N_n} a_{nj}(\overline{x}_n - \overline{x}_j) + b_n\overline{x}_n\right]^T \\
= & [\xi_1, \dots, \xi_n]^T,
\end{aligned} \tag{26}$$

then we have

$$\begin{aligned}
\sum_{i=1}^{n} \xi_i^2 = & ((L + B)\overline{x})^T(L + B)\overline{x} = \left((L + B)^{1/2}\overline{x}\right)^T (L + B) \left((L + B)^{1/2}\overline{x}\right) \\
\geq & c_1 \left((L + B)^{1/2}\overline{x}\right)^T \left((L + B)^{1/2}\overline{x}\right) = c_1\overline{x}^T(L + B)\overline{x} = 2c_1V_1,
\end{aligned} \tag{27}$$

where $c_1 = \lambda_{\min}(L + B)$. It implies that

$$V_1 \leq \frac{1}{2c_1} \sum_{i=1}^{n} \xi_i^2. \tag{28}$$

Secondly, from (8), by a calculation, we get

$$W_i \leq \frac{1}{(2-1/p)2^{1-1/p}k_2^{1+p}} |\overline{v}_i - \overline{v}_i^*||\overline{v}_i^p - \overline{v}_i^{*p}|^{2-1/p} = \frac{1}{(2-1/p)2^{1-1/p}k_2^{1+p}} |\overline{v}_i - \overline{v}_i^*||s_i|^{2-1/p}. \tag{29}$$

Using Lemma A.2 results in

$$W_i \leq \frac{1}{(2-1/p)2^{1-1/p}k_2^{1+p}} 2^{1-1/p}|s_i|^{1/p}|s_i|^{2-1/p} \leq \frac{1}{(2-1/p)k_2^{1+p}} s_i^2. \tag{30}$$

With this inequality in mind, it follows from (28) that

$$V = V_1 + \sum_{i=1}^{n} W_i \leq \frac{1}{2c_1} \sum_{i=1}^{n} \xi_i^2 + \frac{1}{(2-1/p)k_2^{1+p}} \sum_{i=1}^{n} s_i^2 \leq \lambda \left(\sum_{i=1}^{n} \xi_i^2 + \sum_{i=1}^{n} s_i^2 \right), \tag{31}$$

where $\lambda = \max\{\frac{1}{2c_1}, \frac{1}{(2-1/p)k_2^{1+p}}\}$. By Lemma A.4, we have

$$V^{d/2} \leq \lambda^{d/2} \left(\sum_{i=1}^{n} \xi_i^2 + \sum_{i=1}^{n} s_i^2 \right)^{d/2} \leq \lambda^{d/2} \left(\sum_{i=1}^{n} \xi_i^d + \sum_{i=1}^{n} s_i^d \right). \tag{32}$$

Denote $c_2 = \dfrac{k_3}{2\lambda^{d/2}} > 0$. Based on (25) and (32), we have

$$\dot{V} + c_2 V^{d/2} \leq -\frac{k_3}{2} \sum_{i=1}^{n} \xi_i^d - \frac{k_3}{2} \sum_{i=1}^{n} s_i^d \leq 0. \tag{33}$$

By Lemma A.1, it can be concluded that $V(t)$ converges to zero in a finite time, which implies that there exists a time $T \leq \frac{V(0)^{1-d/2}}{c_2(1-d/2)} < +\infty$, such that $V(t) \equiv 0, \forall t \geq T$, which means that $V_1(t) = 0$, $W_i(t) = 0$, $\forall t \geq T$. Based on the definition of V_1, and noticing $L + B > 0$, we can conclude that $x_i(t) = x_d(t), \forall i \in \Gamma, \forall t \geq T$. Thus, the proof is completed. □

Remark 3.2. It seems that the finite-time consensus tracking problem with a leader has been solved in Theorem 3.1. However, from the proposed controller (2), this control law is not distributed, i. e., each agent must use the information of the desired velocity $v_d(t)$. In the sequel, we will employ the technique of design of distributed finite-time convergent observers to solve the problem of distributed finite-time cooperative tracking control.

The distributed finite-time convergent observer is proposed as

$$\dot{\widehat{v}}_i = -\eta \mathrm{sign}\left(\sum_{j \in N_i} a_{ij}(\widehat{v}_i - \widehat{v}_j) + b_i(\widehat{v}_i - v_d) \right), \quad i \in \Gamma, \tag{34}$$

where $\widehat{v}_i$ is the estimate of the desired velocity for the ith agent, and $\eta > L_1$. Based on this observer, substituting the estimate value into (2), we obtain the final distributed finite-time consensus tracking controller, i. e., each agent only use the information from its neighbor and itself. Now, we present the main result of this paper.

Theorem 3.3. For the nonlinear multi-agent systems (1) under Assumptions 2.1 – 2.3, if u_i is designed as

$$\begin{aligned} u_i &= -\frac{k_1}{\underline{a}_i} s_i^{2/p-1} - \frac{F(x_i, v_i) + L_1 + L_2}{\underline{a}_i} \operatorname{sign}(s_i), \\ s_i &= (v_i - \hat{v}_i)^p + k_2^p \Big(\sum_{j \in N_i} a_{ij}(x_i - x_j) + b_i(x_i - x_d) \Big), \qquad i \in \Gamma, \end{aligned} \tag{35}$$

where $\hat{v}_i$ is given in (34) and the other parameters are the same as that of Theorem 3.1, then each agent can track the desired position in a finite time, i. e., $x_i(t) \to x_d$ in a finite time.

P r o o f. For the observer system (34), we first prove that $\widehat{v}_i \to v_d, i \in \Gamma$, in a finite time. Define $e_i = \widehat{v}_i - v_d, i \in \Gamma$, as the observer error, which leads to

$$\begin{aligned} \dot{e}_i =& -\eta \operatorname{sign} \left(\sum_{j \in N_i} a_{ij} [(\widehat{v}_i - v_d) - (\widehat{v}_j - v_d)] + b_i(\widehat{v}_i - v_d) \right) - \dot{v}_d \\ =& -\eta \operatorname{sign} \left(\sum_{j \in N_i} a_{ij}(e_i - e_j) + b_i e_i \right) - \dot{v}_d, \qquad i \in \Gamma. \end{aligned} \tag{36}$$

Define $\varepsilon_i = \sum_{j \in N_i} a_{ij}(e_i - e_j) + b_i e_i, i \in \Gamma$. Let $e = [e_1, \ldots, e_n]^T$ and $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_n]^T$. Based on the definitions of L and B, we know

$$(L + B)e = \varepsilon. \tag{37}$$

Choose Lyapunov candidate

$$V_e = \frac{1}{2} e^T (L + B) e. \tag{38}$$

Based on (36) and noticing that $|\dot{v}_d| \le L_1$, we get

$$\begin{aligned} \dot{V}_e =& e^T (L + B) \Big[\eta \Big(-\operatorname{sign}(\varepsilon_1), \ldots, -\operatorname{sign}(\varepsilon_n) \Big) - \Big(\dot{v}_d, \ldots, \dot{v}_d \Big) \Big] \\ \le& -(\eta - L_1) \sum_{i=1}^n |\varepsilon_i|. \end{aligned} \tag{39}$$

By Lemma A.4, we have

$$\Big(\sum_{i=1}^n |\varepsilon_i|^2 \Big)^{1/2} \le \sum_{i=1}^n |\varepsilon_i|. \tag{40}$$

Substituting (40) into (39) yields

$$\begin{aligned}\dot{V}_e \leq & -(\eta - L_1)\Big(\sum_{i=1}^{n} |\varepsilon_i|^2\Big)^{1/2} = -(\eta - L_1)\Big(\varepsilon^T(L+B)(L+B)\varepsilon\Big)^{1/2} \\ \leq & -(\eta - L_1)\sqrt{2\lambda_{\min}(L+B)}\Big(\frac{1}{2}\varepsilon^T(L+B)\varepsilon\Big)^{1/2} \\ \leq & -(\eta - L_1)\sqrt{2\lambda_{\min}(L+B)}V_e^{1/2}. \end{aligned} \tag{41}$$

Noticing that $\eta - L_1 > 0$, by Lemma A.1, it can be concluded that V_e converges to zero in a finite time, which implies that there exists a time T_e such that $V_e(t) = 0, \forall t \geq T_e$, which implies that $\widehat{v}_i - v_d = 0, \forall t \geq T^*$, $\forall i \in \Gamma$.

Hence, when $t \geq T^*$, the control law (35) will reduce to the control law (2). Based on the results of Theorem 3.1, we can complete the proof. □

4. FINITE-TIME COOPERATIVE TRACKING CONTROL FOR MULTIPLE BTT MISSILES

In this section, we will use one example to illustrate the efficiency of the proposed theoretic results in the above section. Consider the problem of distributed finite-time attitude cooperative control of roll channels of multiple bank-to-turn (BTT) missiles. Without loss of generality, we consider 4 BTT missiles and each missile is regarded as an agent. By [8, 32], the mathematical model for ith BTT missile is described as

$$\begin{aligned}\dot{\gamma}_i &= \omega_i \\ \dot{\omega}_i &= -a_i(t)\omega_i - c_i(t)\delta_i + d_i(t), \quad i = 1,2,3,4,\end{aligned} \tag{42}$$

where γ_i and ω_i are the roll angle and roll rate respectively, δ_i is roll control deflection angle, to be designed. Coefficients $a_i(t)$ and $c_i(t)$ are time-varying aerodynamic parameters of the missile systems. $d_i(t)$ is time-varying bounded external disturbance. By [8], we know that the parameters $a_i(t)$ and $c_i(t)$ are usually bounded and satisfy $a_i(t) \in [0.491, 1.673], c_i(t) \in [584.220, 3045.292]$, which means Assumption 2.1 holds. Assume the desired signal of the roll angle for the group missiles is $\gamma_d(t) = 10 - \sin(t)$. Figure 1 shows the communication topology graph among these missiles where $a_{12} = a_{13} = a_{34} = 1$ and $b_1 = 1$. It means that only the first agent can obtain the information of reference signal.

According to Theorem 3.3, we can design distributed finite-time cooperative tracking controller in the form of

$$\begin{aligned}\delta_i &= \frac{k_1}{\underline{c}_i}s_i^{2/p-1} + \frac{\overline{a}_i|\omega_i| + L_1 + L_2}{\underline{c}_i}\mathrm{sign}(s_i), \\ s_i &= (\omega_i - \hat{\omega}_i)^p + k_2^p\Big(\sum_{j\in N_i} a_{ij}(\gamma_i - \gamma_j) + b_i(\gamma_i - \gamma_d)\Big), \\ \dot{\widehat{\omega}}_i &= -\eta\mathrm{sign}\left(\sum_{j\in N_i} a_{ij}(\widehat{\omega}_i - \widehat{\omega}_j) + b_i(\widehat{\omega}_i - \dot{\gamma}_d)\right), \quad i = 1,\ldots,4,\end{aligned} \tag{43}$$

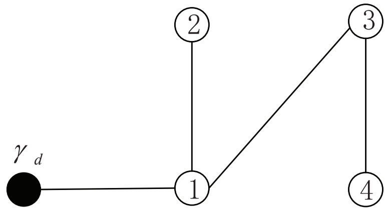

Fig. 1. The graph of communication topology for the multiple BTT missiles.

where k_1, k_2, η are appropriate gains.

In simulation, to simulate the time-varying aerodynamic parameters, the time-varying parameters are given as follows:

$$a_i(t) = \begin{cases} a_i(t_1), & 0 \le t \le t_1; \\ a_i(t_{j-1}) + \frac{a_i(t_j)-a_i(t_{j-1})}{t_j - t_{j-1}}(t - t_{j-1}), & t_{j-1} \le t \le t_j, \quad j = 2, \dots, 7, \end{cases} \tag{44}$$

where $a_i(t_i), c_i(t_i)$ are the parameters' value at different operation points given in Table 1.

Operating points	a_i	c_i
t_1(4.4s)	1.264	1787.048
t_2(11.7s)	1.600	1832.067
t_3(19.5s)	1.636	2128.877
t_4(23s)	1.635	2231.985
t_5(28s)	1.607	3045.292
t_6(35s)	0.936	1329.481
t_7(40s)	0.644	818.706

Tab. 1. Model parameters for different operation points [8].

In addition, as that in [18], the external disturbances are given as: $d_1(t) = 1.1\sin(8t - 1)$, $d_2(t) = -\cos(2t)$, $d_3(t) = 0.8\sin(t) + 0.7\cos(t)$, $d_4(t) = 0.2\cos(11t - 4)$, which implies L_2 can be selected as $L_2 = 1.1$. By a careful calculation, the controller gains can be chosen as $k_1 = 40$, $k_2 = 20$, $p = 7/5$ and $\eta = 2$. Under the proposed controller, the response curves of the closed-loop system are shown in Figures 2−4, respectively, where the initial conditions are chosen as $\gamma(0) = [0, 20, -3, 12.5]$, $\omega(0) = [-1, 0, 1, -2], \hat{\omega}(0) = [0, 0, 0, 0]$. It is easy to see that all the roll angles will reach consensus and the final consensus state is the desired angle.

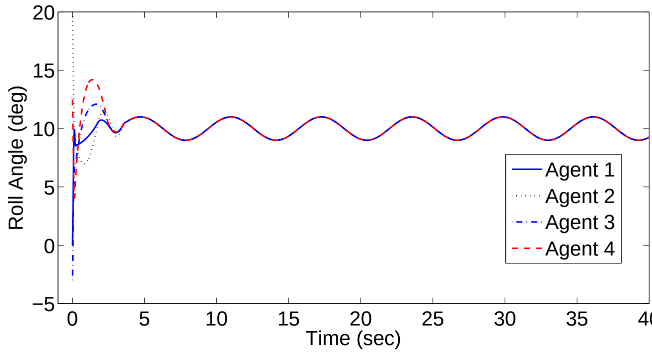

Fig. 2. The response curves of roll angle for the multiple BTT missiles under finite-time controller (43).

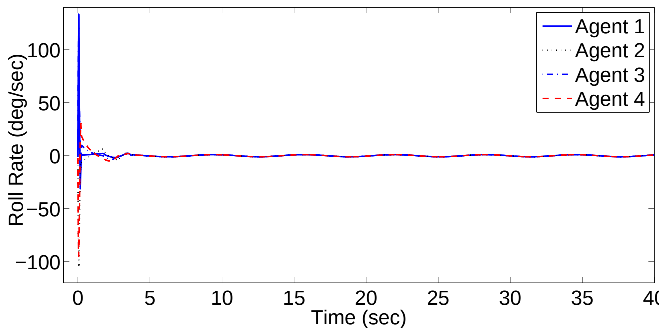

Fig. 3. The response curves of roll rate for the multiple BTT missiles under finite-time controller (43).

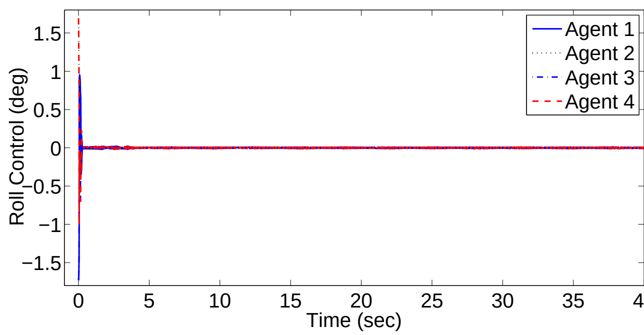

Fig. 4. The response curves of control input for the multiple BTT missiles under finite-time controller (43).

5. CONCLUSIONS

In this paper, we have studied the problem of distributed finite-time cooperative tracking for a class of second-order nonlinear multi-agent systems. Rigorous theoretic analysis shows that the proposed finite-time consensus algorithm can deal with the case with uncertain time-varying control coefficient and unknown nonlinear perturbations. Future works include how to extend the result of this paper to high-order nonlinear multi-agent systems.

APPENDIX A

This Appendix collects some useful lemmas.

Lemma A.1. (Bhat and Bernstein [1]) Consider system $\dot{x} = f(x), f(0) = 0, x \in R^n$, where $f(\cdot) : R^n \to R^n$ is a continuous vector function. Suppose there exists a continuous, positive definite function $V(x) : U \to R$ defined on an open neighborhood U of the origin such that $\dot{V}(x) + c(V(x))^\alpha \leq 0$ on U for some $c > 0$ and $\alpha \in (0, 1)$. Then the origin is a finite-time stable equilibrium of system $\dot{x} = f(x)$ and the finite settling time T satisfies $T \leq \frac{V(x(0))^{1-\alpha}}{c(1-\alpha)}$.

Lemma A.2. (Qian and Lin [25]) If $0 < p = p_1/p_2 \leq 1$, where $p_1 > 0, p_2 > 0$ are positive odd integers, then

$$|x^p - y^p| \leq 2^{1-p}|x - y|^p.$$

Lemma A.3. (Qian and Lin [25]) For $x \in R, y \in R$, $c > 0, d > 0$, then

$$|x|^c|y|^d \leq \frac{c}{c+d}|x|^{c+d} + \frac{d}{c+d}|y|^{c+d}.$$

Lemma A.4. (Hardy et al. [11]) For $x_i \in R, i = 1, \ldots, n$, and a real number $0 < p \leq 1$, then

$$(|x_1| + \ldots + |x_n|)^p \leq |x_1|^p + \ldots + |x_n|^p.$$

Lemma A.5. (Olfati-Saber and Murray [22]) For an undirected connected graph G, the corresponding Laplacian matrix L satisfies:
$x^T Lx = \frac{1}{2} \sum\limits_{i,j=1}^{n} a_{ij}(x_i - x_j)^2 = \frac{1}{2} \sum\limits_{i=1}^{n} \sum\limits_{j \in N_i} a_{ij}(x_i - x_j)^2$ for any $x = [x_1, \ldots, x_n]^T \in R^n$.

Lemma A.6. (Hong et al. [13]) For the multi-agent system (1), if Assumption 2.3 holds, then $L + B$ is positive definite, where L is the Laplacian matrix of graph G.

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