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Christian Doppler (1803 – 1853) and the Impact of Doppler Effect in Astronomy

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1 Biographical information about Christian Doppler (1803 – 1853)

Christian Doppler (1803 – 1853), born in Salzburg, visited the primary school in Salzburg and the Gymnasium in Linz.[1] From 1825 to 1829 Doppler studied mathematics, astronomy and mechanics at Vienna University. From 1829 to 1833 he was assistent at the Polytechnikum in Vienna (founded in 1815). He started his career in Prague in 1835 as teacher in a technical secondary school, then in 1841 he became professor for mathematics at the polytechnical institute. In 1835 Doppler taught mathematics and physics at the Technical School in Prague, then in 1841 he was offered the professorship for elementary mathematics and geometry at the Polytechnical Institute Institute in Prague (old building in Husova street). In 1843 he became a member of Royal Bohemian Society of Sciences in Prague. In 1847 Doppler was professor at the Bergakademie (mining university) in Schemnitz (Banská Štiavnica). In 1848 Doppler became a member of the Academy of Sciences in Vienna. In 1849 he was professor at the Polytechnikum in Vienna, from 1850 to 1852 director of the Physical Institute of Vienna University. Finally in 1853 he died in Venice.

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[1]For more biographical information about Doppler see [18].

In 1842 Christian Doppler (1803–1853) published a paper with the title ÜBER DAS FARBIGE LICHT DER DOPPELSTERNE UND EINIGER ANDERER GESTIRNE DES HIMMELS (Fig. 1) [3] suggesting a relation between the colours of the stars and

Ueber das

farbige Licht der Doppelsterne

und einiger anderer

Gestirne des Himmels.

Versuch einer das Bradley'sche Aberrations-Theorem als integrirenden Theil in sich schliessenden allgemeineren Theorie.

Von

Christian Doppler,

Professor der Mathematik und praktischen Geometrie am technischen Institute und ausserordentl. Mitglied der königl. böhm. Gesellschaft der Wissenschaften.

Fig. 1 Doppler, Christian: ÜBER DAS FARBIGE LICHT DER DOPPELSTERNE UND EINIGER ANDERER GESTIRNE DES HIMMELS (Prag 1842).

their velocities. This idea was wrong: nevertheless, his transfer of the idea to acoustics led to important results: Here he postulated that you hear a higher pitched sound if a source is approaching, if it is receding you hear a lower pitched sound. The theory can be proved in acoustics because a well trained ear is able to hear small changes in sound, about a quarter of a note. The acoustic »Doppler effect« was first checked with trumpet players on a locomotive by Christoph H. D. Buys-Ballot (1817–1890) in 1845. Thirty years later, the astronomer Hermann Carl Vogel (1841–1907) also started successful experiments on the railway from Cologne to Minden for testing the the Doppler theory by using the steam whistle as a stronger source of sound. [20] The test was carried out with a Borsig locomotive between Cologne and Minden in 1875. If the source of sound (locomotive with steam whistle) is at rest with respect to the observers A and B, both hear a sound of the same frequency (pitch). If the source of sound is moving away from observer A and towards observer B, A hears a lower pitched sound, and B a higher pitched sound. The higher the speed, the greater the change in pitch. This effect can also be transferred to optics. In optics the lower pitched sound correspond to a longer wavelength, and hence a shift towards the red end of the spectrum (»redshift«); the higher pitched sound corresponds to a shorter wavelength, and hence a shift towards the blue end of the spectrum (»blueshift«).

Concerning optics Doppler was convinced that not light sources but only astronomical objects might have velocities large enough to prove the effect. Doppler's theory was not accepted at all because his explanation of stellar colours was wrong: He supposed that the normal stellar colour is white indicating no motion. The different colours of the stars would then indicate different velocities. For testing this, Benedetto Sestini (1816–1890) compiled a catalogue with colours of stars already in 1847. [17]

The application of the Doppler effect to optics and astronomy was controversial for a longer time. A first hint for using the Doppler effect in order to get the velocity of the stars from the line shifts in their spectra, came from Armand Hippolyte Louis Fizeau (1819–1890) in 1848. [4] But this lecture remained unknown till it was republished when Ernst Mach (1838–1916) started again the discussion about astronomical applications. Gustav Robert Kirchhoff (1824–1887) recognized in a letter in 1868 to Mach that not the colours are important for determining the velocities of the stars but the line shifts: "Ich bezweifle aber nicht, da die Beobachtung der Sternspektren mit Sicherheit wird erkennen lassen, ob die Bewegung einer Lichtquelle Einfluß auf ihre Farbe hat ... In der Tat hat man ... nur zu beachten, ob die dunklen Linien des Spetrums eine Verschiebung erleiden." [13] In 1860 Ernst Mach succeeded to show the acustic Doppler effect with his rotation apparatus. Since the 1860s astonomers accepted that the Doppler effect can be used also for light sources, i.e. stars. In addition the Italian astrophysicist Angelo Secchi (1818–1878) claimed in 1863 as aim of his spectroscopic observations to get information about the cosmic order and to measure the motion of the stars. But his experiments in 1868 remained unsucessful for the lack of good instruments. [16, p. 199.]

3 The First Spectroscopis Determination of the Solar Rotation

The first successful verification of the Doppler effect in astronomy was made by H. C. Vogel in 1871 by determining the solar rotation. With a recommendation of the Leipzig astrophysicist J. C. F. Zöllner (1834–1882), Vogel got the position as a director of the private observatory of F. G. von Bülow at Bothkamp near Kiel. There he had for his disposal an excellent instrument: the second largest refractor of the continent after the Pulkovo refractor. In addition Vogel designed a spectroscope in 1871 which was constructed by Schroeder, Hamburg (Fig. 2). A Geissler tube filled with hydrogen enabled him to get comparison lines at both sides of

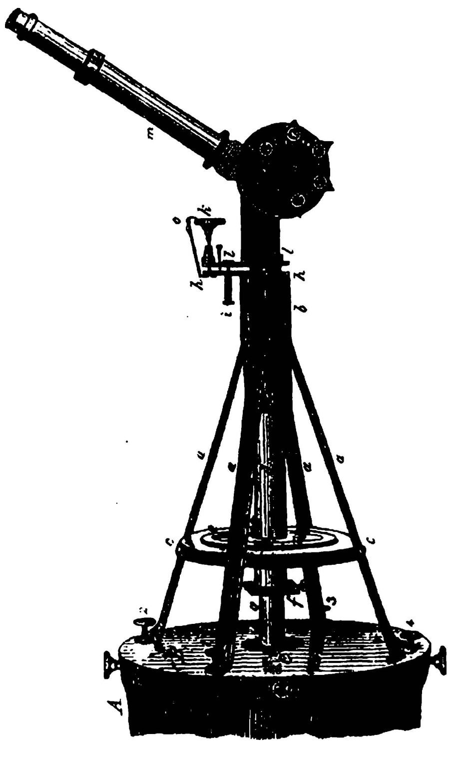

Fig. 2 Bothkamp's direct-vision spectroscope.
Development and production: Hugo Schröder, Hamburg 1871.
Vogel, H. C.: Beschreibung der Sternwarte. Bothkamper Beobachtungen, HeftI, Leipzig 1872.
Bildstelle Deutsches Museum R 4092/17.

the stellar spectrum. Furthermore Vogel used the reversion spectroscope designed by Zöllner (Fig. 3). Two Amici prism systems à vision directe made by Merz in Munich were used for light dispersion. By composing two spectra in a different way, one could measure a double value of the Doppler shift for getting more accuracy.

With these instruments, Vogel succeeded in demonstrating the solar rotation spectroscopically for the first time. [29] The additional prism, mounted on a circle, made it possible to accomplish stronger dispersin of the light into spectral colours for solar observations. Observing the limb of the Sun which is approach-

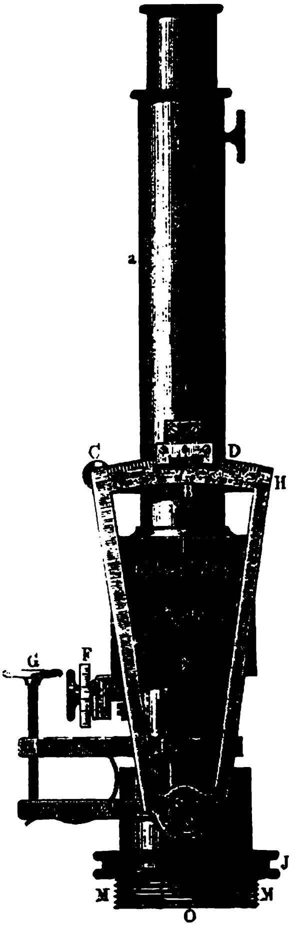

Fig. 3 Reversion spectroscope designed by Zöllner.
Zöllner, J. C. F.. Über ein neues Spectroscop nebst Beiträgen zur Spectralanalyse der Gestirne. Berichte der königlich sächsischen Gesellschaft der Wissenschaften zu Leipzig, math.-phys. Classe, 6. 2. 1869, p. 2. Bildstelle Deutsches Museum 41526.

ing, one recognizes a blue shift of the dark lines in the solar spectrum, observing the other limb which is receding, one recognizes a red shift. For checking the result, one could compare Richard C. Carrington's (1826–1875) exact measurements of the solar rotation based on the motion of the sun spots. [1] Two important questions were then being discussed and could only be solved by spectroscopy: [30, p. 73]

- First, one was interested in the determination of the solar rotation in different heliographic latitudes;
- second, Vogel wanted to decide which absorption lines in the solar spectrum originate from the atmosphere of the Earth. These should not have the Doppler shift of the Sun's limb.

Vogel's success was the reason that astronomers in some other countries started to measure the solar rotation in the same way, for example in Princeton, New Jersey, USA, Paris, France and Sweden. In addition, after the famous solar eclipse in 1868, Joseph Norman Lockyer (1836–1920) and Secchi tried to measure velocities of solar prominences. They got acceptable results (several hundreds of km/s); but it was not possible to check them through independent measurement.

4 Visual Radial Velocities of the Stars

William Huggins (1824–1910) made in 1867 the first practical realisation of the measurement of visual stellar radial velocities. [10] "From the beginning of our work upon spectra of the stars, I saw in vision the application of the new knowledge to the creation of a great method of astronomical observation which could not fail in future to have a powerful influence on the progress of astronomy." [9, p. 195] Huggins used a spectroscope with five prisms to get a good dispersion but had the disadvantage of lower light intensity. The difficult measurements led to contradicting results. He could only state that the line shifts were far smaller than the difference between the sodium D lines corresponding to a maximum velocity of 300 km/s.

In 1871 Vogel with his assistent Oswald Lohse (1845–1915) started to measure visually radial velocities of the stars.

> „Die höchst interessanten Versuche aus der Verschiebung bekannter Linien im Spectrum eines Sterns die Geschwindigkeit zu bestimmen, sind bekanntlich von Huggins am Sirius ausgeführt worden. ... Wegen Schwäche der Sternspectra, Unruhe der Luft und Kleinheit der wahrzunehmenden und zu messenden Grössen, gehören diese Beobachtungen wohl zu den schierigsten.“ [19, p. 33]

Vogels's measured values were about the same order of magnitude as the errors.

The resulting values were not reproduceable even with the same instrument, and the seeing made exact measurements impossible. Thus, in 1886, Vogel, then the director of the Astrophysical Observatory Potsdam, had the idea to make use of the new technique of photography:

> „Das, was für das Auge so schwer ist und so ausserordentlich ermüdend wirkt, einen Mittelwerth aus den oszillierenden Bewegungen der Sternspectrallinien sich zu bilden und diesen mit der ruhenden Linie des künstlichen Spectrums in Vergleich zu bringen, wird aber auf photographischen Wege voraussichtlich leichter gelingen." [21, p.142]

In order to get actual informations he visited the first international meeting for astrophotography in Paris in 1887. With this experience Vogel started the construction of a prototype with a photographic plate.

> „Meine ... Vermuthung, dass die Anwendung der Photographie bei der Lösung dieser Aufgabe wesentliche Vortheile bieten würde, hat sich im vollsten Masse bestätigt. ... Ich glaube daher, ... aussprechen zu können, dass diese hier zum ersten Male gemachte Anwendung der Photographie eine der bedeutsamsten genannt werden kann." [22, p. 122–123]

According to his skill Vogel could chose suitable dimensions for the prisms, collimator and camera objective lens of his spectrograph model A (1888) in order to get enough light although the dispersion was high. The maximum exposure times were only about one hour because of the changing of the refraction index of the prisms caused through fluctuation of the temperature. To allow longer exposure times, heating equipment was needed. Apart from the idea to use photography as measuring method Vogel tried two further important improvements: the application of gratings instead of prisms and the introduction of iron as comparison spectrum which has the advantage to offer a lot of lines. Vogel claimed for his spectrograph the following additional features for an increase of measuring accuracy:

- Large stability of the spectrograph; achieved by using cast-iron.
- Low weight; achieved by using a framework construction.
- Exact focussing of the photographic plate in the focal plane.
- Exact pointing of the star to the slit of the spectrograph, achieved by using an additional small telescope. For the comparison spectrum Vogel used either a Geissler tube filled with hydrogen or an iron spark-spectrum.

The spectrograph model A (Fig. 4) was suitable for measurements of stars to the third magnitude. [23] A new important epoque of stellar astronomy had begun. Important improvements were made by H. C. Vogel and the Potsdam staff in the further development of spectrographs: Ten years after model A, Vogel and Johan-

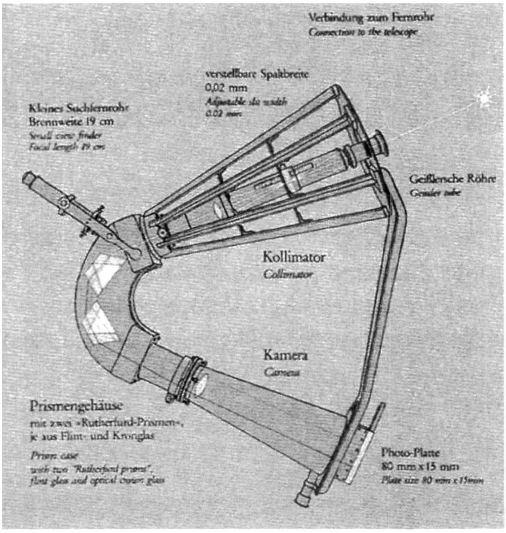

Fig. 4 Vogel's spectrograph model I (1888).
First Stellar Spectrograph Model A.
Development: Hermann Carl Vogel, Potsdam 1888.
Production: Mechanics: Otto Toepfer, Potsdam 1888.
Optics: Steinheil, Munich.
Bildstelle Deutsches Museum BN 41528.

nes Hartmann (1865–1936) built a heating equipment for the three-prism-spectrograph model III (1898) in order to avoid temperature fluctuations during the exposure time. The three-prism-spectrograph model IV (1900) with improved accuracy – made by Vogel and Gustav Eberhard (1867–1940) – still had a small error caused by sagging. The spectrograph may not be flexible during the long exposures. To get a stable but not a heavy instrument, Vogel and Eberhard used for model V (1905) a better cast-iron framework construction and new optics.

6 First Photographic Radial Velocities

In 1892, H. C. Vogel published his results, a catalogue with 51 stellar radial velocities, in the Prussian Academy of Sciences – 50 years after the discovery of the Doppler effect. The basis of his success was the application of photography in order to measure the small line shifts in the spectra.

> „Es ist bekannt, welche epochemachende Förderung die Astrophysik, und im besonderen die Spectralanalyse der Fixsterne durch die Anwendung der Photographie erfahren hat. Unter Benutzung derselben optischen Hülfsmittel gewährt die Spectralphotographie etwa die zwanzigfache Genauigkeit der Messung gegenüber der directen Beobachtung am Fernrohr, ...“ [24, p. 602]

David Gill (1843 – 1914), astronomer at the Cape Observatory, praised in 1891 the precision of Vogel's measurements: "His measurements, made with only a small prism spectrograph, are comparable in precision with line-of-sight velocities obtained with much better instruments today." [5, p. 189] From the improvement and perfection of the spectrographs resulted an increase in measuring accuracy. William Wallace Campbell (1862 – 1938), director of the Lick Observatory since 1901, made remarkable advances: an increase of a factor of ten – but one decade after Vogel. Then the center of research activities moved to the American observatories since the beginning of the 20th century.

7 Analysis of the Spectra

For the evaluation of the spectra the measuring microscopes of Otto Toepfer, Potsdam, were first used. Because the measuring of the spectra took more time than photographing (several days in comparison to one hour), at the turn of the century many spectra remained unworked. In 1906 Hartmann made an important contribution to the development of spectroscopic instruments with his spectrocomparator – constructed by Carl Zeiss Jena. [8] Now it was possible to measure the line displacements relative to any (e.g. solar or stellar) spectrum even with different dispersions. This caused a reduction of the evaluation time to one third. Previously it was necessary to know the exact wave length of the laboratory comparison lines.

8 The Discovery of the Spectroscopic Binaries and the Galactic Rotation

The first two »spectroscopic binaries« were discovered in 1889: H. C. Vogel happens to discover the spectroscopic binaries Algol (ß Persei) and shortly afterwords Spica (α Virginis). [28] At the same time but independently from the Astrophysical Observatory in Potsdam, Mizar (ζ Ursae Maioris) and ß Aurigae were recognized by Edward Charles Pickering (1846 – 1919) and Miss Antonia C. Maury (1866 – 1952), Harvard Observatory, Cambridge, Mass., by a periodic shift of the lines in their spectra. [15], [25], [27] This Doppler shift is towards the blue or the red. By means of good resolving telescopes these spectroscopic binaries cannot be separated. They are only noticable from the periodic changes in their spectra.

Combined with the light curve of Algol, an eclipsing binary, Hermann Carl Vogel (1841 – 1907) could get the dimensions of the system. On this basis he constructed a model of the binary system Algol in 1889. With this model he wanted to demonstrate the dimensions of the stellar system, as well as the diameters and masses of the stars. He obtained these results in a long study of the brightness changes in the light curve as well as line shifts in the spectrum. The importance of the spectroscopic and eclipsing binaries is that we can get the masses of the components, and the mass is fundamental for the theory of stellar evolution.

Another success was the investigation of the spectroscopic binary δ Orionis (discovered by Deslandres) [2] with the spectrograph model I at the Potsdam Astrophysical Observatory which led to the discovery of interstellar gas in 1904. [7] Johannes Hartmann (1865 – 1936) recognized stationary singly-ionized Calcium absorption lines (3934 Å) in the spectra besides the periodic shifted lines. He interpreted them correctly: They originate in an absorbing gaseous layer between the star and the Earth. This was the first proof of the existence of interstellar matter. In 1929, astronomers discovered that interstellar absorption lines are also subject to galactic rotation. This (differential) rotation of our Milky Way was determined by Jan Hendrik Oort (1900 – 1992) in 1927.

9 Further Applications of the Doppler Effect

A further example of the application of the Doppler effect was the determination of the rotation of the large planets, especially Jupiter and Saturn, based on radial velocities. James E. Keeler's (1857 – 1900) measurements of Saturn's ring in Pittsburgh/Allegheny in 1895 could prove the constitution of the ring consisting of small particles revolving the planet in circular orbits; the inclination of the lines is due to the differential rotation of the rings. [11]

With the spectrograph model IV, following an idea of H. C. Vogel, Friedrich Küstner (1856 – 1936) at Bonn Observatory in 1905 made the first astrophysical determination of the solar parallaxe by means of radial velocities of some Hyades stars. [14] An stellar statistic application of the Doppler effect is the deduction of the solar motion towards an apex point from the radial velocities of the stars which was first made by Wilhelm Herschel (1738 – 1822) in 1783 with the aid of the proper motion.

Finally one tried to get information about the rotation of the stars from the broadening of their lines. The idea came from William de Wiveleslie Abney (1843 – 1920) in 1877 and was discussed by Vogel controversally. The prove was not possible earlier than in the 1920s. For example, the pronounced lines in Vega in the constellation Lyra indicate a slow rotation of about 15 km/s. ζ Aquilae in the constellation Eagle rotates 23 times faster and exhibits a clear broadening of the lines due to the Doppler effect.

The spectral features of novae are the superimposition (heterodyne) of two spectra: bright emission lines superimposed on a stellar spectrum with dark lines. This was explained as being a great explosion within a star during which large amounts of gas were formed that were then visible as a fog-like envelope around the star. This defeated the theory that a nova was a new and developing star.

Around the turn of the century, astronomers noticed the expansion of this envelope. The rate of expansion was able to be measured due to the Doppler effect of the line displacement.

It was extremely difficult to measure radial velocities of nebulae. After finishing the big 80 cm photographic refractor for the Potsdam Observatory in 1899 Hartmann tried to measure radial velocities of gaseous nebulae with the Toepfer spectrograph model I and III. [6] Later Vogel und Eberhard used the three-prism-spectrograph model IV in combination with the small 32,5 cm photographic refractor because it had a better aperture to focal length ratio (1 : 10) for observing nebulae. [26] These early photographic measurements of nebulae confirmed rather good the American visual measured values. Due to the Doppler effect, the radial velocity of the galaxies can be calculated from the red shift. Astronomers recognized in the 1920s that most galaxies have large red shifts. Edwin Powell Hubble (1889 – 1953) discovered: the greater the red shift within a spectrum, the further away the galaxy. He published his theory on the connection between distance and the radial velocity of galaxies in 1929. From this he deduced the expansion of the universe, thus substantiating the theory that the space in which the stellar systems are fixed expand.

10 Conclusion

In the 19th century the application of the Doppler effect brought great success to astronomy and culminated with the work of H. C. Vogel: The first spectroscopic measurement of the solar rotation, the first photographic stellar radial velocities and some applications, the discovery of spectroscopic binaries, the discovery of interstellar gas, the rotation of planets and stars, and radial velocities of nebulae. This successful development was continued in the 20^{th} century when in the 1920s the red shift of the galaxies was interpreted to indicate an expansion of the universe.

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