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Simulation of Indoor Humidity[*]

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Abstract

The paper deals with the moisture in the internal air of a modeling room. The problems connected with the undesired relative humidity of the indoor air are introduced, the reviewing of the hygric situation in the modeling room is described, the less or more precise models for relative humidity estimation depending on the influencing parameters are performed and the reasoning—when, why and how the particular models should be used, is added. As in particular hygric conditions some of the influencing parameters can be neglected, the models differ each from other by the parameters being involved or neglected.

Key words: Heat equation, moisture, saturation line, hygric model, building engineering, FEM modeling.

2010 Mathematics Subject Classification: 80A20, 65L05

1 Introduction

Nowadays when the energy consumption fell into rigorous control, it is necessary to build up energetically unassuming buildings. As the building, especially its envelope separating the indoor and outdoor surroundings, interacts with outdoor environment and at the same time it should upkeep the indoor climate of the standard quality stipulated by technical norms, it is necessary to project the building carefully, with optimal thermal properties. Among the monitored values of the rooms in buildings where people reside, the indoor air temperature and relative humidity belong. The temperature on the inner surface of the walls (especially that separating the outdoor and indoor surroundings), windows and

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doors have to be kept above the temperature enabling the mould and condensation occurrence. As the thermal and hygric phenomenon influence each other, they have to be considered interactively.

The thermal properties of the particular components can vary more or less with varying moisture content. Also the phenomenon called moisture buffering can influence the temperature field.

The first investigations concerning the heat and moisture transfer (HAM models) influencing each other are dated to late 80s and early 90s of the last century. [12] That time the first retrofitted buildings appeared after the world oil crisis in 70s and as the retrofitting often was not done properly, the problems with undesirable indoor dampness broadened out.

The researchers [4] compared the theoretical results with experimental data and concluded that in some cases (rooms with a higher moisture load, hygroscopic (hygroscopically-active) surfaces presented, and surfaces with the condensed water) there is a need to involve the moisture storage into the physical models predicting the indoor climate in the room.

This paper compares three models described in [12]. The computations are done within a two week time interval for a testing room where the surfaces are optionally all hydrophobic or some of them are hydrophobic (finished with hydrophobic paint or tiles, floor of linoleum or paving) and some are hygroscopic (brick wall finished by lime and cement plaster).

The first model is eligible for the rooms with usually lower moisture load, or where the higher vapor production does not last for a longer periods, or the air exchange with the outside air is sufficiently set, e.g. the moisture buffering is negligible and need not to be involved in the model.

The second model (suitable for kitchens at school or bathroom) includes the condensation/evaporation on/from the surfaces impact into the lumping hygric situation as the hydrophobic surfaces are present predominantly. Moreover, the latent heat of condensation and evaporation influences the thermal situation. These two phenomena are coupled in each time step. Therefore solving the heat equation requires the immediate calculating of the amount of the condensed water on the surfaces, quantification of the corresponding latent heat; incorporating the latent heat as the source function (right hand side) into the non-homogenous heat equation, solving the heat equation in next step, etc. In this study the latent heat of evaporation/condensation was not included in the temperature field calculation.

The third model is the most complex and also the most complicated one. It is adequate to use the model in the rooms where besides the moisture load and the moisture buffering due to condensation/evaporation the storage of the moisture in the hygroscopic materials is considered. This physical phenomenon is governed by Luikov's equations. Coupling of the moisture and thermal potentials is expressed exactly, see e.g. [11]. This complex approach was used for example in [7,8]. But the equations are nonlinear and not easy to solve and in addition, the huge amount of physical properties is needed even for approximate solution. In such a way the judging a room would be too much time consuming. That is why the more simple methods are optimal for the purpose of moisture problems preventing.

The comparison of the models is discussed for particular hygric situations under the same moisture load course and the ventilation rate. The models are more less general and the other (arisen in engineering) internal air humidity situations under various moisture loads and ventilation rates can be derived easily by the same way. The input parameters in all three models are time dependent piecewise constant functions of vapor production, ventilation rate, temperature field (from FEM software computation) on the inside surface of the focused fragment of building construction and the external temperature and relative humidity during the two weeks typical for the region [10].

2 Humidity in the air

Humidity in the air propagates by diffusion and mostly by convection. In our investigation we consider a throughout mixing of the internal air. Three quantities can describe the amount of the water in the air: c [kg$\cdot$m^{-3}] vapor concentration, p [Pa] is the partial pressure of the water vapor in the air, and φ [%] relative humidity. The relation between c and p results from the ideal gas law with $c = m/V$:

$$c = \frac{p}{RT} \tag{2.1}$$

with R [J/(kg $\cdot$ K)] being specific gas constant, ($R = 462$ for water vapor) and T [K] temperature. Relative humidity (2.2) expresses the ratio between current water content amount (current partial pressure of the water vapor in the air) and the maximal water content amount (saturated partial pressure of the water vapor) at the same temperature.

$$\varphi = \frac{p}{p_{sat}(T)}\, 100\,\% \tag{2.2}$$

where p_{sat} [Pa] is the saturated partial pressure of the water vapor in the air. Here p_{sat} is the maximal possible pressure of the vapor in the air at that temperature. The dependence of the psat on the temperature (saturation line) is illustrated on the Fig. 2.1 (line denoted 100 %).

$$p_{sat} = \exp(65.8094 - 7066.27/T - 5.976 \ln T) \tag{2.3}$$

It is the regression line acquired from measured data by a data handling method [1].

On the Fig. 2.2 there is an illustration of the relative humidity increase up to the saturation value by increasing the moisture production at the same temperature or by the decreasing temperature with moisture content kept at the same value, or both. This obviously happens before the moisture problem arises.

If we want to avoid the moisture related problems (mold occurrence, damage of the plaster, changing the insulating parameters of the materials due to their wetting), we have to evaluate hygric performance even before the building is built up or retrofitted. The theoretical calculations can prevent the future dampness problems.

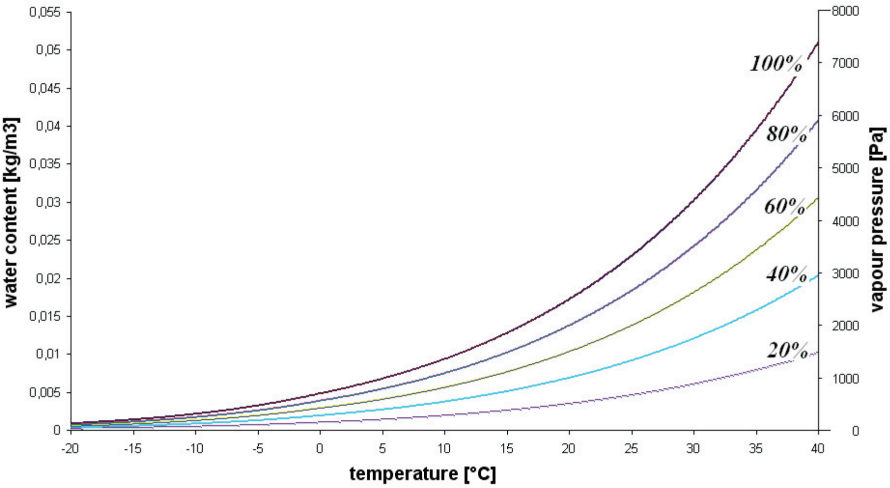

Fig. 2.1 Saturation line and other relative humidity lines [9]. The water concentration (left y axis) and the corresponding partial pressures of the water vapor in the air (right y axis) dependence of the temperature

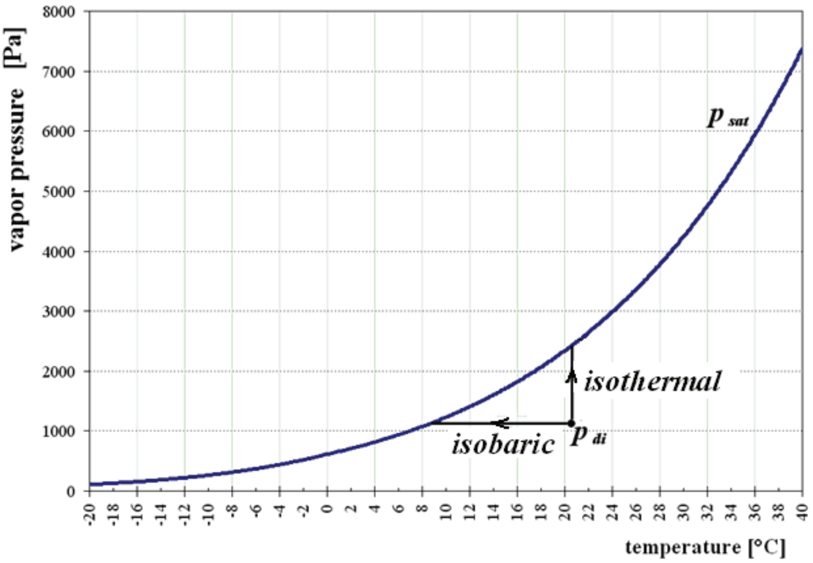

Fig. 2.2 Saturated value of partial pressure is reached from p_{di} by increasing moisture production (constant temperature) or by decreasing the temperature (constant partial pressure of water vapor)

If we want to avoid the moisture related problems (mold occurrence, damage of the plaster, changing the insulating parameters of the materials due to their wetting), we have to evaluate hygric performance even before the building is built up or retrofitted. The theoretical calculations can prevent the future dampness problems.

The moisture load of the room is based on the mass balance in the zone that says the amount of moisture incoming to zone + the vapor amount moisture produced in the zone = moisture going out of the zone + moisture stored there. As it is evident e.g. from Fig. 2.1, 2.2, the relative humidity is strongly linked to the temperature. So on the beginning of the hygric performance evaluation there is a thermal calculation. Having a building design before us at first we should point out to the weak places—thermal bridges, see Fig. 2.3 and afterwards to examine and treat them properly.

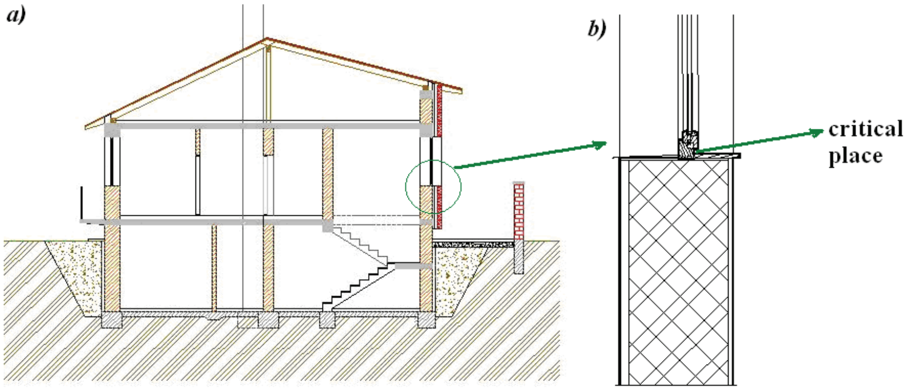

Fig. 2.3 Thermal bridges—critical places of a house; a) section of a designed house with one thermal bridge circled, b) thermal bridge detailed [14]

The saturated pressure on the coldest place of the room (the critical one), see Fig. 2.3, is essential for the indoor climate (thermal and hygric) parameters stipulation. That is why all hygric and thermal calculation have to be done in interactivity. The spatial temperature distribution of the inner surface temperature plays the crucial role here. 3D heat transfer in the thermal bridge is governed by the linear transient heat equation

$$-\nabla(\lambda\nabla T) = \varrho c\frac{\partial T}{\partial t} \quad \text{in } \Omega \subset \mathbb{R}^3 \tag{2.4}$$

where Ω is the fragment of the building construction (circled in the Fig. 2.3). The heat transfer between the bulk air and the domain can be described by the Newton boundary conditions on the external/internal surfaces

$$-q_i = h_i(T - T_i) \quad \text{on } \Gamma_i \tag{2.5}$$

$$q_e = h_e(T - T_e) \quad \text{on } \Gamma_e \tag{2.6}$$

The cutting edges should be situated sufficiently far from the temperature field being deformed. Zero fluxes through cutting edges of the fragment are represented by the homogenous Neumann boundary condition

$$q_h = 0 \quad \text{on } \Gamma_h. \tag{2.7}$$

The initial condition represents the initial spatial temperature distribution in the domain

$$T(x, y, 0) = T_0(x, y), \tag{2.8}$$

with q_e, q_i $[\mathrm{J\,m^{-2}\,s^{-1}}]$ being the density of the thermal flux between the bulk air and the domain by the inner/outer surfaces, h_e, h_i surface film coefficients for heat transfer $[\mathrm{W/(m^2\,K)}]$ on the inner/outer surfaces, T_e, T_i temperatures of the bulk external/internal air [K].

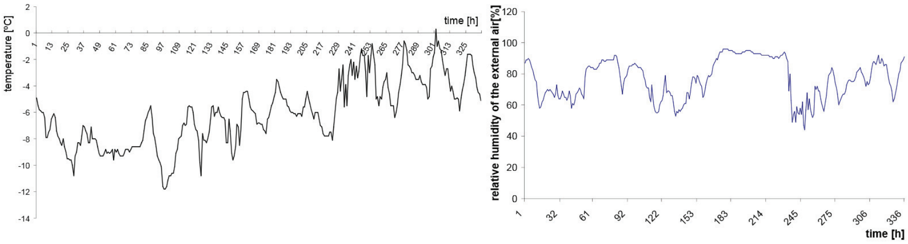

Fig. 2.4 External temperature and relative humidity during first two weeks of reference year [2]

3 Moisture in porous building materials

When a dry material (component of a building construction) is situated in the surroundings of the certain relative humidity (it is exposed to the moisture load), it starts to absorb the moisture. The process of the material wetting is called sorption and it is quantified by the sorption isotherm for each specific material. If the surrounding air remains on the same humidity during a sufficiently long period, the sorption goes on until the equilibrium stage—the material acquires water content corresponding to the relative humidity value. The equilibrium moisture content depends on the temperature, atmospheric pressure and the relative humidity of the surrounding air; see Fig. 3.1 (lower curve). The dependence is nonlinear.

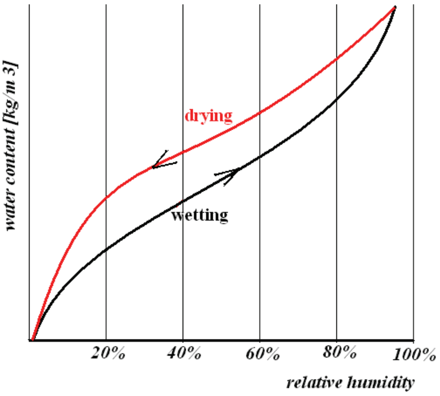

Fig. 3.1 Equilibrium water content of a material at the specific bulk air relative humidity during sorption (lower curve) and during desorption (upper curve)

Each material has its own sorption isotherm. Porous materials are characterized by a difference between the adsorption and desorption isotherm. This phenomenon is called hysteresis.

The sorption is governed by the diffusion equation which can be written either using the relative humidity—equation (3.1) or using of partial pressure of the vapor in the air, equation (3.2).

$$-\operatorname{div}(\delta \nabla (p_{sat}\varphi)) = \frac{\partial w_h}{\partial \varphi}\frac{\partial \varphi}{\partial t} \tag{3.1}$$

$$-\operatorname{div}(\delta \nabla p) = \frac{a_h}{p_{sat}}\frac{\partial p}{\partial t} \tag{3.2}$$

where δ [s] is the water vapor permeability, φ [-] is the relative humidity of the air, $a_h = \frac{\partial w_h}{\partial \varphi}$ [$\mathrm{kg} \cdot \mathrm{m}^{-3}$] is a specific moisture content in material (moisture capacity of the material), the slope of the sorption isotherm that depends on the relative humidity.

4 Simulation of the indoor air humidity in a single zone room

The simulation of the indoor humidity is focused to the possible moisture related problems revealing. The testing room is an empty single zone (the air in the room is mixed immediately, e.g. homogenous) of dimensions 3.5 m × 4 m × 2.8 m with one external wall. There is a window of 2 m^2 in the internal wall there is a door of the same area.

At the beginning we evaluate the amount (partial pressure) of the water vapor in the air. It is based on the mass balance in a zone: *the amount of the water vapor produced in the zone + amount of the incoming water vapor = amount of the outgoing water vapor + amount of the water vapor stored.*

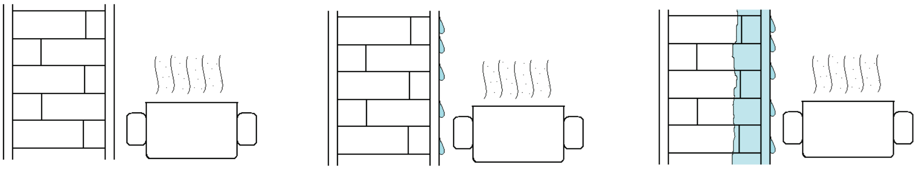

Fig. 4.1 Three models illustration: a) Model 1: indoor air humidity model neglecting condensation and sorption on the surfaces, b) Model 2: involving the surface condensation/evaporation, neglecting the hygroscopicity of the surfaces, c) Model 3: involving the condensation/evaporation and hygroscopic influences of the surfaces

In all three models the initial value of partial pressure of the water vapor in the indoor air of temperature 20 °C and 50 % relative humidity was taken.

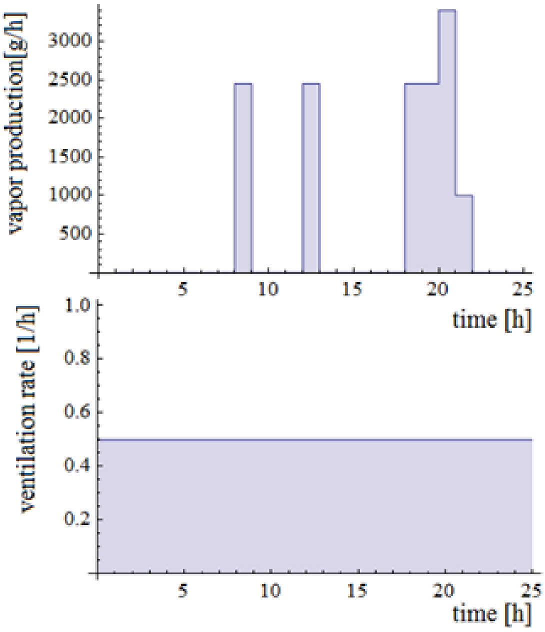

Fig. 4.2 The input parameters are indoor water vapor production G_p in the room, volume V [m^3] of the room, air temperature T_I [K], outdoor humidity represented by the partial pressure p_e [Pa] that can be easily calculated by using formulas (2.2) and (2.3), from the provided temperature and relative humidity data, see Fig. 2.4, ventilation intensity n [s^{-1}], condensation on the surfaces and sorption capability of the surfaces. Table 4.1 provides the typical production of the water vapor dependence on the activity in the room.

Activity of	**Vapor production G in kg/h**
Adult person – at rest	0.05
brainwork	0.07
light manual work	0.1
hard manual work	0.2
Child under 12 year	0.023
Flaming candle	0.011
Flow heater	6
Boiling water (3 l pot)	0.175
Animal (horse or cow)	0.3

Table 4.1 The vapor production depending of the activity in the room

The humidity transport in the air is driven by convection mostly; the transport of humidity by the diffusion is almost always negligible. The diffusion goes into account just in the very thin layer of the air very close to the surface of the solid material as this air does not move.

Model 1

The mass balance form described by the formula (4.1) represents the simplest model of the room hygric stage. It says that the condensation/evaporation at the surfaces and sorption/desorption of the surfaces does not occur or they are negligible (not high vapor production or short term of moisture load or sufficiently high ventilation rate).

$$G_p + G_i = G_o + G_s \tag{4.1}$$

where G_p [kg/s] quantifies the vapor per a time unit produced in the room, see Table 4.1, G_i incoming vapor, $G_i = \frac{nVp_E}{RT_I}$; G_o outgoing vapor, $G_o = \frac{nVp_I}{RT_I}$, G_s vapor stored in the room, $G_s = \frac{V}{RT_I}\frac{dp_I}{dt_I}$. Here n [s^{-1}] is a ventilation rate, p_I/p_E [Pa] partial pressure of the water vapor of the internal/external air, T_I [K] temperature of the indoor air. Ordinary differential equation (4.1) together with the initial condition $p_I(t_0) = p_{I0}$ can be solved analytically [12].

$$p_I(t) = \frac{e^{-nt}(-RG_pT_I + e^{nt}RG_pT_I - p_E(t)nV + e^{nt}p_E(t)nV + p_I(t_0)nV)}{nV} \tag{4.2}$$

As mentioned before, the input data—outdoor air properties (temperature and relative humidity) are given in a discrete form—given per each hour of the reference year [10]. Mathematically these data are interpreted as a stepwise function with time step 1 hour. That is why the analytical solution (4.2) is applied in particular time steps one by one, taking the resulting partial pressure as the initial value of pressure in the next step.

The model is not appropriate for the high moisture load under low ventilation rate as it does not reflect the physical reality that exceeding saturated pressure is impossible due to the effect of condensation.

Model 2

All surfaces hydrophobic (e.g. linings on the walls), window (2 m^2) of the temperature 10 °C, external wall 14 °C, other walls 18 °C.

The model where the condensation on the surfaces is taken into account is governed by the mass balance is of the form (sudden high relative humidity of the air within a short time period):

$$G_p + G_i = G_o + G_s + \sum_j \overline{G}_j \tag{4.3}$$

where $\sum_j \overline{G}_j$ is the contribution of the condensation/evaporation on all surfaces. Here

$$\overline{G}_j = \int_{A_j} \beta_j (p_I - p_{sat,j}) \tag{4.4}$$

with β_j [$\mathrm{s} \cdot \mathrm{m}^{-1}$] being an inside surface film coefficient diffusion for vapor diffusion and $p_{sat,j}$ the saturated partial pressure of the water vapor as a function of the surface temperature (computed by using the formula (2.3)). Integral (4.4) can be rewritten as the sum of two integrals $\beta_j p_I \int_{A_j} dA_j = \beta_j p_I A_j$ and $\beta_j \int_{A_j} p_{sat,j}\, dA_j$ where $p_{sat,j}$ is the known stepwise function. Averaging the temperature on each surface A_j where the condensation/evaporation takes place (denoting it $t_{av,j}$), also this integral can be simplified:

$$\beta_j \int\limits_{A_j} p_{sat,j} dA_j = \beta_j p_{sat,j}(t_{av,j}) A_j$$

Remembering the fact that the input parameters are the stepwise functions, it is sufficient for our purpose to evaluate the solution of the differential equation (4.4) approximately, by using the backward approximation of time derivative of indoor air vapor pressure function $p_I'(t) \approx \frac{p_I(t+\Delta t)-p_I(t)}{\Delta t}$

$$\begin{aligned} p_I(t+\Delta t) = p_I(t) + \Delta t \bigg(\frac{RT_I}{V} \Big(G_p + \sum_j \beta_j A_j p_{sat,j}(t_{av}) \\ - \sum_j \beta_j A_j p_j(t) \Big) + n(p_E - p_I(t)) \bigg) \end{aligned} \tag{4.5}$$

Condensate buffering is performed on the Fig. 4.3.

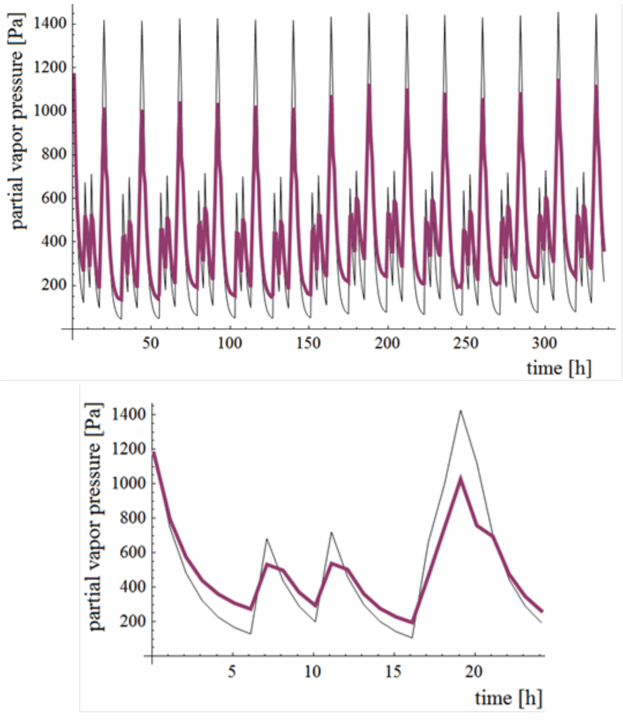

Fig. 4.3 The comparison of the resulting partial pressure of the water vapor in the indoor air of the model 1 and 2 within the investigated time interval (2 weeks)—upper graph, detailed for 24 hours—lower graph

The resulting partial pressure involving the condensation/evaporation is illustrated on the Fig. 4.4. The amplitude of the partial pressure with condensation/evaporation included in the model is smaller than in the model 1 without condensation/evaporation included. The dumping effect here is evident.

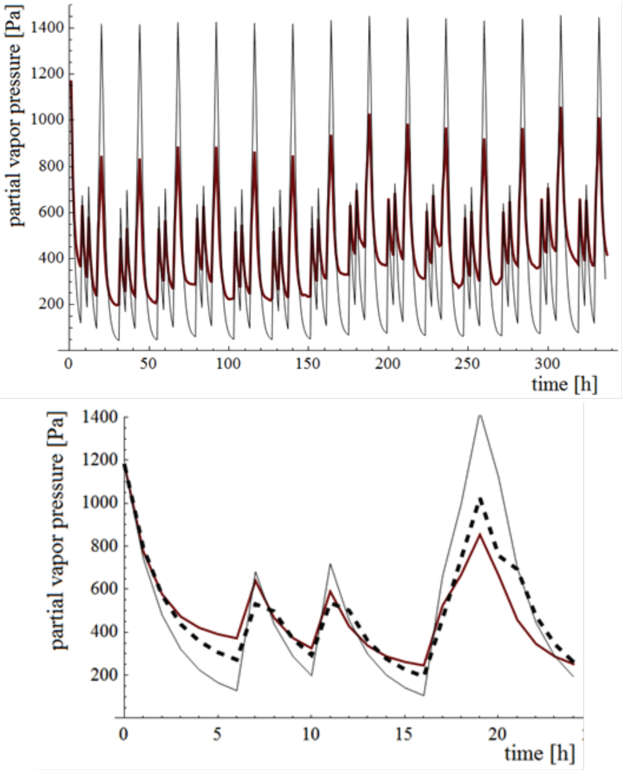

Fig. 4.4 The comparison of the resulting partial pressure of the water vapor in the indoor air of the model 1 and 3 within the investigated time interval (2 weeks)—upper graph, detailed for 24 hours, all three models—lower graph with thin line representing the 1st model value of partial pressure, thicker line—3rd model, dashed line the 2nd model.

Model 3

Window and doors hydrophobic, the rest surfaces hygroscopic.

The complex model gives the condensation/evaporation contribution and the sorption/release of the surrounding surfaces into the consideration:

$$G_p + G_i = G_o + G_s + \sum_j \overline{G}_j + \sum_k \overline{\overline{G}}_k \tag{4.6}$$

where

$$\overline{\overline{G}}_k = A_k \delta_p \nabla p_k = A_k \frac{1}{\mu N} \nabla p_k \tag{4.7}$$

A_k [m^2] being the k-th hygroscopic surface area, μ [-] water vapor resistance factor, N [s^{-1}] diffusion constant, $N = RT/D_{va} \approx 5.4\mathrm{e}9$ s^{-1} (with D_{va} [m^{2}s^{-1}] water vapor diffusion coefficient in the air).

By using of these relations, the equation (4.6) can be written

$$p_I(t+\Delta t) = p_I(t) + \Delta t\bigg(\frac{RT_I}{V}\Big(G_p + \sum_j \beta_j A_j p_{sat,j}(t_{av}) - \sum_j \beta_j A_j p_j(t) + \sum_k A_k \delta_k \nabla p_k(t)\Big) + n(p_E - p_I(t))\bigg) \tag{4.8}$$

Here ∇p_k was evaluated from the mass balance equation (3.2) using the simplified concept of penetration depth [12]. The applied material parameters were 5.7 for the slope of the sorption isotherm and 9 for the water vapor resistance factor of plaster, and 0.936 being the slope of the sorption isotherm and 15 being the diffusion resistance factor of brick. Model results are presented in the Fig. 4.4.

5 Conclusion

For the sake of conclusive comparison of the three presented models the calculations are performed with the same vapor production and ventilation intensity. By using any other input data, various cases hygric situations can be treated.

The vapor uptake by hygroscopic surface acts as a dumping factor in the resulting internal moisture represented by p_i.

The calculation based on the hygric models should be done even before the building being constructed—retrofitted with the aim of avoiding local condensation on the thermal bridges during long periods and following mold occurrence that is strongly undesirable.

The models presented in the paper are less or more complex. The second and the third models illustrate the dumping effect of condensate on the surfaces and the absorbent in the wall.

The usage of the appropriate model is preconditioned by the specific hygric situation (supposed vapor production and the ventilation intensity). The simplest model (Model 1) is suitable for using in the room where moisture load is not high or the ventilation intensity is sufficiently high, so it is not necessary to consider the condensation on the surrounding surfaces and sorption into them. Nevertheless, the condensation or mold can occur. The room of the high moisture production (kitchen or bathroom with hydrophobic surfaces) is better to be treated by the Model 2. The Model 3 is the most complex and suitable to be used in rooms with both hydrophobic and hygroscopic surfaces and where the relative humidity is obviously higher for long period. In such a case the moisture buffering (condensation/evaporation and sorption/desorption) is a significant factor influencing the entire environment. The phenomenon of moisture buffering is not seldom used for the intently ambient humidity correction.

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