

# Aktuárské vědy

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J. F. Steffensen

More multiplicity

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## More multiplicity.

By *J. F. Steffensen* (Copenhagen).

Mr. Misra's interesting article in „Aktuárské vědy“, 1937, p. 22, „More Multiplicity in the Rate of Interest, etc.“ contains, as already pointed out in the Editorial Notes, more algebra than finance and therefore only partly calls for comments on my part. I find it, however, desirable to make the following few observations.

In his first paper<sup>1)</sup> Mr. Misra confined his attention to „purely financial transactions“, meaning transactions which are independent of mortality, involving only the rate of interest (I, p. 71). In my remarks in *J. I. A.*, vol. LXIV, p. 169, I therefore only had purely financial transactions in view, and shall still confine myself to these, without occupying myself with Mr. Misra's later references (II, pp. 25 and 41) to certain transactions involving mortality.

Further, I thought that I had made it perfectly clear that I was only speaking of transactions where the payments on both sides are fixed beforehand,<sup>2)</sup> in which case the general level of interest which can come into consideration is given beforehand, so that the possibility of a very wide range, not to speak of the whole range from zero to infinity, may be ignored. It is therefore practically excluded that an outstanding balance which is positive at one (reasonable) valuation rate of interest may be negative at another. It is even in most cases easy, if desired, to indicate very wide limits to the rate within which the balance does not change its sign.

Besides, in the most frequent of all financial transactions, where *A* lends a sum of money to *B* who pays back according to a certain plan, the balance is always in *A*'s favour, no matter what rate of interest is employed in the valuation of the outstanding balance. Hence, when Mr. Misra says (II, p. 23) that „the hypothetical condition, i. e. the outstanding evaluated balance is always in favour of one and the same party, can be (true)<sup>3)</sup> only for a rate of interest involved in the transaction and possibly for certain others but not for all the rates of interest in the universe“, this assertion needs modification.

But it is true that if *A* only lends part of the money at once, promising to lend the remainder at a later date, then, if the valuation rate of interest is allowed to range to infinity, some value of *i* can strictly be found for which the balance, just before *A* has paid the second instalment of the loan, is in *B*'s favour, because, with such a freedom in the choice of *i*, we can always obtain that

$$|S_r| > |vS_{r+1} + v^2S_{r+2} + \dots|.$$

<sup>1)</sup> *J. I. A.*, vol. LXIV, 1933, p. 71. This paper will be quoted as „I“, and the paper in *Aktuárské vědy* as „II“.

<sup>2)</sup> These two words are cursivated in my Note in *J. I. A.*

<sup>3)</sup> The text has, by a misprint, „tone“.

In practice, however, the rate of interest does not range to infinity, and not even to 100%, corresponding to  $v = \frac{1}{2}$  (used by Mr. Misra „for simplicity“).

The example given by Mr. Misra (II, p. 23)

$$200 - 106v - 3v^2 + 297v^3 - 412v^4 = 0$$

is a construction which would hardly occur in real finance. And yet, although in such an invented transaction one must be prepared for the worst, it is clear that the balance will be constantly in  $A$ 's favour provided only that  $297v^3 < 412v^4$ , or  $i < 38,72\%$ ; and this is sufficient for concluding that there can be at most one rate of interest, worth the name, involved in the transaction. As a matter of fact, there is only one positive root.

But this discussion is really obsolete, since Mr. Lidstone's proof, given in his additional remarks to my Note in the *J. I. A.*, shows that, if the transaction is in equilibrium at rate  $i$ , and if, at that particular rate, one of the parties is always the creditor of the other party, then no other rate can be a solution. Another proof has in the meantime been given by G. S. Diwan and V. V. Narlikar in their paper „A practical financial transaction“<sup>4)</sup> which seems to have escaped Mr. Misra's attention.

Finally, I cannot follow Mr. Misra's objections (II, p. 24) to my differentiating the infinite power series  $f(v) = \sum v^n S_n$ . It belongs to the elements of analysis that the derived series  $\sum n v^{n-1} S_n$  has the same circle of convergence as the original series and represents  $f'(v)$ . What happens, if  $v$  is a point on the circle of convergence itself, is another and more delicate question; but then, financial operations do not occur on the circle of convergence.

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## Die Anwendung der Theorie der Elementarwahrscheinlichkeit zweier Abweichungen auf die Darstellung der Differentialgleichung der Frequenzfunktion zweier Variablen.

Von Hans Koepler, Berlin.

In Anlehnung an Bachelier<sup>1)</sup> stellen wir zunächst folgende Betrachtungen an:

Es sei  $f(s, x, y) dx dy$  die Wahrscheinlichkeit, daß bei  $s$  Beobachtungen die Abweichung der einen Art zwischen  $x$  und  $x + dx$  und die Abweichung der anderen Art zwischen  $y$  und  $y + dy$  falle.

<sup>4)</sup> Proc. of the Indian Ac. of Sc., Vol. II (1935), No 2, Sec. A, with Addendum in Vol. VI (1937), No 2, Sec. A.