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AN IMPROVED DELAY-DEPENDENT STABILIZATION CRITERION OF LINEAR TIME-VARYING DELAY SYSTEMS: AN ITERATIVE METHOD

Venkatesh Modala, Sourav Patra and Goshaidas Ray

This paper presents delay-dependent stabilization criteria for linear time-varying delay systems. A less conservative stabilization criterion is derived by invoking a new Lyapunov–Krasovskii functional and then, extended reciprocally convex inequality in combination with Wirtinger's inequality is exploited to obtain an improved stabilization criterion where a set of nonlinear matrix inequalities is solved by applying the cone complementarity algorithm. The proposed stabilization technique transforms a non-convex problem into a nonlinear trace minimization problem which is solved by an iterative approach. Numerical examples are considered to demonstrate the effectiveness of the proposed stabilization criteria and the presented iterative algorithm outperforms some existing results.

Keywords: time-delay systems, state feedback controller, Lyapunov–Krasovskii functional, Wirtinger's inequality, reciprocally convex inequality, linear matrix inequality

Classification: 93Dxx, 93B52

1. INTRODUCTION

The stability of time-delay systems is a fundamental issue to be addressed both from theoretical and practical points of view. Time-delay is found in many practical systems such as networked control systems [12], neutral systems [25], process control systems [13], etc., and it may lead to degradation of performance or even to instability of the system. In the last two decades ([6] and references therein), much effort has been paid to the delay-dependent stability and stabilization problems for time-delayed systems. It is well-known that the Lyapunov–Krasovskii (LK) functional plays an instrumental role in deriving a delay-dependent criteria. For reducing the conservatism in stability criteria, various effective methods such as Jensen's inequality, cross-term method [15], free weighting matrices method [14], the integral inequalities [19], [20], the reciprocally convex approach [21], etc., have been introduced in the literature along with the choice of an appropriate Lyapunov–Krasovskii (LK) functional. In [36], a matrix-based quadratic convex approach has been introduced in the chosen LK functional to analyze the stability of linear time-varying delay systems.

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The choice of an LK functional combined with Wirtinger's integral inequalities is discussed in [26] to achieve an improved delay upper bound in the sense of stability. An LMI based stability criteria for linear time-varying delayed system has been discussed using different approaches such as uncertain limits of integration [4], without neglecting useful terms [22], Bessel-Legendre inequality [27], extended reciprocally convex inequality [35]. The integral cross-terms appear in the time-derivative of LK functional are approximated with linear matrix inequalities plays important roles to reduce the conservatism of stability criteria. In [29], it is shown that an improved delay upper bound has been obtained using the augmented LK functional in combination with the extended reciprocally convex inequality.

In addition, the delay-dependent stabilization criteria for linear time-varying delay systems has been discussed at a length in the literature ([5, 8, 34] and [10]). In [30, 38] and [32], a delay-dependent stabilization criterion of linear systems with time-varying delay has been established to achieve a less conservative delay upper bound using Finsler's lemma and Wirtinger's inequalities. Various bounding inequalities and lemmas such as Park's inequality [9], reciprocally convex combination lemma, the free-weighting matrix-based inequality ([17, 37], zero inequalities [16], relaxation based approach with slack variables [23] etc., have extensively been used for deriving the stabilization conditions of time-delay systems. The stability and stabilization problems have also been tackled by many authors [3, 24, 31, 33] and [1]. In [2], for neutral time-delay systems, delay-dependent stabilization conditions have been obtained using LK functional for neutral time-delay systems with actuator saturation via an auxiliary time-delay feedback.

To the best of our knowledge, no results are available to design a controller for linear interval time-varying delay systems where a set of Nonlinear Matrix Inequality (NLMI) constraints appearing in the derivative of LK functional is solved using an iterative approach. The contributions of this paper are summarized as follows:

- A new LK functional is introduced.
- To reduce conservatism, the Wirtinger's and extended reciprocal inequalities are used for the cross-integral terms appear in time-derivative of LK functional, and the proposed stabilization conditions are derived in terms of nonlinear matrix inequalities (NLMIs).
- The derived set of NLMIs is solved using the cone complementarity algorithm (CCA) i. e., by transforming a non-convex problem into a nonlinear trace minimization problem which is solved through an iterative approach, to ensure the asymptotic stability of the closed-loop system.
- The proposed iterative method provides improved delay upper bounds compared to the existing results for various delay lower bounds with different delay derivatives.

The established stabilization conditions are solved using standard LMI toolboxes [11]. Finally, three numerical examples are considered to demonstrate the advantages of the proposed technique over the existing results.

This paper is organized as follows: Section 2 gives some preliminary lemmas that are successfully used to establish the stabilization criteria. The statement of problem is considered in Section 3, while the main results of the paper based on iterative algorithm are established in Section 4. In Section 5 the simulation results are presented to illustrate the results of the paper. Finally, Section 6 concludes with a summary of the obtained results.

2. NOTATIONS AND PRELIMINARIES

Let $\mathbb{R}$ and $\mathbb{C}$ denote the set of real and complex numbers, respectively. $\mathbb{R}^{n\times m}$ denotes the set of all real matrices with dimension $n \times m$. I_n is the identity matrix of dimension $n \times n$. A square symmetric matrix $A < 0\,(> 0)$ indicates that it is a negative (positive) definite matrix, A^T and A^{-1} represent the transpose and the inverse of matrix A, respectively. $\lambda_i(A)$ represent the eigenvalues of matrix A. diag$(\ldots)$ represent the block diagonal matrix. $\mathbf{0}_{m\times n}$ is the null matrix of $m \times n$ dimensions. $(*)$ represents the symmetric element in a symmetric matrix. $col\{x_1, x_2, \ldots, x_n\} = [x_1^T, x_2^T, \ldots, x_n^T]^T$ where $x_1, x_2, \ldots, x_n$ are column vectors. $tr(A)$ represents the trace of matrix A. $\bigotimes$ represents Kronecker product.

Following inequalities and lemmas are given which have been utilized to develop the main results of this paper for time-delay systems.

Wirtinger's inequality. (Seuret and Gouaisbaut [26]) For given symmetric positive-definite matrix $N \in \mathbb{R}^{n\times n}$, two scalars a and b with $b > a$ and a vector valued continuously differentiable function $g : [a, b] \to \mathbb{R}^n$, the following inequality holds:

$$\begin{aligned} \int_a^b \dot{g}^T(u) N \dot{g}(u)\,\mathrm{d}u \quad &\geq \quad \frac{1}{b-a}(g(b)-g(a))^T N (g(b)-g(a)) \\ &+ \quad \frac{3}{b-a}\bar{\Lambda}^T N \bar{\Lambda}, \end{aligned} \tag{1}$$

where $\bar{\Lambda} = g(b) + g(a) - \frac{2}{b-a}\int_a^b g(u)\,\mathrm{d}u$.

Extended reciprocal convex inequality. (Zhang et al. [35]) For real symmetric matrices $Y_1 > 0$ and $Y_2 > 0$, a real scalar $\alpha \in (0, 1)$ and any matrices S_1 and S_2, the following matrix inequality holds:

$$\begin{bmatrix} \frac{1}{\alpha}Y_1 & 0 \\ (*) & \frac{1}{1-\alpha}Y_2 \end{bmatrix} \geq \begin{bmatrix} Y_1 + (1-\alpha)U_1 & (1-\alpha)S_1 + \alpha S_2 \\ (*) & Y_2 + \alpha U_2 \end{bmatrix}, \tag{2}$$

where $U_1 = Y_1 - S_2 Y_2^{-1} S_2^T$ and $U_2 = Y_2 - S_1^T Y_1^{-1} S_1$.

Schur Complement Lemma. For symmetric matrices Q and R, and a matrix S of appropriate dimension, the inequality

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \quad < \quad 0 \tag{3}$$

is equivalent to

$$Q \;<\; 0, \qquad R - S^T Q^{-1} S \;<\; 0. \tag{4}$$

$R - S^T Q^{-1} S$ is called the Schur complement of Q.

Lemma 2.1. (Li and Jia [18]) For any given matrix X and $R = R^T > 0$ of appropriate dimensions, the following inequality holds:

$$X^T R^{-1} X \geq X + X^T - R. \tag{5}$$

The above inequality follows from $(R - X)^T R^{-1} (R - X) \geq 0$.

3. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

Consider a linear system with time-varying delay:

$$\dot{x}(t) = Ax(t) + A_d x(t - d(t)) + Bu(t), \quad \forall \; t \geq 0, \tag{6}$$

with $d(t)$ satisfies the following conditions:

$$0 \leq d_m \leq d(t) \leq d_M, \qquad \mu_1 \leq \dot{d}(t) \leq \mu_2, \tag{7}$$

where μ_1 and μ_2 are scalars. And then,

$$x(t) = \phi(t), \qquad \forall \; t \in [-d_M, 0], \tag{8}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, A, A_d and B are real matrices of appropriate dimensions; the pair (A, B) is assumed to be stabilizable; $\phi(t)$ is a continuous-time initial function defined on $[-d_M, 0]$.

Problem statement. Consider the linear time-delay system (6) which satisfies the conditions given in (7). The main objective is to design a static state feedback control law

$$u(t) = Kx(t), \qquad K \in \mathbb{R}^{m \times n}, \tag{9}$$

such that the closed-loop system

$$\dot{x}(t) = (A + BK)x(t) + A_d x(t - d(t)), \quad \forall \quad t \geq 0, \tag{10}$$

is asymptotically stable *i. e.,* the system response approaches to its stable equilibrium point as time $t \to \infty$.

4. STABILIZATION OF LINEAR TIME-VARYING DELAY SYSTEM

In this section, the state feedback design problem of linear system with time-varying delay is solved by introducing a new Lyapunov–Krasovskii functional. A less conservative result compared to existing literature is established in the sense of delay upper bound using the Wirtinger's inequality in combination with the extended reciprocally convex inequality, and the effectiveness of the proposed technique is demonstrated through numerical examples.

Novel LK functional. A new Lyapunov–Krasovskii (LK) functional is introduced to provide improved delay upper bound compared to existing literature which is given below:

$$
\begin{aligned}
V(t) &= x^T(t)P_{11}x(t) + \tilde{x}_1^T(t)\hat{P}\tilde{x}_1(t) \\
&+ \int_{t-d(t)}^{t} x^T(s)Q_1x(s)\,\mathrm{d}s + \int_{t-d_m}^{t} x^T(s)Q_2x(s)\,\mathrm{d}s \\
&+ \int_{t-d_M}^{t} x^T(s)Q_3x(s)\,\mathrm{d}s + \int_{t-d(t)}^{t-d_m} x^T(s)Q_4x(s)\,\mathrm{d}s \\
&+ d_m\int_{t-d_m}^{t}\int_{\theta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{t-d_M}^{t-d_m}\int_{\theta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)\,\mathrm{d}s\mathrm{d}\theta, \qquad (11)
\end{aligned}
$$

where

$$
\tilde{x}_1(t) = \left[\int_{t-d_m}^{t} x^T(s)\,\mathrm{d}s \quad \int_{t-d(t)}^{t-d_m} x^T(s)\,\mathrm{d}s \quad \int_{t-d_M}^{t-d(t)} x^T(s)\,\mathrm{d}s\right]^T, \qquad (12)
$$

$P_{11}, \hat{P}, Q_1, Q_2, Q_3, Q_4, Z_1, Z_2$ are symmetric positive-definite matrices, and Wirtinger's and extended reciprocally convex inequalities are used to differentiate the last two double integral terms of LK functional (11).

Remark 4.1. In the proposed LK functional (11), the two terms $\left[\int_{t-d(t)}^{t-d_m} x^T(s)\,\mathrm{d}s\right]^T$ and $\left[\int_{t-d_M}^{t-d(t)} x^T(s)\,\mathrm{d}s\right]^T$ are newly augmented compared to $\tilde{x}_1(t)$ of LK functional [30].

The time-derivative of the proposed LK functional (11) along the trajectories of (6) leads to:

$$
\begin{aligned}
\dot{V}(t) &= \dot{x}^T(t)P_{11}x(t) + x^T(t)P_{11}\dot{x}(t) \\
&+ \tilde{x}_1^T(t)\hat{P}\dot{\tilde{x}}_1(t) + \dot{\tilde{x}}_1^T(t)\hat{P}\tilde{x}_1(t) \\
&+ x^T(t)Q_1x(t) - (1-\dot{d}(t))x^T(t-d(t))Q_1x(t-d(t)) \\
&+ x^T(t)Q_2x(t) - x^T(t-d_m)Q_2x(t-d_m) \\
&+ x^T(t)Q_3x(t) - x^T(t-d_M)Q_3x(t-d_M) \\
&+ x^T(t-d_m)Q_4x(t-d_m) \\
&- (1-\dot{d}(t))x^T(t-d(t))Q_4x(t-d(t)) \\
&+ d_m^2\dot{x}^T(t)Z_1\dot{x}(t) - d_m\int_{t-d_m}^{t} \dot{x}^T(s)Z_1\dot{x}(s)\,\mathrm{d}s \\
&+ (d_M-d_m)\dot{x}^T(t)Z_2\dot{x}(t) - \int_{t-d_M}^{t-d_m} \dot{x}^T(s)Z_2\dot{x}(s)\,\mathrm{d}s. \qquad (13)
\end{aligned}
$$

The terms $d_m\int_{t-d_m}^{t} \dot{x}^T(s)Z_1\dot{x}(s)\,\mathrm{d}s$ and $\int_{t-d_M}^{t-d_m} \dot{x}^T(s)Z_2\dot{x}(s)\,\mathrm{d}s$ in (13) are bounded with tighter inequalities using Wirtinger's and extended reciprocal convex inequalities.

Substituting the state-equations given in (10) and $\tilde{x}_1(t)$ into (13), we get

$$\begin{aligned}
\dot{V}(t) &= \Big[(A+BK)x(t)+A_d x(t-d(t))\Big]^T P_{11}x(t) \\
&+ x^T(t)P_{11}\Big[(A+BK)x(t)+A_d x(t-d(t))\Big] \\
&+ \begin{bmatrix} x(t)-x(t-d_m) \\ x(t-d_m)-(1-\dot{d}(t))x(t-d(t)) \\ (1-\dot{d}(t))x(t-d(t))-x(t-d_M) \end{bmatrix}^T \hat{P} \begin{bmatrix} \int_{t-d_m}^{t} x(s)\,\mathrm{d}s \\ \int_{t-d(t)}^{t-d_m} x(s)\,\mathrm{d}s \\ \int_{t-d_M}^{t-d(t)} x(s)\,\mathrm{d}s \end{bmatrix} \\
&+ \begin{bmatrix} \int_{t-d_m}^{t} x(s)\,\mathrm{d}s \\ \int_{t-d(t)}^{t-d_m} x(s)\,\mathrm{d}s \\ \int_{t-d_M}^{t-d(t)} x(s)\,\mathrm{d}s \end{bmatrix}^T \hat{P} \begin{bmatrix} x(t)-x(t-d_m) \\ x(t-d_m)-(1-\dot{d}(t))x(t-d(t)) \\ (1-\dot{d}(t))x(t-d(t))-x(t-d_M) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
&+ x^T(t)Q_1x(t)-(1-\dot{d}(t))x^T(t-d(t))Q_1x(t-d(t)) \\
&+ x^T(t)Q_2x(t)-x^T(t-d_m)Q_2x(t-d_m) \\
&+ x^T(t)Q_3x(t)-x^T(t-d_M)Q_3x(t-d_M) \\
&+ x^T(t-d_m)Q_4x(t-d_m)-(1-\dot{d}(t))x^T(t-d(t))Q_4x(t-d(t)) \\
&+ d_m^2\Big[(A+BK)x(t)+A_d x(t-d(t))\Big]^T \\
&\quad Z_1\Big[(A+BK)x(t)+A_d x(t-d(t))\Big]-d_m\int_{t-d_m}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\,\mathrm{d}s \\
&+ d_D\Big[(A+BK)x(t)+A_d x(t-d(t))\Big]^T Z_2 \\
&\quad \Big[(A+BK)x(t)+A_d x(t-d(t))\Big]-\int_{t-d_M}^{t-d_m}\dot{x}^T(s)Z_2\dot{x}(s)\,\mathrm{d}s, \qquad (14)
\end{aligned}$$

where $d_D = d_M - d_m$.

Now defining

$$\begin{aligned}
\zeta_1(t) = \ & col\Big\{x(t), x(t-d_m), x(t-d(t)), \\
& \frac{1}{d_m}\int_{t-d_m}^{t} x(s)\,\mathrm{d}s, \frac{1}{d(t)-d_m}\int_{t-d(t)}^{t-d_m} x(s)\,\mathrm{d}s, \frac{1}{d_M-d(t)}\int_{t-d_M}^{t-d(t)} x(s)\,\mathrm{d}s\Big\}, \qquad (15)
\end{aligned}$$

$$G_1(d(t)) = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & -(1-\dot{d}(t))I & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\dot{d}(t))I & -I & 0 & 0 & 0 \end{bmatrix}, \quad G_0(\dot{d}(t)) = \begin{bmatrix} 0 & 0 & 0 & 0 & d_mI & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (d(t)-d_m)I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (d_M-d(t))I \end{bmatrix}, \qquad (16)$$

$$e_1 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 & 0 & I & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (17)$$

and

$$\hat{Q}(\dot{d}(t)) = diag(Q_1 + Q_3, -Q_2 + Q_4, -(1 - \dot{d}(t))(Q_1 + Q_4), -Q_3, 0, 0, 0), \tag{18}$$

we can rewrite (14) as

$$\dot{V}(t) = \zeta_1^T(t) \underbrace{\begin{bmatrix} e_1^T A^T P_{11} e_1 + e_1^T K^T B^T P_{11} e_1 + e_3^T A_d^T P_{11} e_1 + e_1^T P_{11} A e_1 + e_1^T P_{11} B K e_1 + e_1^T P_{11} A_d e_3 \\ + G_1^T(d(t)) \hat{P} G_0(\dot{d}(t)) + G_0^T(\dot{d}(t)) \hat{P} G_1(d(t)) + \hat{Q}(\dot{d}(t)) \\ + d_m^2 \Big[(A+BK)e_1 + A_d e_3\Big]^T Z_1 \Big[(A+BK)e_1 + A_d e_3\Big] \\ + d_D \Big[(A+BK)e_1 + A_d e_3\Big]^T Z_2 \Big[(A+BK)e_1 + A_d e_3\Big] \end{bmatrix}}_{\Psi_1(d(t),\dot{d}(t))} \zeta_1(t)$$
$$-d_m \int_{t-d_m}^{t} \dot{x}^T(s) Z_1 \dot{x}(s)\, \mathrm{d}s - \int_{t-d_M}^{t-d_m} \dot{x}^T(s) Z_2 \dot{x}(s)\, \mathrm{d}s. \tag{19}$$

Now using Wirtinger's integral inequality, the middle term of (19) satisfies the following inequality constraint:

$$-d_m \int_{t-d_m}^{t} \dot{x}^T(s) Z_1 \dot{x}(s)\, \mathrm{d}s \le -\zeta_1^T(t) E_1^T \hat{Z}_1 E_1 \zeta_1(t), \tag{20}$$

where $E_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & 0 & -2I & 0 & 0 \end{bmatrix}$ and $\hat{Z}_1 = diag(Z_1, 3Z_1)$. The last term of (19) can be written as

$$-\int_{t-d_M}^{t-d_m} \dot{x}^T(s) Z_2 \dot{x}(s)\, \mathrm{d}s = -\int_{t-d_M}^{t-d(t)} \dot{x}^T(s) Z_2 \dot{x}(s)\, \mathrm{d}s - \int_{t-d(t)}^{t-d_m} \dot{x}^T(s) Z_2 \dot{x}(s)\, \mathrm{d}s. \tag{21}$$

Again using the Wirtinger's integral inequality (1) and defining $E_2 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 & -2I & 0 \end{bmatrix}$, $E_3 = \begin{bmatrix} 0 & 0 & I & -I & 0 & 0 & 0 \\ 0 & 0 & I & I & 0 & 0 & -2I \end{bmatrix}$ and $\hat{Z}_2 = diag(Z_2, 3Z_2)$, from (21) we get

$$-\int_{t-d_M}^{t-d(t)} \dot{x}^T(s) Z_2 \dot{x}(s)\, \mathrm{d}s \le -\frac{1}{d_M - d(t)} \zeta_1^T(t) E_3^T \hat{Z}_2 E_3 \zeta_1(t), \tag{22}$$

$$-\int_{t-d(t)}^{t-d_m} \dot{x}^T(s) Z_2 \dot{x}(s)\, \mathrm{d}s \le -\frac{1}{d(t) - d_m} \zeta_1^T(t) E_2^T \hat{Z}_2 E_2 \zeta_1(t). \tag{23}$$

Due to (22) and (23), (21) can be written as

$$-\int_{t-d_M}^{t-d_m} \dot{x}^T(s) Z_2 \dot{x}(s)\, \mathrm{d}s \le -\zeta_1^T(t) \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}^T \begin{bmatrix} \frac{1}{d(t)-d_m} \hat{Z}_2 & 0 \\ (*) & \frac{1}{d_M - d(t)} \hat{Z}_2 \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix} \zeta_1(t). \tag{24}$$

Using extended reciprocally convex inequality (2), if there exist matrices S_1 and S_2, then the right-hand term of (24) can be written as

$$-\zeta_1^T(t) \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}^T \begin{bmatrix} \frac{1}{d(t)-d_m} \hat{Z}_2 & 0 \\ (*) & \frac{1}{d_M - d(t)} \hat{Z}_2 \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix} \zeta_1(t) \le -\frac{1}{d_D} \zeta_1^T(t) \Psi_{2,[d(t)]} \zeta_1(t), \tag{25}$$

where

$$\Psi_{2,[d(t)]} = \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}^T \begin{bmatrix} \hat{Z}_2 + \frac{d_M - d(t)}{d_D}(\hat{Z}_2 - S_2\hat{Z}_2^{-1}S_2^T) & \frac{d_M - d(t)}{d_D}S_1 + \frac{d(t)-d_m}{d_D}S_2 \\ (*) & \hat{Z}_2 + \frac{d(t)-d_m}{d_D}(\hat{Z}_2 - S_1^T\hat{Z}_2^{-1}S_1) \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}$$

$$\dot{V}(t) \leq \zeta_1^T(t) \underbrace{\begin{bmatrix} \Psi_1(d(t),\dot{d}(t)) - E_1^T\hat{Z}_1E_1 \\ -\frac{1}{d_D}\Psi_{2,[d(t)]} \end{bmatrix}}_{\Psi_3(d(t),\dot{d}(t))} \zeta_1(t).$$

Now using the Schur complement lemma for $\Psi_3(d(t),\dot{d}(t))$, a set of sufficient conditions for stability can be written as

$$\begin{bmatrix} \Phi_{11}(d(t),\dot{d}(t)) & \Phi_{12} & \Phi_{13} \\ (*) & -Z_1 & 0 \\ (*) & (*) & -d_DZ_2 \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned} \Phi_{11(d(t),\dot{d}(t))} &= e_1^TA^TP_{11}e_1 + e_1^TK^TB^TP_{11}e_1 + e_3^TA_d^TP_{11}e_1 \\ &+ e_1^TP_{11}Ae_1 + e_1^TP_{11}BKe_1 + e_1^TP_{11}A_de_3 \\ &+ G_1^T(d(t))\hat{P}G_0(\dot{d}(t)) + G_0^T(\dot{d}(t))\hat{P}G_1(d(t)) \\ &+ \hat{Q}(\dot{d}(t)) - E_1^T\hat{Z}_1E_1 - \frac{1}{d_D}\Psi_{2,[d(t)]}, \\ \Phi_{12} &= d_m\Big[(A+BK)e_1 + A_de_3\Big]^TZ_1, \\ \Phi_{13} &= d_D\Big[(A+BK)e_1 + A_de_3\Big]^TZ_2. \end{aligned}$$

Pre- and post-multiplying (26) respectively with *blockdiag*$(\bar{M}, Z_1^{-1}, Z_2^{-1})$ where $\bar{M} = I_{7\times7}\bigotimes M$ and $M = P_{11}^{-1}$, we get

$$U(d(t),\dot{d}(t)) = \begin{bmatrix} U_{11}(d(t),\dot{d}(t)) & U_{12} & U_{13} \\ (*) & -MR_1^{-1}M & 0 \\ (*) & (*) & -d_DMR_2^{-1}M \end{bmatrix} < 0. \tag{27}$$

In above,

$$\begin{aligned} U_{11}(d(t),\dot{d}(t)) &= e_1^T(MA^T + N^TB^T + AM + BN)e_1 \\ &+ e_3^TMA_d^Te_1 + e_1^TA_dMe_3 + G_1^T(d(t))\bar{P}G_0(\dot{d}(t)) \\ &+ G_0^T(\dot{d}(t))\bar{P}G_1(d(t)) + \bar{\hat{Q}}(\dot{d}(t)) - E_1^T\hat{R}_1E_1 - \frac{1}{d_D}\hat{\Psi}_{2,[d(t)]}, \\ U_{12} &= d_m\Big[(AM+BN)e_1 + A_dMe_3\Big]^T, \end{aligned}$$

$$\begin{aligned}
U_{13} &= d_D\Big[(AM+BN)e_1 + A_d M e_3\Big]^T,\\
N &= KM, \quad \bar{P} = \bar{M}_2\hat{P}\bar{M}_2, \quad \bar{M}_2 = diag(M,M,M),\\
\bar{\hat{Q}}(\dot{d}(t)) &= \bar{M}\hat{Q}\bar{M}\\
&= diag(\bar{Q}_1+\bar{Q}_2+\bar{Q}_3, -\bar{Q}_2+\bar{Q}_4, -(1-\dot{d}(t))(\bar{Q}_1+\bar{Q}_4), -\bar{Q}_3, 0,0,0),\\
\bar{Q}_i &= MQ_iM, \ i=1,2,3,4, \quad \hat{R}_1 = diag(R_1, 3R_1), \qquad \hat{R}_2 = \begin{bmatrix} R_2 & 0 \\ 0 & 3R_2 \end{bmatrix},\\
R_1 &= MZ_1M, \qquad R_2 = MZ_2M,\\
\hat{\Psi}_{2,[d(t)]} &= \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}^T \begin{bmatrix} \hat{R}_2 + \frac{d_M-d(t)}{d_D}(\hat{R}_2 - \bar{S}_2\hat{R}_2^{-1}\bar{S}_2^T) & \frac{d_M-d(t)}{d_D}\bar{S}_1 + \frac{d(t)-d_m}{d_D}\bar{S}_2 \\ (*) & \hat{R}_2 + \frac{d(t)-d_m}{d_D}(\hat{R}_2 - \bar{S}_1^T\hat{R}_2^{-1}\bar{S}_1) \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}\\
\bar{S}_1 &= \bar{M}_1S_1\bar{M}_1, \quad \bar{S}_2 = \bar{M}_1S_2\bar{M}_1, \quad \bar{M}_1 = diag(M,M).
\end{aligned}$$

Using the Schur complement lemma, conditions in (27) are equivalent to the conditions given in (28) that hold for all $d(t) \in [d_m, d_M]$ and $\dot{d}(t) \in [\mu_1, \mu_2]$. This ensures $\dot{V} < 0$ leading to asymptotic stability of the closed-loop system satisfying (7).

Theorem 4.2. Given non-negative scalars d_m, d_M and scalars μ_1 and μ_2 satisfying the conditions (7), the time-delayed system (6) is asymptotically stable by using the state feedback control law (9), if there exist real symmetric positive-definite matrices $M \in \mathbb{R}^{n\times n}$, $\bar{P} \in \mathbb{R}^{3n\times 3n}$, $\bar{Q}_1 \in \mathbb{R}^{n\times n}$, $\bar{Q}_2 \in \mathbb{R}^{n\times n}$, $\bar{Q}_3 \in \mathbb{R}^{n\times n}$, $\bar{Q}_4 \in \mathbb{R}^{n\times n}$, $R_1 \in \mathbb{R}^{n\times n}$, $R_2 \in \mathbb{R}^{n\times n}$ and the matrices $N \in \mathbb{R}^{m\times n}$, $\bar{S}_1 \in \mathbb{R}^{2n\times 2n}$ and $\bar{S}_2 \in \mathbb{R}^{2n\times 2n}$ such that the following NLMIs hold for $d(t) \in [d_m, d_M]$ and $\dot{d}(t) \in [\mu_1, \mu_2]$.

$$\begin{bmatrix} \Xi_{11}(d(t),\dot{d(t)})|_{d(t)=0,\dot{d}(t)=0} - \frac{1}{d_D}\hat{\Psi}_4 & \Xi_{12} & \Xi_{13} & E_2^T\bar{S}_2 \\ (*) & \Xi_{22} & 0 & 0 \\ (*) & (*) & \Xi_{33} & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Xi_{11}(d(t),\dot{d(t)})|_{d(t)=d_m,\dot{d}(t)=\mu_2} - \frac{1}{d_D}\hat{\Psi}_4 & \Xi_{12} & \Xi_{13} & E_2^T\bar{S}_2 \\ (*) & \Xi_{22} & 0 & 0 \\ (*) & (*) & \Xi_{33} & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Xi_{11}(d(t),\dot{d(t)})|_{d(t)=d_M,\dot{d}(t)=0} - \frac{1}{d_D}\hat{\Psi}_5 & \Xi_{12} & \Xi_{13} & E_3^T\bar{S}_1^T \\ (*) & \Xi_{22} & 0 & 0 \\ (*) & (*) & \Xi_{33} & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Xi_{11}(d(t),\dot{d(t)})|_{d(t)=d_M,\dot{d}(t)=\mu_1} - \frac{1}{d_D}\hat{\Psi}_5 & \Xi_{12} & \Xi_{13} & E_3^T\bar{S}_1^T \\ (*) & \Xi_{22} & 0 & 0 \\ (*) & (*) & \Xi_{33} & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0, \tag{28}$$

where

$$
\begin{aligned}
\Xi_{11}(d(t),\dot{d}(t)) &= e_1^T AMe_1 + e_1^T MA^T e_1 + e_1^T BNe_1 + e_1^T N^T B^T e_1 \\
&+ e_1^T A_d Me_3 + e_3^T MA_d^T e_1 + G_1^T(d(t))\bar{P}G_0(\dot{d}(t)) \\
&+ G_0^T(\dot{d}(t))\bar{P}G_1(d(t)) + \bar{\bar{Q}}(\dot{d}(t)) - E_1^T\hat{R}_1E_1, \\
e_i &= \left[\ 0_{n\times(i-1)n},\ I,\ 0_{n\times(7-i)n}\ \right], i = 1,2,\ldots,7, \\
G_1(d(t)) &= col\Big\{d_m e_5,\ (d(t)-d_m)e_6,\ (d_M - d(t))e_7\Big\}, \\
G_0(\dot{d}(t)) &= col\Big\{e_1 - e_2,\ e_2 - (1-\dot{d}(t))e_3,\ (1-\dot{d}(t))e_3 - e_4\Big\}, \\
\bar{\bar{Q}}(\dot{d}(t)) &= diag\Big(\bar{Q}_1+\bar{Q}_2+\bar{Q}_3, -\bar{Q}_2+\bar{Q}_4, -(1-\dot{d}(t))(\bar{Q}_1+\bar{Q}_4), -\bar{Q}_3, 0, 0, 0\Big), \\
\hat{R}_1 &= diag\{R_1, 3R_1\}, \quad d_D = d_M - d_m, \\
\hat{\Psi}_4 &= \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}^T \begin{bmatrix} 2\hat{R}_2 & \bar{S}_1 \\ (*) & \hat{R}_2 \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}, \\
\hat{\Psi}_5 &= \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}^T \begin{bmatrix} \hat{R}_2 & \bar{S}_2 \\ (*) & 2\hat{R}_2 \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}, \\
E_i &= col\Big\{e_i - e_{i+1},\ e_i + e_{i+1} - 2e_{i+4}\Big\}, i = 1,2,3, \\
\Xi_{12} &= d_m\Big[(AM+BN)e_1 + A_dMe_3\Big]^T, \\
\Xi_{13} &= d_D\Big[(AM+BN)e_1 + A_dMe_3\Big]^T, \\
\Xi_{22} &= -MR_1^{-1}M, \quad \Xi_{33} = -d_D MR_2^{-1}M. \qquad (29)
\end{aligned}
$$

The state feedback controller gain is $K = NM^{-1}$.

Remark 4.3. The LK functional (11) proposed in Theorem 4.2 for linear interval time-varying delayed system (6) is different from the LK functional given in [38]. It is worth noting that the terms associated with $\hat{P}$, Q_2, Q_3 and Q_4 are introduced to obtain improved delay upper bound. The augmented vector $\tilde{x}_1(t)$ in (12) associated with the quadratic term $\tilde{x}_1^T(t)\hat{P}\tilde{x}_1(t)$ of (11) is substantially different from that of LK functional considered in Theorem 1 of [30]. To obtain the stabilization conditions in LMI form, the LK functional (11) is obtained by setting $P_{12} = P_{13} = P_{14} = 0$ associated with P=$\left[\begin{array}{c|ccc} P_{11} & P_{12} & P_{13} & P_{14} \\ \hline (*) & & \hat{P} & \end{array}\right]$, which involves the first quadratic term of LK functional of Theorem 2 in [29], where $\hat{P}$=$\begin{bmatrix} P_{22} & P_{23} & P_{24} \\ (*) & P_{33} & P_{34} \\ (*) & (*) & P_{44} \end{bmatrix}$. Subsequently, a tight bound inequality constraint for the last two terms in (19), $-d_m \int_{t-d_m}^{t} \dot{x}^T(s)Z_1\dot{x}(s)\,\mathrm{d}s$ and $-\int_{t-d_M}^{t-d_m} \dot{x}^T(s)Z_2\dot{x}(s)\,\mathrm{d}s$, is obtained by applying the Wirtinger's inequality in combination with the extended reciprocally convex inequality. This, in turn, facilitates the development of a tighter bounding inequality condition and simultaneously ensures $\dot{V}(t)$ to be negative definite. Thus, the proposed delay-dependent stabilization criteria provide less conservative delay upper bound compared to [38] and [30].

Remark 4.4. Owing to the presence of nonlinear terms $-MR_1^{-1}M$ and $-d_D MR_2^{-1}M$ in the proposed stabilization criterion (see the expressions for Ξ_{22} and Ξ_{33} in Theorem 4.2), the NLMIs (28) cannot be solved readily for computing the controller gain K. To resolve these nonlinear terms, one can use Lemma 3 (see (5)) as follows:

$$-MR_1^{-1}M \leq R_1 - 2M, \tag{30}$$

$$-d_D MR_2^{-1}M \leq d_D(R_2 - 2M). \tag{31}$$

Using above inequalities, we can rewrite U in (27) as

$$\begin{aligned} U(d(t),\dot{d}(t)) &\leq \bar{U}(d(t),\dot{d}(t)) \\ &= \begin{bmatrix} U_{11}(d(t),\dot{d}(t)) & U_{12} & U_{13} \\ (*) & \bar{U}_{22} & 0 \\ (*) & (*) & \bar{U}_{33} \end{bmatrix}, \end{aligned} \tag{32}$$

$$\bar{U}_{22} = R_1 - 2M, \quad \bar{U}_{33} = d_D(R_2 - 2M),$$

that, in turn, yields the stabilization criterion given in Theorem 4.5 below which can be solved readily by using the standard numerical packages.

Theorem 4.5. Given non-negative scalars d_m, d_M and scalars μ_1 and μ_2 satisfying the conditions in (7), the time-delayed system (6) is asymptotically stable by using the state feedback control law (9), if there exist real symmetric positive-definite matrices $M \in \mathbb{R}^{n\times n}$, $\bar{P} \in \mathbb{R}^{3n\times 3n}$, $\bar{Q}_1 \in \mathbb{R}^{n\times n}$, $\bar{Q}_2 \in \mathbb{R}^{n\times n}$, $\bar{Q}_3 \in \mathbb{R}^{n\times n}$, $\bar{Q}_4 \in \mathbb{R}^{n\times n}$, $R_1 \in \mathbb{R}^{n\times n}$, $R_2 \in \mathbb{R}^{n\times n}$ and the matrices $N \in \mathbb{R}^{m\times n}$, $\bar{S}_1 \in \mathbb{R}^{2n\times 2n}$ and $\bar{S}_2 \in \mathbb{R}^{2n\times 2n}$ such that the following LMIs hold.

$$\begin{gathered}
\begin{bmatrix} \Xi_{11}|_{d(t)=0,\dot{d}(t)=0} - \frac{1}{d_D}\hat{\Psi}_4 & \Xi_{12} & \Xi_{13} & E_2^T\bar{S}_2 \\ (*) & \bar{\Xi}_{22} & 0 & 0 \\ (*) & (*) & \bar{\Xi}_{33} & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0, \\
\begin{bmatrix} \Xi_{11}|_{d(t)=d_m,\dot{d}(t)=\mu_2} - \frac{1}{d_D}\hat{\Psi}_4 & \Xi_{12} & \Xi_{13} & E_2^T\bar{S}_2 \\ (*) & \bar{\Xi}_{22} & 0 & 0 \\ (*) & (*) & \bar{\Xi}_{33} & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0, \\
\begin{bmatrix} \Xi_{11}|_{d(t)=d_M,\dot{d}(t)=0} - \frac{1}{d_D}\hat{\Psi}_5 & \Xi_{12} & \Xi_{13} & E_3^T\bar{S}_1^T \\ (*) & \bar{\Xi}_{22} & 0 & 0 \\ (*) & (*) & \bar{\Xi}_{33} & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0, \\
\begin{bmatrix} \Xi_{11}|_{d(t)=d_M,\dot{d}(t)=\mu_1} - \frac{1}{d_D}\hat{\Psi}_5 & \Xi_{12} & \Xi_{13} & E_3^T\bar{S}_1^T \\ (*) & \bar{\Xi}_{22} & 0 & 0 \\ (*) & (*) & \bar{\Xi}_{33} & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0,
\end{gathered} \tag{33}$$

where

$$\bar{\bar{\Xi}}_{22} = R_1 - 2M, \quad \bar{\bar{\Xi}}_{33} = d_D(R_2 - 2M)$$

and the other elements of matrix variables are same as given in Theorem 4.2.
The state feedback controller is $K = NM^{-1}$.

Remark 4.6. As the conditions given in the right-hand side of (32) are affine functions of $d(t)$ and $\dot{d}(t)$, the sufficient conditions for negative definiteness of the right-hand side of (32) are obtained as $\bar{U}_{[0,0]} < 0$, $\bar{U}_{[d_m,\mu_2]} < 0$, $\bar{U}_{[d_M,0]} < 0$ and $\bar{U}_{[d_M,\mu_1]} < 0$ ([29]), and using Schur complement lemma, the sufficient conditions for the stabilization of system (6) are given in (33) with the delay satisfying (7).

Using the terms $(-R_1 + 2M)$ and $(-R_2 + 2M)$ in place of nonlinear terms $MR_1^{-1}M$ and $MR_2^{-1}M$ makes the stabilization criterion of Theorem 4.5 conservative. Hence to reduce the conservatism, we employ cone complementarity algorithm (CCA) [7] for solving the NLMIs stated in Theorem 4.2. Using CCA, the nonlinear inequalities ($MR_1^{-1}M$ and $MR_2^{-1}M$) in Theorem 4.2 are solved by converting them into nonlinear trace minimization problem with inequality constraints, thus, the state feedback controller for system (6) can be designed with an improved delay upper bound.

The CCA algorithm is presented in the next section.

4.1. Cone complementarity algorithm

In cone complementarity algorithm [7] for solving the NLMIs stated in Theorem 4.2, two new symmetric positive definite matrix variables O_1 and O_2 are defined such that

$$MR_1^{-1}M \geq O_1, \tag{34}$$
$$MR_2^{-1}M \geq O_2. \tag{35}$$

Since the inequalities (34) and (35) are more generalized than (30) and (31), replacing the nonlinear terms $MR_1^{-1}M$ and $MR_2^{-1}M$ respectively with O_1 and O_2 would yield less conservative stabilization criterion than that of Theorem 4.5. Using Schur lemma, the conditions (34) and (35) can be expressed as follows:

$$\begin{bmatrix} O_1^{-1} & M^{-1} \\ M^{-1} & R_1^{-1} \end{bmatrix} \geq 0, \qquad \begin{bmatrix} O_2^{-1} & M^{-1} \\ M^{-1} & R_2^{-1} \end{bmatrix} \geq 0. \tag{36}$$

Now, by letting $D_1 = O_1^{-1}$, $J = M^{-1}$, $Y_1 = R_1^{-1}$, $D_2 = O_2^{-1}$, $Y_2 = R_2^{-1}$, Theorem 4.2 is restated as follows:

Theorem 4.7. Given non-negative scalars d_m, d_M and scalars μ_1 and μ_2 satisfying the conditions (7), the time-delayed system (6) is asymptotically stable by using the state feedback control law (9), if there exist real symmetric positive-definite matrices $M \in \mathbb{R}^{n\times n}$, $\bar{P} \in \mathbb{R}^{3n\times 3n}$, $\bar{Q}_1 \in \mathbb{R}^{n\times n}$, $\bar{Q}_2 \in \mathbb{R}^{n\times n}$, $\bar{Q}_3 \in \mathbb{R}^{n\times n}$, $\bar{Q}_4 \in \mathbb{R}^{n\times n}$, $R_1 \in \mathbb{R}^{n\times n}$, $R_2 \in \mathbb{R}^{n\times n}, O_1 \in \mathbb{R}^{n\times n}, O_2 \in \mathbb{R}^{n\times n}, D_1 \in \mathbb{R}^{n\times n}, D_2 \in \mathbb{R}^{n\times n}, Y_1 \in \mathbb{R}^{n\times n}, Y_2 \in$

$\mathbb{R}^{n\times n}$, $J \in \mathbb{R}^{n\times n}$ and the matrices $N \in \mathbb{R}^{m\times n}$, $\bar{S}_1 \in \mathbb{R}^{2n\times 2n}$ and $\bar{S}_2 \in \mathbb{R}^{2n\times 2n}$ such that the following LMIs hold.

$$\begin{bmatrix} \Xi_{11}(d(t),\dot{d(t)})|_{d(t)=0,\dot{d}(t)=0} - \frac{1}{d_D}\hat{\Psi}_4 & \Xi_{12} & \Xi_{13} & E_2^T\bar{S}_2 \\ (*) & -O_1 & 0 & 0 \\ (*) & (*) & -d_D O_2 & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Xi_{11}(d(t),\dot{d(t)})|_{d(t)=d_m,\dot{d}(t)=\mu_2} - \frac{1}{d_D}\hat{\Psi}_4 & \Xi_{12} & \Xi_{13} & E_2^T\bar{S}_2 \\ (*) & -O_1 & 0 & 0 \\ (*) & (*) & -d_D O_2 & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Xi_{11}(d(t),\dot{d(t)})|_{d(t)=d_M,\dot{d}(t)=0} - \frac{1}{d_D}\hat{\Psi}_5 & \Xi_{12} & \Xi_{13} & E_3^T\bar{S}_1^T \\ (*) & -O_1 & 0 & 0 \\ (*) & (*) & -d_D O_2 & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Xi_{11}(d(t),\dot{d(t)})|_{d(t)=d_M,\dot{d}(t)=\mu_1} - \frac{1}{d_D}\hat{\Psi}_5 & \Xi_{12} & \Xi_{13} & E_3^T\bar{S}_1^T \\ (*) & -O_1 & 0 & 0 \\ (*) & (*) & -d_D O_2 & 0 \\ (*) & (*) & (*) & -d_D\hat{R}_2 \end{bmatrix} < 0, \tag{37}$$

with

$$\begin{bmatrix} D_1 & J \\ J & Y_1 \end{bmatrix} \geq 0, \tag{38}$$

$$\begin{bmatrix} D_2 & J \\ J & Y_2 \end{bmatrix} \geq 0, \tag{39}$$

$$D_1O_1 = I,\ JM = I,\ Y_1R_1 = I,\ D_2O_2 = I,\ Y_2R_2 = I \tag{40}$$

and the other elements of matrix variables are same as given in Theorem 4.2.
The controller is then given by $K = NM^{-1}$.

Since the conditions given in (40) are not LMIs, Theorem 4.7 cannot be solved easily. Using the CCA of [7], this non-convex problem is transformed into nonlinear minimization problem subject to the following LMI constraints:

$$\begin{bmatrix} D_1 & I \\ I & O_1 \end{bmatrix} \geq 0,\quad \begin{bmatrix} J & I \\ I & M \end{bmatrix} \geq 0,\quad \begin{bmatrix} Y_1 & I \\ I & R_1 \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} D_2 & I \\ I & O_2 \end{bmatrix} \geq 0,\quad \begin{bmatrix} Y_2 & I \\ I & R_2 \end{bmatrix} \geq 0. \tag{41}$$

If the conditions hold for $D_1 > 0$, $O_1 > 0$, $J > 0$, $D_2 > 0$, $O_2 > 0$, $Y_1 > 0$ and $Y_2 > 0$, then $tr(D_1O_1) \geq n$, $tr(JM) \geq n$, $tr(Y_1R_1) \geq n$, $tr(D_2O_2) \geq n$, and $tr(Y_2R_2) \geq n$. The equality conditions $tr(D_1O_1) = n$, $tr(JM) = n$, $tr(Y_1R_1) = n$, $tr(D_2O_2) = n$, and $tr(Y_2R_2) = n$ hold if and only if $D_1O_1 = I$, $JM = I$, $Y_1R_1 = I$, $D_2O_2 = I$, and $Y_2R_2 = I$. The equality constraint (first term in (40)), *i. e.,* $D_1O_1 = I$ can be equivalently converted into nonlinear minimization problem with LMI $\begin{bmatrix} D_1 & I \\ I & O_1 \end{bmatrix} \geq 0$ satisfying $D_1 > 0$, $O_1 > 0$ and $tr(D_1O_1) = n$. Hence, by using CCA the non-convex feasibility problem of Theorem 4.7 is transformed into nonlinear trace minimization problem as follows:

$$\min tr\left(JM + \sum_{i=1}^{2} \left(D_iO_i + Y_iR_i \right) \right)$$

subject to constraints from (37) to (39) and (41).

Remark 4.8. Using Schur lemma, the first term in (41) can be equivalently written as $D_1 - O_1^{-1} \geq 0$, then pre-and post multiplying with $O_1^{\frac{1}{2}}$ we obtain $O_1^{\frac{1}{2}} D_1 O_1^{\frac{1}{2}} - O_1^{\frac{1}{2}} O_1^{-1} O_1^{\frac{1}{2}} \geq 0$. Applying trace, we get $tr\{O_1^{\frac{1}{2}} D_1 O_1^{\frac{1}{2}} - O_1^{\frac{1}{2}} O_1^{-1} O_1^{\frac{1}{2}}\} \geq 0$ which implies $tr\{D_1O_1\} \geq n$ (by using $tr\{AB\} = tr\{BA\}$ having compatible dimensions of A and B). When trace is equal to n, it is equivalent to the first equality constant in (41). Similarly, all the remaining terms in (40) have been obtained as given in (41).

The algorithmic steps of CCA are given below.

4.1.1. Algorithm for finding maximum delay upper bound for time-varying delay system

Step 1 For given values of $d_m \neq 0$, μ_1 and μ_2 using Theorem 4.7, obtain the maximum allowable value of d_M. Set $d_{\max} = d_M$.

Step 2 Using $d_{\max}$, find at *zeroth* iteration, a feasible set $(M,\ N,\ \bar{Q}_1,\ \bar{Q}_2,\ \bar{Q}_3,\ \bar{Q}_4,\ R_1,\ R_2, O_1, O_2,\ Y_1,\ Y_2,\ D_1,\ D_2,\ J,\ \bar{S}_1,\ \bar{S}_2)^k$ satisfying the constraints (37) to (39) and (41) with $M > 0$, $\bar{Q}_i > 0$, $i = 1,\ 2,\ 3,\ 4$, $R_j > 0$, $O_j > 0$, $D_j > 0$, $Y_j > 0$, $j = 1,\ 2$. Set $k = 0$.

Step 3 Using the feasible set, solve the following trace minimization problem:

$$\min \ tr\left(\textstyle\sum_{i=1}^{2}(D_iO_i^k + O_iD_i^k + Y_iR_i^k + R_iY_i^k) + JM^k + MJ^k \right) \tag{42}$$

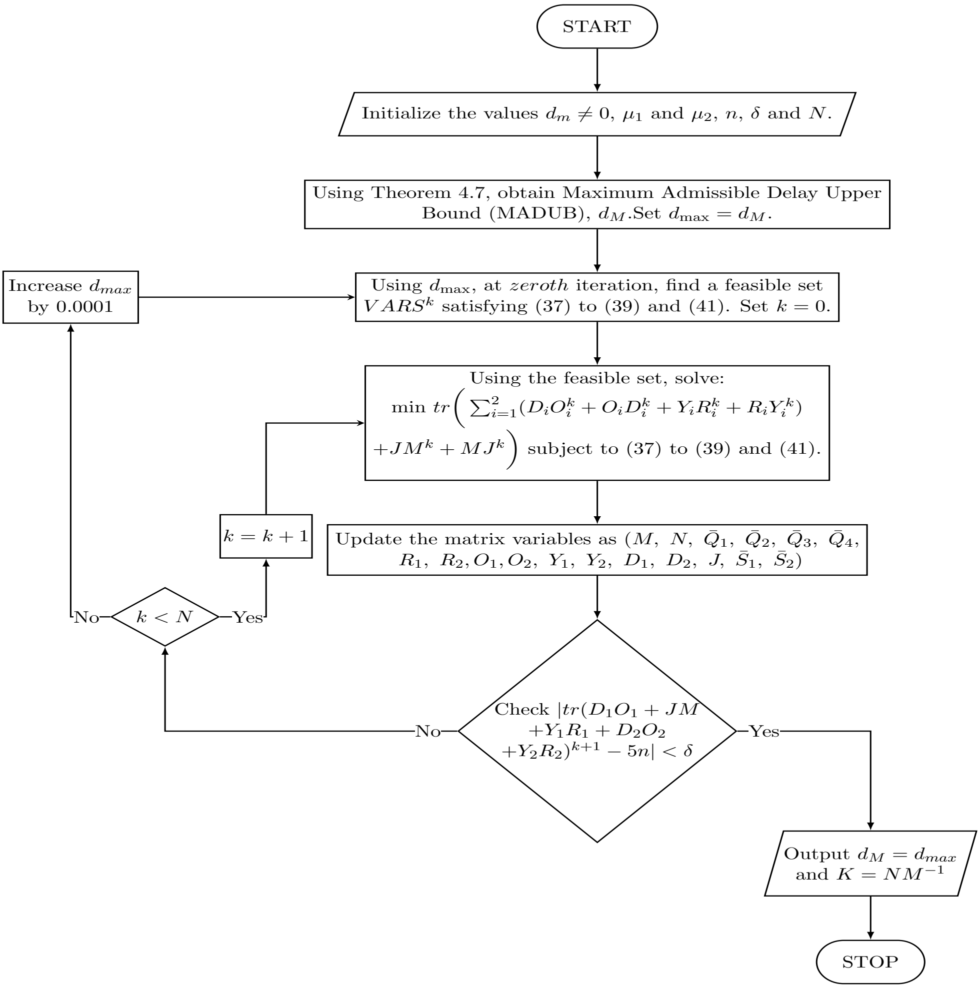
subject to (37) to (39) and (41). Obtain the corresponding solution $(M,\ N,\ \bar{Q}_1,\ \bar{Q}_2,\ \bar{Q}_3,\ \bar{Q}_4,\ R_1,\ R_2, O_1, O_2, Y_1,\ Y_2,\ D_1,\ D_2,\ J,\ \bar{S}_1,\ \bar{S}_2)$.

Step 4 Update the matrix variables as $(M,\ N,\ \bar{Q}_1,\ \bar{Q}_2,\ \bar{Q}_3,\ \bar{Q}_4,\ R_1,\ R_2, O_1, O_2,\ Y_1,\ Y_2,\ D_1,\ D_2,\ J,\ \bar{S}_1,\ \bar{S}_2)$.

If $|tr(D_1O_1 + JM + Y_1R_1 + D_2O_2 + Y_2R_2)^{k+1} - 5n| < \delta$, where δ is a given sufficiently small positive number, output $d_M = d_{\max}$, and $K = NM^{-1}$; exit.

Step 5 If $k < N$, where N is the maximum number of iterations pre-specified, set $k = k + 1$, and then go to Step 3 considering the feasible solution obtained in Step 4, otherwise $d_{\max} = d_{\max} + 0.0001$, and go to Step 2.

The flowchart of CCA is shown in Figure 1:



where

(i) $VARS^k$=$\{\ (M,\ N,\ \bar{Q}_1,\ \bar{Q}_2,\ \bar{Q}_3,\ \bar{Q}_4,\ R_1,\ R_2,\ O_1, O_2,\ Y_1,\ Y_2,\ D_1,\ D_2,\ J,\ \bar{S}_1,\ \bar{S}_2)^k\}$ and these variables are updated based on the solution of the optimization problem given in Step 2.

Fig. 1. Flowchart of CCA

Remark 4.9. Various bounding inequalities and lemmas such as Park's inequality [9], Finsler's lemma [38], reciprocally convex combination lemma [30], the free-weighting matrix-based inequality ([17, 37]), zero inequalities [16], relaxation based approach with slack variables [23] etc., have extensively been used for deriving the stabilization conditions of time-delay systems. Whereas, in Theorem 4.2, we have used Wirtinger's inequality along with the extended reciprocally convex inequality to derive the stabilization conditions in NLMIs, where Lemma 5 is utilized to convert the conditions into LMI form given in Theorem 4.5 and the conservatism is reduced further using CCA by converting non-convex problem into nonlinear trace minimization problem given in Theorem 4.7. This, in turn, facilitates to obtain a less conservative stabilization criterion in LMI framework but with some reasonable computational burden.

Below in Table 1 the number of decision variables (N_c) involved in the proposed stabilization criteria and the existing methods is calculated and depicted. It is shown that with given number of the decision variables, less conservative results are obtained compared to existing methods.

Method	N_c
[10]	$13n^2 + 5n + 2nm$
[38]	$2n^2 + 2n + nm$
[30]	$8n^2 + 4n + nm$
[37]	$6n^2 + 2n + nm$
[24]	$16n^2 + 8n + nm$
[17]	$10n^2 + 3n + nm$
[23]	$9n^2 + 5n + nm$
[28]	$11.5n^2 + 6.5n + nm$
[16]	$75.5n^2 + 9.5n + nm$
Theorem 4.5	$16n^2 + 8n + nm$
Theorem 4.7	$19.5n^2 + 11.5n + nm$

Tab. 1. Number of decision variables.

5. NUMERICAL EXAMPLES

To illustrate the effectiveness of the proposed stabilization criteria, three numerical examples are considered in this section. The superiority of the proposed result has been shown compared to existing literature.

Example 5.1. For the linear time-delayed system (6), the following state-space matrices are considered [38]:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \qquad A_d = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{43}$$

Note that the delay-free system (43) is unstable with $u(t) \equiv 0$. When state-feedback control is applied, the system becomes stable and from Tables 2 and 3, it follows that the matrix $A + BK - A_d$ is not Hurwitz stable and the closed-loop stability becomes delay-dependent ([12]). Thus, the maximum admissible delay upper bound (MADUB) d_M and the corresponding controller gain K for various delay lower bounds with different delay derivatives are computed using the $feasp$ and $mincx$ solver of LMI toolbox for the proposed stability criteria, listed in Tables 2 and 3.

Method	d_m	d_M	Iterations	K	$\lambda_i(A+BK-A_d)$
[38]	0.2	0.846	-	$\begin{bmatrix}-0.5707 & -2.7524\end{bmatrix}$	-0.1947, 0.5223
[30]	0.2	1.385	-	$\begin{bmatrix}-0.1354 & -1.7510\end{bmatrix}$	0.7882, 0.3608
Theorem 4.5	0.2	1.637	-	$\begin{bmatrix}-0.1732 & -1.7366\end{bmatrix}$	0.6238, 0.5396
Theorem 4.2 (Using CCA)	0.2	1.8	8	$\begin{bmatrix}-1.0826 & -2.9990\end{bmatrix}$	0.0881 , -0.1871
	0.2	2.0	11	$\begin{bmatrix}-1.2884 & -3.5322\end{bmatrix}$	0.3500 , -0.9822
	0.2	2.5	26	$\begin{bmatrix}-1.7340 & -4.5645\end{bmatrix}$	0.4558, -3.5233
	0.2	3.2	110	$\begin{bmatrix}-6.7429 & -12.7797\end{bmatrix}$	0.4023, -10.2821
	0.2	3.4	169	$\begin{bmatrix}-9.1306 & -16.4263\end{bmatrix}$	0.3878, -13.9141
[38]	0.5	0.967	-	$\begin{bmatrix}-0.5290 & -2.7166\end{bmatrix}$	0.6358, -0.4524
[30]	0.5	1.535	-	$\begin{bmatrix}-0.2172 & -2.0351\end{bmatrix}$	0.7564, 0.1085
Theorem 4.5	0.5	1.765	-	$\begin{bmatrix}-0.1588 & -1.7195\end{bmatrix}$	0.6856, 0.4949
Theorem 4.2 (Using CCA)	0.5	1.8	6	$\begin{bmatrix}-0.9133 & -2.5633\end{bmatrix}$	$0.1684 \pm 0.4709i$
	0.5	2.0	10	$\begin{bmatrix}-1.1142 & -3.2234\end{bmatrix}$	0.3234, -0.6468
	0.5	2.5	17	$\begin{bmatrix}-1.6330 & -4.4275\end{bmatrix}$	0.4519, -1.9794
	0.5	3.2	85	$\begin{bmatrix}-6.1191 & -12.2055\end{bmatrix}$	0.4300, -9.7356
	0.5	3.5	168	$\begin{bmatrix}-10.7524 & -19.6133\end{bmatrix}$	0.4066, -17.1199

− refers to controller parameters obtained by solving LMIs only.

Tab. 2. MADUB d_M and K for two d_m values with $\mu_1 = 0$ and $\mu_2 = 0.5$ for time-varying delay system (Example 5.1).

From the Table 2 and 3, it is shown that the proposed criteria (Theorem 4.2 with CCA and Theorem 4.5) provide less conservative result compared to existing techniques due to the new augmented state vector (12) of the LK functional (11). It is clearly seen that by using CCA the conservatism is further reduced for linear time-delay system.

Example 5.2. Consider the linear time-delayed system (6) with the following state-space matrices as given in [30]:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \qquad A_d = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{44}$$

The maximum admissible delay upper bound d_M and the controller gain K are listed in Table 4 for two delay lower bounds with $\dot{d}(t) = 0.9$. It is apparent that the proposed theorems provide less conservative delay upper bound compared to the existing techniques using the designed state feedback controller that stabilizes the system (44).

Method	d_m	d_M	Iterations	K	$\lambda_i(A+BK-A_d)$
[38]	0.2	0.653	-	$\begin{bmatrix}-0.3349 & -2.7029\end{bmatrix}$	0.7897 -0.5926
[30]	0.2	1.340	-	$\begin{bmatrix}-0.1262 & -1.5921\end{bmatrix}$	0.6540 ± 0.0803i
Theorem 4.5	0.2	1.610	-	$\begin{bmatrix}-0.1378 & -1.6387\end{bmatrix}$	0.6306 ± 0.0372i
Theorem 4.2 (Using CCA)	0.2	1.8	24	$\begin{bmatrix}-0.7756 & -3.0502\end{bmatrix}$	0.5416, -0.6918
	0.2	1.9	26	$\begin{bmatrix}-0.9478 & -3.6155\end{bmatrix}$	0.5886, -1.3042
	0.2	2.3	93	$\begin{bmatrix}-3.7697 & -10.9727\end{bmatrix}$	0.6107, -8.6835
	0.2	2.4	138	$\begin{bmatrix}-5.1701 & -14.2078\end{bmatrix}$	0.5994, -11.9072
[38]	0.5	0.776	-	$\begin{bmatrix}-0.2628 & -2.4470\end{bmatrix}$	0.8057, -0.3527
[30]	0.5	1.476	-	$\begin{bmatrix}-0.1930 & -1.8036\end{bmatrix}$	0.6537, 0.4427
Theorem 4.5	0.5	1.722	-	$\begin{bmatrix}-0.1164 & -1.6121\end{bmatrix}$	0.7458, 0.5421
Theorem 4.2 (Using CCA)	0.5	1.8	16	$\begin{bmatrix}-0.7035 & -2.6119\end{bmatrix}$	0.3148, -0.0267
	0.5	1.9	17	$\begin{bmatrix}-0.7185 & -2.6885\end{bmatrix}$	0.3908, -0.1793
	0.5	2.5	79	$\begin{bmatrix}-4.7606 & -12.9012\end{bmatrix}$	0.5893, -10.5905
	0.5	2.65	116	$\begin{bmatrix}-9.3188 & -22.6406\end{bmatrix}$	0.5626, -20.3032

Tab. 3. MADUB d_M and K for two d_m values with $\mu_1 = 0$ and $\mu_2 = 0.9$ for time-varying delay system (Example 5.1).

Method	d_m	d_M	Iterations	K	$\lambda_i(A+BK-A_d)$
[38]	0.2	0.484	-	$\begin{bmatrix}-12.4665 & -7.2081\end{bmatrix}$	0.9952, -4.2033
[30]	0.2	0.893	-	$\begin{bmatrix}-4.0419 & -3.0188\end{bmatrix}$	0.9979, -0.0167
Theorem 4.5	0.2	0.926	-	$\begin{bmatrix}-2.5791 & -2.2868\end{bmatrix}$	0.9901, 0.7231
Theorem 4.2 (Using CCA alg.)	0.2	0.975	58	$\begin{bmatrix}-4.8648 & -6.1363\end{bmatrix}$	1.5740, -3.7103
	0.2	1.158	149	$\begin{bmatrix}-24.8845 & -20.3340\end{bmatrix}$	1.2630, -17.5970
	0.2	1.189	179	$\begin{bmatrix}-31.0580 & -24.1438\end{bmatrix}$	1.3387, -21.4825
[38]	0.5	0.565	-	$\begin{bmatrix}-9.7549 & -5.8548\end{bmatrix}$	0.9941, -2.8489
[30]	0.5	0.936	-	$\begin{bmatrix}-6.8641 & -3.8576\end{bmatrix}$	0.6081, -0.4657
Theorem 4.5	0.5	0.985	-	$\begin{bmatrix}-3.4939 & -2.7430\end{bmatrix}$	0.9946, 0.2624
Theorem 4.2 (Using CCA alg.)	0.5	1.045	46	$\begin{bmatrix}-7.2442 & -7.7505\end{bmatrix}$	1.5005, -5.2510
	0.5	1.225	89	$\begin{bmatrix}-20.9740 & -15.6069\end{bmatrix}$	1.2963, -12.9032
	0.5	1.325	142	$\begin{bmatrix}-36.5664 & -23.9152\end{bmatrix}$	1.2094, -21.1246

Tab. 4. MADUB d_M and K for two values of d_m with $\mu_1 = 0$ and $\mu_2 = 0.9$ (Example 5.2).

The closed-loop state trajectories of the system (44) with the stabilizing controller $K = \begin{bmatrix}-36.5664 & -23.9152\end{bmatrix}$ obtained from proposed criterion (see Table 4) for $d(t) = 0.5+0.825|sin(1.0909t)|$ are shown in Figure 2 and the control input is shown in Figure 3 considering $x(t) = \phi(t) = [1 \quad -1]^T$ for $t \in [-d_M, 0]$. The computational time is calculated as 8.4048 sec to obtain d_M=1.325 for d_m=0.5 and $\dot{d}(t)$=0.9.

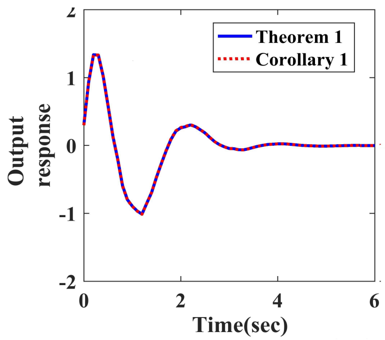


Fig. 2. State trajectories of the stabilized system for $d_m = 0.5, d_M = 1.325$ and $|\dot{d}(t)| = 0.9$. (Example 5.2).

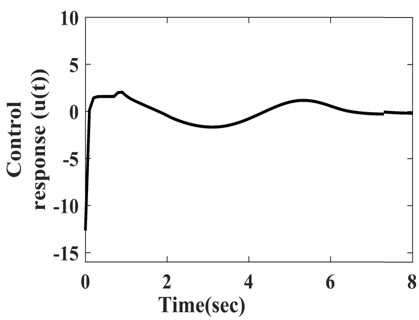


Fig. 3. Control input to the system (Example 5.2).

6. CONCLUSION

The stabilization problem of linear time-varying delay systems has been addressed by constructing a new LK functional to provide an improved delay upper bound *i. e.,* to obtain less conservative stability conditions compared with the earlier reported results. The preferable inequalities (Wirtinger's and Extended reciprocal convex inequalities) are employed to obtain a tighter bound for the integral terms in the derivative of the LK functional, which helps us to reduce further conservatism. The proposed delay-dependent stabilization criterion has been derived in NLMI framework and it has been solved through CCA using nonlinear trace minimization problems. This, in turn, yields a state-feedback controller that ensures the asymptotic stability of the time-varying delayed system with less control effort. Finally, three numerical examples have been considered to demonstrate that the proposed criteria provide less conservative results compared with the existing results. As future work, the presented method will be extended to observer-based control scheme for linear time-varying delay systems.

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