PREFACE

This book contains the foundations of the theory of groupoids and groups. A groupoid is a nonempty set on which there is defined a binary operation, called multiplication, associating with every two-membered sequence of elements of the set again an element of the latter. Generally there are no postulates as regards the multiplication. The concept of a groupoid forms the basis of an extensive theory of groupoids which, though rather general, considerably approximates the properties of groups. The theory of groupoids is founded on the theory of the decompositions in sets, on the one hand, and on the concept of homomorphic mapping, on the other hand.

The theory of decompositons in sets was founded by the author in about 1939, independently of the theory of equivalence relations developed at about the same time by P. DUBREIL and M.-L. DUBREIL-JACOTIN (1937) and O. ORE (1942). Between the two theories there is no essential difference; the theory of decompositions in sets can, however, in certain cases be more conveniently applied because its basis, the concept of a decomposition in a set is purely of set-theoretical character and so less complicated than the concept of an equivalence relation. Since their origin, both the theories have been considerably developed and often applied to problems of various branches of mathematics. Employing the concepts and methods relative to sets and lattices, the theory of decompositions in sets describes situations occurring in connection with its basic concept. The decompositions in sets implied in the theory of groupoids are mostly of algebraic character, that is to say, are bound by certain relations with the multiplication, as—for example—the decompositions corresponding to homomorphic mappings. The theory of such decompositions is, in fact, the essence of the theory of groupoids. This is, naturally, true even for groups, which are groupoids with special properties of the multiplication.

The present book is based on two editions of my text-book "Introduction into the theory of groups" which met with most favourable criticism in the literature. It has, however, been largely extended and contains a number of genuine results due to the mentioned concept of the subject; the latter are, for the most part, closely connected with the classical theorems of the theory of groups. That applies, in particular, to the theory of series of decompositions in sets and their application to scientific classifications as well as to the corresponding algebraic theories of the series of factoroids and factor groups.

The book consists of three chapters of about the same length: I. Sets, II. Groupoids, III. Groups. The chapters are, so to say, simply mapped onto one another, since to the single situations concerning sets and dealt with in Chapter I there correspond, in Chapters II and III, analogous algebraic situations concerning groupoids and groups, respectively. This method of exposition seems particularly useful from the didactic point of view because the simple notions relative to sets take on more complicated forms in case of groupoids and groups; that leads to a better understanding of the structure of the concepts and methods of the algebraic theories in question and helps to formulate the most satisfactory proofs. The book also suggests many new ways of developing the mentioned theory and leads the reader to independent scientific work.

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