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Years Ago

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Eduard Čech

Eduard Čech (1893–1960), the internationally famous topologist, was the leading Czech mathematician of his generation. He was born in Stracov in northeastern Bohemia, and went to the Charles University in Prague to study mathematics in 1912. But his university studies were interrupted by the First World War, and he only graduated in 1920. By then he had become interested in projective differential geometry, which studies those features of the geometry of embedded curves, surfaces, and higher-dimensional spaces that are projectively invariant. Čech's work emphasised results about tangency, correspondences between manifolds, and (notable for his later work in topology) the systematic theory of duality in projective spaces. His developing interests in this relatively new field soon earned him a scholarship to study with Fubini, its acknowledged leader, in Turin in 1921–22. The visit was fruitful, and they went on to write two books on the subject, in 1927 and 1931. Čech returned to his native country and, after his habilitation (the degree that qualified him to be a university teacher), took up an extraordinary professorship at the Masaryk University in Brno. There he lectured on analysis and algebra for 12 years; this may well have deepened his interest in topology. In 1928 he became a full professor, and, inspired by the papers in the Polish journal *Fundamenta Mathematica*, he turned increasingly to topology: all his papers after 1931 were on that subject.

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The subject of topology was already dividing into two branches: point-set topology, which studies topological spaces directly, and algebraic topology, which studies spaces by passing to algebraic objects that may be attached to them. One might also distinguish between the study of the topological properties of most spaces and the study of important special cases, or, if you prefer, the properties of spaces in general and the special properties of perverse spaces. Most topological spaces which arise naturally in other branches of mathematics have simple properties. The difficulty arises in giving suitable, intrinsic foundations for the subject of topology, and the role of the otherwise strange, artificial spaces studied by point-set topologists is to test the utility of definitions and the generality of proofs. The Polish school excelled in work of this kind. Algebraic topology, on the other hand, aims to distinguish between, even to characterise topological spaces by computing algebraic objects, such as groups or rings, that can be attached to them. It is not restricted to the study of well-behaved spaces, but such are the computational problems which arise that in practice most interest attaches to the study of tractable examples. Happily, these have tended to be the spaces that arise naturally in other contexts. However, the tools of algebraic topology are often insensitive. For example, there are many contractible spaces that are not homeomorphic, but all necessarily have the same homology groups. Throughout the 1930s there was considerable interest in extending the tools of algebraic topology to deal with such problems.

Čech's first contributions in topology were characteristically broad and aimed at keeping the subjects of algebraic and point-set topology together. In his first two papers he developed a homology theory for arbitrary spaces, and established general duality theorems

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for manifolds, generalising the classical duality of subspaces of projective spaces. Čech's approach to homology theory was deliberately intended to be very general, as the title of his paper in *Fundamenta Mathematica* for 1932 makes clear ("General homology theory in an arbitrary space"). It is based on the idea of studying all finite open coverings of a given space. The paper introducing this idea has become one of his best known, although many of the ideas in it can be traced back to earlier work of Vietoris and Alexandrov. Čech's originality lay in introducing the concept of an inverse limit to obtain homology groups independent of the choice of covering. The approach turns out to work very well for compact spaces, and this paper is the origin of what is now called the Čech homology theory. The corresponding cohomology theory worked less well for noncompact spaces, giving unexpected results even for the first cohomology group of the real line. A better way forward was found by C.H. Dowker, who replaced Čech's finite coverings with arbitrary coverings. With this modification, Čech cohomology was shown by Eilenberg and Steenrod to satisfy all the axioms they postulated for a cohomology theory (unlike the Čech homology theory with integer coefficients, which fails the excision axiom).

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In another paper, presented to the International Congress of Mathematicians in Zürich in 1932, Čech presented his ideas on the definition of the higher homotopy groups of a space. Unhappily, the report on his talk was brief and obscure, and Čech never returned to the subject himself, thus leaving it open for the decisive contributions of Hurewicz. Hurewicz later wrote that his definition agreed with Čech's, and Alexandrov was to single out Čech's contribution in 1961, when commemorating Čech's life and work, and to lament that it had been misunderstood. In 1934 Čech extended his approach to homology theory to obtain a theory of local homology, appropriate to the study of neighbourhoods of a topological space.

His reputation grew, and when he reported on his results at an international conference on topology held in Moscow in 1935 Lefschetz invited him to visit the newly-founded Institute for Advanced Study in Princeton. Upon his return, Čech founded a topology seminar at Brno which applied itself to the work of Alexandrov and Urysohn. In three years the seminar published 26 papers, including Čech's "On Bicompact spaces" which appeared in *Annals of Mathematics*. In this paper he introduced the idea of the (Stone)-Čech compactification of a regular topological space. The seminar continued until 1939, when the Germans invaded Czechoslovakia and closed all the universities. Thereafter it continued in the flat of Čech's student B. Pospíšil, until 1941, when the Gestapo arrested Pospíšil. The seminar had a lasting influence on the development of mathematics in Czechoslovakia, because it marked the introduction of the method of working on mathematical problems in groups.

Eduard Čech

After the War, Čech moved to the Charles University in Prague. Now in his fifties, he began an intensive administrative career. In 1947 he was appointed Director of the Mathematical Research Institute of the Czech Academy of Sciences and Arts and in 1950 a Director of the Central Mathematical Institute. In 1952 the Central Institute was incorporated into the Mathematical Institute of the Czechoslovak Academy of Sciences, with Čech as its first Director. That year he returned to the Charles University to head its newly-founded Mathematical Institute. But he also found time to redirect his mathematical interests; in the 1950s he wrote 17 papers on differential geometry. He also deepened his interests in the teaching of mathematics. He wrote seven textbooks for secondary schools and held seminars on elementary mathematics at both Brno and Prague.