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EXISTENCE OF UNIVERSAL CONNECTIONS

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This is a report on a paper, written in collaboration with S. RAMANAN, which will appear in a forthcoming issue of the American Journal of Mathematics.

The purpose of the paper is to prove the existence of universal connections for principal bundles with a compact Lie group as structure group. We prove that, given a compact Lie group G and a positive integer n, there exist a differentiable principal G-bundle E and a connection γ_0 on E such that any connection on a differentiable principal G-bundle P with base of dimension $\leqq n$ can be obtained as the inverse image of the connection γ_0 by a differentiable bundle homomorphism of P into E. As is well-known, the analogous problem for bundles without connections is treated in the topology of fibre bundles.

It is also known that the Stiefel bundles play the role of universal bundles for the unitary groups $U(k)$. One can define in a natural way a connection on every Stiefel bundle. We prove that these connections themselves are universal for connections in $U(k)$-bundles.

In the unitary case the problem is first solved locally by explicit construction. The local solutions are then pieced up with the help of a special type of covering by coordinate cells.

In the general case, the compact Lie group G is identified with a closed subgroup of a unitary group. Starting from a universal connection for this unitary group, a universal connection for G is constructed by generalizing the usual method of construction of an invariant connection in the principal bundle associated with a Lie group and a closed subgroup.

A theorem of A. WEIL asserts that the cohomology classes of the base of a principal G-bundle obtained by substitution of the curvature form of a connection on P in the invariant polynomials of G is independent of the connection. Our result seems to explain this invariance and furnishes an alternate proof in the case of compact Lie groups.