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On envelopes in a certain category of ordered topological vector spaces

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ON ENVELOPES IN A CERTAIN CATEGORY OF ORDERED TOPOLOGICAL VECTOR SPACES

by

Jürgen FLACHSMEYER

The lecture is on a paper which will soon appear under the title: Envelopes in the category of Kakutami-M-spaces (Mannheim Conference on Categorial Topology, 1975). Considered are the following categories:

1. Bool Alg (Boolean algebras and their Boolean homomorphisms)

2. $Comp_0$ (Zero-dim. compact Hausdorff spaces and their cont. maps)

3. Kaku MI (Kakutani M-spaces with order unit and their homomorphisms)

4. Comp (Compact Hausdorff spaces and their cont. maps)

What is a Kakutani-M-space ?

This is a Banach space $\mathbb{M}$ over $\mathbb{R}$ equipped with a lattice order $\leq$ such that these structures are related as follows:

$|x| \leq |y| \Longrightarrow \| x \| \leq \| y \|$. This means $\mathbb{M}$ is a Banach-lattice.

The M-condition holds:

$$x \geq 0, \ y \geq 0 \Longrightarrow \| x \vee y \| = \| x \| \vee \| y \|$$

order unit? This means an element $\breve{u}$ in the unit ball $\{x \mid \|x\| \leq 1\}$ such that this ball is equal to an interval $[-\breve{u},\breve{u}]$, i.e. $\breve{u}$ is a greatest (unique!) element in the ball.

The homomorphisms are the continuous linear lattice homomorphisms which preserve the ur .

There is a dual equivalence of

Bool Alg and $Comp_0$ by the

Stone-functor and its converse:

$\mathbb{B}$ - - - - - - - - - -> Spec ($\mathbb{B}$) .

Stone representation

There is a dual equivalence of

Kaku MI and Comp by the

Kakutani-functor and its converse:

$\mathbb{M}$ - - - - - - - - - -> Spec ($\mathbb{M}$) .

Kakutani representation

The talk now is about the categorical notions of envelopes and injective hulls in Kaku MI.

These things are described as special order extensions.

It is shown that generalizations of theorems of Sikorski given for Bool Alg concerning envelopes and injective hulls there can be given for Kaku MI here.