Kazimierz Musiał

# The weak Radon-Nikodym property

In: Zdeněk Frolík (ed.): Abstracta. 4th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1976. pp. 123–124.

Persistent URL: http://dml.cz/dmlcz/701056

FOURTH WINTER SCHOOL (1976)

# THE WEAK RADON-NIKODYM PROPERTY

by

K. MUSIAL

Def. A Banach space X has the weak RNP iff given any finite complete measure space $(S, \Sigma, \mu)$ and a $\mu$-continuous measure $\nu : \Sigma \longrightarrow X$ of $(\sigma)$-finite variation there exists a weakly measurable $f \cdot S \longrightarrow X$ such that

$$\forall E \in \Sigma \qquad \nu(E) = \text{Pettis} - \int_E f d\mu \ .$$

Theorem. If X is separable then $X^*$ has WRNP iff $X \not\ni \ell_1$ (isomorphically).

Corollary 1. If X is separable and $X^{**} = \bigcup_{\alpha < \omega_1} X_\alpha$ ($X_\alpha = w^*$-sequential closure of $X_{\alpha-1}$ whenever $\alpha$ is non limit and $X_\alpha = \bigcup_{\beta < \alpha} X_\beta$ if $\alpha$ is limit, $X_0 = X$), then X is weak$^*$ sequentially dense in $X^{**}$.

Corollary 2. If X is separable, $X \not\ni \ell_1$ and $X^*$ is non-separable, then given a non purely atomic finite measure space $(S, \Sigma, \mu)$ there exists a bounded Pettis integrable function $f: S \longrightarrow X^*$ which is not weakly equivalent to any strongly measurable $g : S \longrightarrow X^*$ .

Corollary 3 (Rybakov). If the set of extreme points of the unit ball of $X^*$ is norm separable, then $X^*$ is separable.

Remark. There exists X with WRNP and without RNP.