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A new result of G. A. Edgar on representing points in a convex bounded subset of Banach spaces with the Radon-Nikodym property as barycentres of Radon measures

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A NEW RESULT OF G.A. EDGAR ON REPRESENTING POINTS IN A CONVEX BOUNDED SUBSET OF BANACH SPACES WITH THE RADON-NIKODYM PROPERTY AS BARYCENTRES OF RADON MEASURES

by

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A well-known theorem of Choquet states that if K is a convex metrizable compact in a locally convex space then for each $x \in K$ there exists a Borel measure μ supported by the set Extr (K) (i.e. extremal points of K) such that

$x = \int_K \mathrm{Id}\, d\mu$ (Id stands for identity map).

Two years ago, Edgar proved the following

Theorem: If K is a convex closed bounded subset of a Banach space with the RNP then for each $x \in K$ there exists a Borel measure μ supported by Extr (K) such that $x =$ $= \int_K \mathrm{Id}\, d\mu$

The case when K is non separable remained open. Recently, Edgar has defined a partial order relation $\prec$ between measures defined on a given convex set such that we have the following:

Theorem (Edgar): If K is a closed convex bounded subset of a Banach space with the RNP, then for each $x \in K$ there exists a Radon measure μ on K, maximal with respect to $\prec$ such that $x = \int_K \mathrm{Id}\, d\mu$. If in addition K is separable then "maximal measure" means just the same as "supported by extremal points of K ".

In general (i.e. when K is not separable) it could happen that the support of a maximal measure is disjoint with the set of extreme points of K. This has been shown by an example due to W.J. Davis, G.A. Edgar and W.B. Johnson.