

Luděk Zajíček

On the differentiation of convex functions

In: Zdeněk Frolík (ed.): Abstracta. 4th Winter School on Abstract Analysis.  
Czechoslovak Academy of Sciences, Praha, 1976. pp. 159.

Persistent URL: <http://dml.cz/dmlcz/701068>

## Terms of use:

© Institute of Mathematics of the Academy of Sciences of the Czech Republic,  
1976

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project  
*DML-CZ: The Czech Digital Mathematics Library* <http://dml.cz>

FOURTH WINTER SCHOOL (1976)

## ON THE DIFFERENTIATION OF CONVEX FUNCTIONS

by

L. ZAJÍČEK

If  $f$  is a real continuous convex function which domain  $D_f$  is an open convex subset of a Banach space  $B$ , we denote by  $N(f)$  the set of all points  $x \in D_f$  at which  $f$  is not Gâteaux differentiable.

Definition: A set  $X \subset B$  is called  $(C - C)$ -hypersurface if there exist a closed hyperplane  $H \subset B$ ; a vector  $v \notin H$  and two continuous convex functions  $f, g$  defined on  $H$  such that

$$X = \{x + (f(x) - g(x)) \cdot v, x \in H\}$$

Theorem (i) If  $f$  is a continuous convex function in a separable Banach space then  $N(f)$  can be covered by a countable union of  $(C - C)$ -hypersurfaces.

(ii) If  $A \subset B$  is a countable union of  $(C - C)$ -hypersurfaces then there exists a continuous convex function  $f$  on  $B$  such that  $A \subset N(f)$ .

Remark: It follows from independent results of N. Aronszajn and R. Phelps on Gâteaux differentiation of Lipschitz functions that  $N(f)$  and consequently any  $(C - C)$ -hypersurface in separable Banach space is of measure zero for any Gaussian measure on  $B$ . But I am not able to prove it directly using the above theorem.