

J. P. R. Christensen

Submeasure and measure

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SUBMEASURES AND MEASURES

J.P.R. Christensen

Let (X, \mathcal{A}) be a set with a Boolean algebra \mathcal{A} of subsets. A submeasure φ on \mathcal{A} is a setfunction which has the properties

- i) $\varphi(\emptyset) = 0$; $A \subseteq B \implies \varphi(A) \leq \varphi(B)$
- ii) $\varphi(A \cup B) \leq \varphi(A) + \varphi(B)$.

The submeasure φ is pathological if there does not exist a non trivial finitely additive non negative measure dominated by φ . The existence of pathological submeasures has been shown in [1] and independently by Preiss & Villomovsky and by Popov .

The main open problem in the subject is the control measure problem , which is equivalent with the following problem .

The submeasure φ is called a Maharam submeasure if it is defined on a σ -field and sequentially point continuous (see [1]) . The control measure problem is equivalent with the problem whether or not every Maharam submeasure admits a probability (countably additive) with the same zero sets. It was shown that if a control measure exists then there exists a control measure dominated by φ . In fact the problem is equivalent with whether or not the Maharam submeasure is pathological.

Our main result so far is that a control measure exist if and only if the Maharam submeasure φ defined on the measurable space (X, \mathcal{B}) fulfills the condition:

We consider the unit interval I with usual Borel structure and Lebesgue measure ν . Let $A \subseteq X \times I$ be a measurable subset and suppose

$$\forall x \in X : \nu(\{t \in I \mid (x, t) \in A\}) = 0.$$

Then there exists a $t_0 \in I$ such that

$$\varphi(\{x \in X \mid (x, t_0) \in A\}) = 0.$$

If the above statement is true (for every measurable set A) then there is a control measure. The condition is of course necessary (Fubini theorem). The proof that the condition is sufficient also can be found in [2].

It is not known whether a translation invariant Maharam submeasure defined on the Borel subsets of a compact metrizable abelian group is pathological.

It is not known whether a control measure exists for measures taking values in the space of equivalence classes of measurable functions (with topology of convergence in measure).

- 1) Wojciech Herer & Jens Peter Reus Christensen, On the existence of pathological submeasures and the construction of exotic topological groups, Math. Ann. 213, 203-210 (1975).
- 2) Jens Peter Reus Christensen, Some results with relation to the control measure problem, To appear.