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THE GENERALIZED REGULAR FUNCTIONS OVER CONFORMAL QUATERNIONIC MANIFOLD

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It is well known, that the system of all regular quaternionic functions (in Fueter sense, see [1]) is not closed with respect to composition. Hence we can not define the notion of regular function of quaternionic variable over certain class of quaternionic manifolds as in the theory ao complex variable.

We must define the notion of "regular function" as a section of a special canonical fibre bundle over conformal quaternionic manifold

The set of quaternions will be denoted by H, the set of all regular functions overa open subset U will be denoted by O(U).

Let us denote by G the conformal group of quaternions, it means the group of all mappings of the form $(a + bq)(c + dq)^{-1}$, where $a,b,c,d \in H$ and $ad - cb \neq 0$.

We say, that the real four-dimensional manifold M is a conformal quaternionic manifold, if there exists the atlas on M such, that the transition functions belong to G.

The following theorem contains a "pseudogroup property" of O(U) with respect to G. For proof see [1].

<u>Theorem:</u>

Let $f(q) = (a + bq)(c + dq)^{-1}$ be an element of G. Suppose, that f is continuous on U. Let us denote by $J_f(q)$ the function

$J_f(q) = (c + dq)^{-1} . |c + dq|^{-2}$. Then $F:U \longrightarrow H$ is regular on U if and only if the function $J_f(q) . F \circ f(q)$ is regular on $f^{-1}(U)$.

The main theorem of this paper is the following:

<u>Theorem</u>:

Let $f,g \in G$, then $J_{f\circ g}(q) = J_g(q).J_f(g(q))$.

This theorem contains a "chain law" for the functions J.

Now we can define a line fiber bundle A(M), which forms a suitable space for regular sections over conformal manifold M.

<u>Definition</u>:

Consider the trivialisation (U_i, p_i) of a conformal manifold M.

Denote $p_{ij} = p_j \circ p_i^{-1}$.

Over each U_i we dwfine A(M) to be trivial, isomorphic to $U_i x H$.

The transition functions are the following ones:

$$U_i \times H \longmapsto U_j \times H$$
$$(x, q_i) \longmapsto (x, q_j)$$
$$q_j = J^{-1}_{p_{ij}}(p_i(x)).q_i$$

It can be shown, that this object is well defined.

Now we define a notion of regular section of A(M).

<u>Definition</u>:

We say, that a section $u: M \longrightarrow A(M)$ is regular, if for each trivialisation (U_i, p_i) of M the function $u_i \circ p_i^{-1}$, where u_i is the trivialisation of u over U_i, belongs to O(U).

It can be shown, that this definition is corect, it means, that regularity not depends from a choice of trivialisation.

It is a consequance of a previous theorem.

References

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