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TRANSVERSE G-STRUCTURES ON

FOLIATED MANIFOLDS

Pierre Molino

Let M be a compact connected n -dimensional manifold endowed with a q -codimension foliation \mathcal{F} . All the structures are assumed to be C^∞ .

1. Transverse fields ; transverse G-structures

We denote by P the subbundle of TM tangent to the leaves of the foliation. $Q = TM/P$ is the transverse bundle of (M, \mathcal{F}) .

If X is a foliated vector field, it defines a section \bar{X} of Q . \bar{X} is the transverse field associated to X . The set $\mathcal{L}(M, \mathcal{F})$ of transverse fields has a natural Lie algebra structure.

We denote by $E_T(M, P_T, GL(q, R))$ the principal bundle of frames of Q . E_T is the bundle of transverse frames of (M, \mathcal{F}) . It is endowed with a natural structure form θ_T , which is a R^q -valued tensorial form. Using θ_T , we define on E_T a lifted foliation \mathcal{F}_T in the following way : $X_z \in T_z E_T$ is tangent to the leaf of \mathcal{F}_T at z iff $i_{X_z} \theta_T = i_{X_z} d\theta_T = 0$. We denote by P_T the subbundle of TE_T tangent to the leaves of the lifted foliation.

If $e_T(M, P_T, G)$ is a G -subbundle of E_T such that

$$P_T \subset T_z(e_T) \quad \forall z \in e_T,$$

e_T is a transverse [or foliated] G-structure on (M, \mathcal{F}) [1] [3] [4].

2. Transverse parallelisms ; Lie foliations.

If $G = \{e\}$, a transverse G -structure on (M, \mathcal{F}) is a transver-

se parallelism [1] [5]. Such a structure is determined by q transverse fields $\{\bar{X}_1, \dots, \bar{X}_q\}$ which are linearly independent at each point. In this case, we say that (M, \mathfrak{F}) is a parallelisable foliation.

If, moreover, $\{\bar{X}_1, \dots, \bar{X}_q\}$ is a basis of a q -dimensional Lie subalgebra \mathfrak{g} of $\mathfrak{L}(M, \mathfrak{F})$, one says that (M, \mathfrak{F}) has a structure of Lie \mathfrak{g} -foliation. Lie foliations have been studied by Fedida in [2]. In [7], we obtained

Theorem 1. If (M, \mathfrak{F}) is a parallelisable foliation, then the closures of the leaves are the fibers of a fibration $\pi : M \rightarrow W$. Moreover, there exists a Lie algebra \mathfrak{g} such that \mathfrak{F} induces on each fiber of π a Lie \mathfrak{g} -foliation.

First part of this result may be also obtained from a theorem of Sussmann [8].

If $\pi_*(M) = 0$, the structural Lie algebra \mathfrak{g} is abelian. Using a well-known result of Tischler, the fiber of π admits, in this case, a fibration on $\pi^{\mathfrak{F}}$, where $r = q - \dim W$. From these remarks, we deduce

Theorem 2. If M is a simply connected compact manifold, M admits no 4-codimension parallelisable foliation.

3. Riemannian foliations

If $G = O(q)$, a transverse G -structure on (M, \mathfrak{F}) is a transverse riemannian structure. We say that (M, \mathfrak{F}) , endowed with such a structure $e_{\mathfrak{F}}$, is a riemannian foliation. Riemannian foliations were introduced by B. Reinhart [7].

It is possible, in this case, to introduce a transverse Levi-Civita connection $\omega_{\mathfrak{F}}$ on $e_{\mathfrak{F}}$. Moreover, $\omega_{\mathfrak{F}} + \theta_{\mathfrak{F}}$ defines a transverse

parallelism on (e_T, \mathfrak{F}_T) . This fact allows us to use results of the previous section in order to study riemannian foliations. Several results are obtained in this way ; for example

Theorem 3. If (M, \mathfrak{F}) is a riemannian foliation, and $\pi_1(M) = 0$, then there exists an algebra of transverse fields in the center of $\mathfrak{L}(M, \mathfrak{F})$ whose transverse orbits define the closures of the leaves.

Using the same methods, riemannian foliations are classified in [6] in codimension ≤ 3 .

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