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## SOME REMARKS ON RAMSEY MATROIDS

Jaroslav Nešetřil, Svatopluk Poljak, Daniel Turzík

The purpose of this note is to summarize some of the results which are going to appear in [2] and to complement these results by stating some open problems and related consequences:

1. The following is the main result of [1]:

Theorem 1.1: For every simple matroid  $M = M(X)$  and for every positive integer  $k$  there exists a matroid  $N = N(Y)$  such that for every partition  $Y = Y_1 \cup \dots \cup Y_k$  there exists a matroid  $M' = M'(X')$ ,  $M' \cong M$ , such that  $X' \subseteq Y_i$  for some  $Y_i$ .

The statement of 1.1 may be abbreviated by saying that matroids have singleton-Ramsey property. For many classes of structures (mainly of combinatorial type, such as graphs etc.) the existence of a singleton-Ramsey property may be established by simple means. The proof of 1.1 given in [1] (and also in [2]) is not of such simplicity.

Particularly, the following is not known:

Problem 1.2: Denote by  $F(M)$  the minimal size of a matroid  $N$  which has the property stated in 1.1. Is it true that there exists a constant  $\alpha$  such that  $F(M) \leq |X|^\alpha$  for every matroid  $M = M(X)$ ?

It is also not known which classes of matroids have singleton-Ramsey property.

2. The following is the main result of [2]:

Theorem 2.1: For every simple matroid  $M = M(X)$  and for every positive integer  $k$  there exists a matroid  $N = N(Y)$  such that for every partition  $a_1 \cup \dots \cup a_k$  of the set of all lines of  $N$  with exactly 2 elements there exists a matroid  $M' = M'(X')$ ,  $M' \simeq M$ , such that all 2-point lines of  $M'$  are in one of the classes of the partition.

The statement of 2.1 may be summarized by saying that matroids have edge-Ramsey property (an edge meaning a flat with 2 independent points).

A construction related to 2.1 is even less effective and therefore it is, at present, needless to state an analogy of 1.2.

However, the following seems to be an interesting problem:

Problem 2.2: Which classes of matroids have edge-Ramsey property?

Perhaps this problem will have mostly a negative answer. E.g. it may be seen that the class of all graphical matroids does not have edge-Ramsey property. The same is true for transversal matroids.

The following problem is related to the existence of edge-Ramsey matroids and it seems to require a new technique:

Problem 2.3: Given a matroid  $M = M(X)$  does there exist a matroid  $N = N(Y)$  with the following property:

For every partition  $a_1 \cup \dots \cup a_n$  ( $n$  is a positive integer) of the set of all lines of  $N$  with exactly 2 points there exists a matroid  $M' = M'(X')$ ,  $M' \simeq M$ , such that the partition of all 2-point lines restricted to  $M'$  is canonical.

Here we say that an equivalence  $\sim$  on the set of all 2-lines of  $M = M(X)$  is canonical if there exists an ordering  $\leq$  of  $X$  such that one of the following possibilities holds for all 2-lines  $xy, x'y'$  with  $x < y, x' < y'$  :

1.  $xy \sim x'y'$  iff  $x = x', y = y'$
2.  $xy \sim x'y'$  iff  $x = x'$
3.  $xy \sim x'y'$  iff  $y = y'$
4.  $xy \sim x'y'$

A positive solution to this problem would provide both an analogy of Erdős-Rado canonization lemma for matroids and a strengthening of the selective property of matroids proved in [1] .

3. The above theorems were established by means of amalgams of matroids along a special set systems. The method of the proof has some further consequences.

For example the following may be proved using the basic construction given in [1] , [2]:

Given a matroid  $M = M(X)$  denote by  $\text{Aut } M$  the group of all automorphisms  $f : M \rightarrow M$  .

Theorem 3.1: Let  $M = M(X)$  be a matroid,  $G$  a subgroup of  $\text{Aut}(X)$  . Then there exist a matroid  $N = N(Y)$  with the following properties:

1.  $M$  is a restriction of  $N$  ;
2.  $\text{Aut } N \cong G$  ;
3. every automorphism  $f \in G$  extends uniquely to an automorphism of  $N$  . (I.e. for every  $f \in G$  there exists unique  $\bar{f} \in \text{Aut } N$  such that  $\bar{f}|_X = f$  .)

This generalizes some of the results of Piff and Welsh, see [4], chapter 17 .

Sketch of a proof: We may assume without loss of generality that every point  $M$  lies on a line with at least 4 points. Consider a set  $X' = X \times \{0,1\}$  and

Let  $G'$  be the group of all permutations  $g' : X' \rightarrow X'$  defined by  $g'(x,0) = (g(x),0)$ ,  $g'(x,1) = (g(x),1)$ , for a  $g \in G$ . Let  $(Y',E')$  be a graph which satisfies:

1.  $\text{Aut}(Y',E') \cong G'$  ;
2. every  $g' \in G'$  extends uniquely to an automorphism of  $(Y',E')$  ;
3.  $(Y',E')$  is 3-connected and without triangles;
4. every edge of  $(Y',E')$  belongs to a cycle of length  $\leq 7$  ;
5.  $\{(x,0),(x,1)\}; x \in X\} \subseteq E'$  .

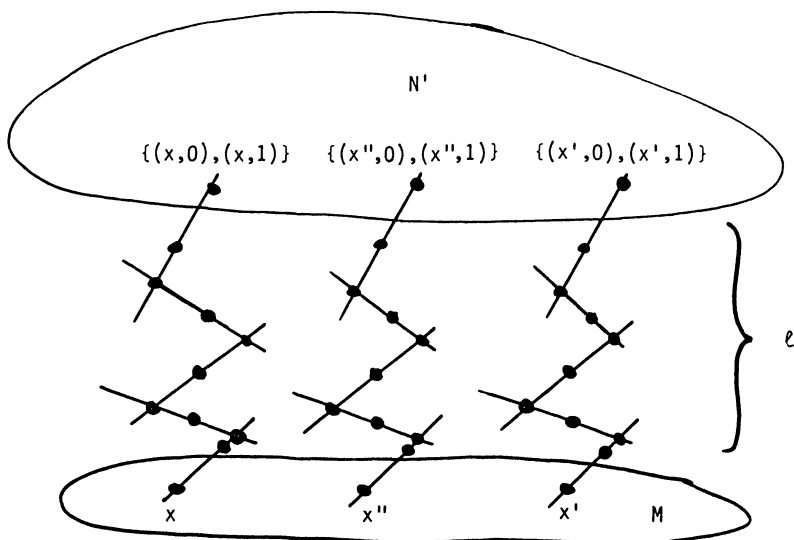
The existence of such graphs follows from techniques given in [3] .

Let  $N' = N(E')$  be the cycle matroid of the graph  $(Y',E')$  and let  $N$  be the amalgam of matroids  $N'$  and  $M$  and "chains of 3-lines" of length  $\ell > \max \{7,r(N'),r(M)\}$  which is constructed in [1].

This is indicated on Fig. 1 .

As the amalgamation given in [1] is locally free, it is easy to see that  $N$  has all the desired properties:

Fig. 1



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