GEOMETRY OF BRST-FORMALISM

Abramov V.

In the present paper we describe the geometrical approach to BRST-formalism. To this end the infinite dimensional supermanifold which was independently introduced in [1] and [5] is used. The base space of this supermanifold is the configuration space of all classical fields. The odd part of this space is built using the infinite dimensional Grassman algebra which was described in [3]. We have the odd vector field X^{BRST} on this supermanifold corresponding to the BRST-transformation. The effectiv action is the even functional on this supermanifold. In the present paper the set of odd and even vector fields which form Lie superalgebra is found. It is shown that X^{BRST} is the element of this superalgebra. In the third part of our paper we use the formal geometry which was introduced by Gelfand and other authors in [4]. In the frames of this formal geometry it is possible to describe the BRST-formalism on the local level.

1. Let φ^i be the boson gauge field and $S(\varphi)$ is the action. In general the index i denotes the set of discret indeces and continuous variables. Under the summation over i we understand the summation over discrete indeces and the integretion over continuous variables. The generators of infinitesimal gauge transformations

$$\Gamma_\alpha = R^i_\alpha(\varphi)\frac{\delta}{\delta\varphi}, \tag{1.1}$$

form the algebra

$$[\Gamma_\alpha, \Gamma_\beta] = f^\gamma_{\alpha\beta}\,\Gamma_\gamma. \tag{1.2}$$

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The action $S(\varphi)$ is invariant under the gauge transformations

$$\Gamma_\alpha \left[S(\varphi) \right] = \mathcal{R}^i_\alpha \frac{\delta S}{\delta \varphi^i} = 0. \tag{1.3}$$

Let M denote the configuration space of all classical fields φ^i. One can consider the operators Γ_α as the vector fields on M and the action $S(\varphi)$ as the functional on M. In order to build the supermanifold over M it will be necessary to consider the infinite dimensional Grassmann algebra $\mathcal{Y}$. Its definition, structure and concrete realization on the space of functions is given in [3]. In [1] this realization is extended to the spaces of sections of some vector bundle, for example $E_{\underline{G}}=P\otimes_G \underline{G}$, where P is the principal fiber bundle, G is the structure group of P and $\underline{G}$ is the Lie algebra of G. The concrete realization allows us to define the generators c^α, $\bar{c}^\alpha$ of $\mathcal{Y}$ which anticommute

$$\{ c^\alpha, c^\beta \} = \{ c^\alpha, \bar{c}^\beta \} = \{ \bar{c}^\alpha, \bar{c}^\beta \} = 0 \tag{1.4}$$

(it should be noted that the indices α, β contain the continuous variables as we said above). Then it is possible to write the arbitrary element f of $\mathcal{Y}$ in the form

$$f = \sum_{m,n} \bar{c}^{\alpha_1} \ldots \bar{c}^{\alpha_n} f_{\alpha_1 \ldots \alpha_n ; \beta_1 \ldots \beta_m} c^{\beta_1} \ldots c^{\beta_m}, \tag{1.5}$$

where $f_{\alpha_1 \ldots \alpha_n ; \beta_1 \ldots \beta_m}$ is the kernel of some operator. In the natural way the variational derivatives $\delta / \delta c^\alpha$, $\delta / \delta \bar{c}^\beta$ is defined.

The construction of supermanifold belonging to Berezin-Leites is most convenient for the infinite dimensional objects such as the space of all classical fields M and the Grassmann algebra $\mathcal{Y}$. Let us denote SM the superspace which we call the superspace of quantified fields. The base space of SM is the space of all classical fields M and $C(SM) = C(M) \otimes \mathcal{Y}$, where C(SM) is the space of the functionals on SM and C(M) is the space of the functional on M. It should be noted that we describe the construction of the superspace SM schematically, in a general way. The theory of infinite dimensional supermanifolds in the form which is most appropriate for our purposes develops in [5].

The local coordinates on SM are $(\varphi^i, \bar{c}^\alpha, c^\alpha)$ where $\bar{c}^\alpha, c^\alpha$ are the generators of Grassmann algebra $\mathcal{Y}$. As follow from (1.5) one can write the arbitrary functional F on SM in the form

$$F=\sum_{n,m} \bar{c}^{\alpha_1}\ldots\bar{c}^{\alpha_n} F_{\alpha_1\ldots\alpha_n;\beta_1\ldots\beta_m}(\varphi)\, c^{\beta_1}\ldots c^{\beta_m}, \tag{1.6}$$

where $F_{\alpha_1\ldots\alpha_n;\beta_1\ldots\beta_m}(\varphi) \in C(M)$. In what follows it will be necessary to consider such functionals on SM which depend on supplementary parameters b^α in a polinomial way, i.e.

$$F=\sum_{n,m} \bar{c}^{\alpha_1}\ldots\bar{c}^{\alpha_n} F_{\alpha_1\ldots\alpha_n;\beta_1\ldots\beta_m}(\varphi,b)\, c^{\beta_1}\ldots c^{\beta_m}; \tag{1.7}$$

where

$$F(\varphi,b)=\sum_{p=0}^{\infty} F_{\alpha_1\ldots\alpha_p}(\varphi)\, b^{\alpha_1}\ldots b^{\alpha_p}. \tag{1.8}$$

The arbitrary element X of the Lie superalgebra of the vector fields Der SM on SM which depends on the parameters b and has the form

$$X = F^i(b)\frac{\delta}{\delta\varphi^i}+\Phi^\alpha(b)\frac{\delta}{\delta c^\alpha}+\Psi^\gamma(b)\frac{\delta}{\delta \bar{c}^\gamma} \;. \tag{1.9}$$

The space of the vector fields

$$X = F^\alpha(b)\Gamma_\alpha + \Phi^\alpha(b)\frac{\delta}{\delta c^\alpha} + \Psi^\gamma(b)\frac{\delta}{\delta \bar{c}^\gamma} \quad , \tag{1.10}$$

on SM is the subalgebra Der^{gauge}SM of Der SM that follows from (1.2).

2.Let $\mathcal{Y}_\alpha(\varphi)$ be the certain functional on M which we call the gauge functional. Let us introduce the following notations

$$\Gamma_\beta(\mathcal{Y}_\alpha) = \mathcal{Y}_{\alpha,\beta} \;, \quad \Gamma_{\alpha_p}(\mathcal{Y}_{\alpha,\alpha_1\ldots\alpha_{p-1}}) = \mathcal{Y}_{\alpha,\alpha_1\ldots\alpha_p} \tag{2.1}$$

Then the quantum effective action $S_Q(b)$ is the even functional on SM which depends on the parameters b and it has the following form

$$S_Q(b)= S(\varphi)+ \frac{1}{2}(b)^2+ b^\alpha\mathcal{Y}_\alpha - \bar{c}^\alpha\mathcal{Y}_{\alpha,\beta}\, c^\beta . \tag{2.2}$$

Let us consider the equation

$$X(S_Q) = 0 \tag{2.3}$$

where $X \in \mathrm{Der}^{gauge}$SM. Substituting (1.10) and (2.2) into (2.3) one can rewrite the equation (2.3) as follows

$$-F^{\beta}(b)b^{\alpha}\mathcal{Y}_{\alpha,\beta}+F^{\gamma}(b)\bar{c}^{\alpha}\mathcal{Y}_{\alpha,\beta\gamma}c^{\beta}_{+}\Phi^{\alpha}(b)\mathcal{Y}_{\alpha,\beta}c^{\beta}_{-}\Psi^{\beta}(b)\bar{c}^{\alpha}\mathcal{Y}_{\alpha,\beta}=0. \qquad (2.4)$$

Taking into account (1.8) the equation (2.4) can be rewritten in the form

$$-F^{\beta}_{\alpha_1\ldots\alpha_{p-1}}\mathcal{Y}_{\alpha_p,\beta}+F^{\gamma}_{\alpha_1\ldots\alpha_p}\bar{c}^{\alpha}\mathcal{Y}_{\alpha,\beta\gamma}c^{\beta}_{+}\Phi^{\alpha}_{\alpha_1\ldots\alpha_p}\mathcal{Y}_{\alpha,\beta}c^{\beta}_{-}\Psi^{\beta}_{\alpha_1\ldots\alpha_p}\bar{c}^{\alpha}\mathcal{Y}_{\alpha,\beta}=0. \qquad (2.5)$$

Hence

$$F^{\beta}_{\alpha_1\ldots\alpha_{p-1}}=\mathcal{Y}^{-1}_{\beta\alpha_p}(F^{\gamma}_{\alpha_1\ldots\alpha_p}\bar{c}^{\alpha}\mathcal{Y}_{\alpha,\beta\gamma}c^{\beta}_{+}\Phi^{\alpha}_{\alpha_1\ldots\alpha_p}\mathcal{Y}_{\alpha,\beta}c^{\beta}_{-}\Psi^{\beta}_{\alpha_1\ldots\alpha_p}\bar{c}^{\alpha}\mathcal{Y}_{\alpha,\beta}) \qquad (2.6)$$

Using the expression (1.6) one can write the functionals F, Φ and Ψ in the local coordinates of the superspace SM.Substituting the obtained expressions for F, Φ and Ψ to (2.6) we get

$$\begin{aligned}F^{\beta}_{(\alpha)_p(\beta)_m(\gamma)_n}=\mathcal{Y}^{-1}_{\beta\alpha_p}\Big[&(-1)^{n-1}\mathcal{Y}_{\beta_m,\gamma_n}F^{\alpha}_{(\alpha)_p(\beta)_{m-1}(\gamma)_{n-1}}+\mathcal{Y}_{\alpha,\underline{\gamma_n}}\Phi^{\alpha}_{(\alpha)_p(\beta)_m\underline{(\gamma)_{n-1}}}+\\&+(-1)^{n-1}\mathcal{Y}_{\underline{\beta_m},\alpha}\Psi^{\alpha}_{(\alpha)_p\underline{(\beta)_{m-1}}(\gamma)_n}\Big],\end{aligned} \qquad (2.7)$$

where $(\alpha)_p=\alpha_1\ldots\alpha_p$ and the underlined indices are alternated. In the present paper we do not cite the intermediate steps because of the awkwardness of the calculations and at once represent the final result.By substituting (2.7) into (1.10) one can distinguish the three families of the vector fields on SM which satisfy the eq. (2.3).They are

$$X^{BRST}_{\alpha\beta}=\mathcal{O}\mathcal{C}_{\alpha\beta\gamma}\Gamma_{\gamma}+b^{\alpha}\frac{\delta}{\delta\bar{c}^{\alpha}}-\mathcal{B}_{\alpha\beta\gamma}\frac{\delta}{\delta c^{\gamma}}, \qquad (2.8)_a$$

where $\mathcal{O}\mathcal{C}_{\alpha\beta\gamma}$ and $\mathcal{B}_{\alpha\beta\gamma}$ are the polinomials which are expressed in terms of the gauge functionals in the following way

$$\begin{cases}\mathcal{O}\mathcal{C}_{\alpha\beta\gamma}=\mathcal{Y}^{-1}_{\gamma\alpha}\mathcal{Y}_{\beta\sigma}c^{\sigma},\\ \mathcal{B}_{\alpha\beta\gamma}=\mathcal{Y}^{-1}_{\gamma\alpha'}\mathcal{Y}_{\alpha',\beta'\gamma}\mathcal{O}\mathcal{C}_{\alpha\beta\gamma'}c^{\beta'},\end{cases} \qquad (2.9)_a$$

$$X^{\overline{BRST}}_{\alpha\beta}=-\mathcal{L}_{\alpha\beta\gamma}\Gamma_{\gamma}+\mathcal{Q}_{\alpha\beta\gamma}\frac{\delta}{\delta\bar{c}^{\gamma}}+b^{\alpha}\frac{\delta}{\delta c^{\beta}} \qquad (2.8)_b$$

where

$$\begin{cases} \mathcal{L}_{\alpha\beta\gamma} = \mathcal{Y}^{-1}_{\gamma\alpha} \mathcal{Y}_{\sigma\beta} \bar{c}^{\sigma}, \\ \mathfrak{N}_{\alpha\beta\gamma} = \mathcal{Y}_{\beta'\gamma} \mathcal{Y}_{\alpha',\beta\gamma} \mathcal{L}_{\alpha\beta\gamma'} \bar{c}^{\alpha'}, \end{cases} \tag{2.9$_b$}$$

$$X^{0}_{\alpha\beta} = \bar{c}^{\alpha} \frac{\delta}{\delta \bar{c}^{\beta}} - \mathfrak{N}_{\alpha\beta\gamma} \frac{\delta}{\delta c^{\gamma}} . \tag{2.8$_c$}$$

THEOREM. The odd vector fields $X^{BRST}_{\alpha\beta}, X^{\overline{BRST}}_{\alpha'\beta'}$ and the even vector fields $X^{0}_{\alpha\beta}$ satisfy the following commutational relations

$$\begin{aligned} &\left[X^{BRST}_{\alpha\beta}, X^{BRST}_{\alpha'\beta'}\right] = 0, && \left[X^{BRST}_{\alpha\beta}, X^{0}_{\alpha'\beta'}\right] = \delta_{\beta\alpha'} X^{BRST}_{\alpha\beta'}, \\ &\left[X^{\overline{BRST}}_{\alpha\beta}, X^{\overline{BRST}}_{\alpha'\beta'}\right] = 0, && \left[X^{\overline{BRST}}_{\alpha\beta}, X^{0}_{\alpha'\beta'}\right] = \delta_{\beta\alpha'} X^{BRST}, \\ &\left[X^{BRST}_{\alpha\beta}, X^{\overline{BRST}}_{\alpha'\beta'}\right] = 0, && \left[X^{0}_{\alpha\beta}, X^{0}_{\alpha'\beta'}\right] = \delta_{\alpha\beta'} X^{0}_{\beta\alpha'} + \delta_{\beta\alpha'} X^{0}_{\alpha\beta'} . \end{aligned} \tag{2.10}$$

The Lie superalgebra described in the theorem contains the odd vector field

$$X^{BRST} = \delta_{\alpha\beta} X^{BRST}_{\alpha\beta} = (R^{i}_{\alpha} c^{\alpha}) \frac{\delta}{\delta \varphi^{i}} + b^{\alpha} \frac{\delta}{\delta \bar{c}^{\alpha}} - (\tfrac{1}{2} f^{\gamma}_{\alpha\beta} c^{\alpha} c^{\beta}) \frac{\delta}{\delta c^{\gamma}},$$

where we use the relation

$$\mathcal{Y}_{\alpha,\beta\gamma} c^{\beta} c^{\gamma} = \frac{1}{2} f^{\sigma}_{\beta\gamma} \mathcal{Y}_{\alpha,\sigma} c^{\beta} c^{\gamma} .$$

The vector field X^{BRST} corresponds to the famous BRST-transformations.It follows from (2.10) that $(X^{BRST})^2=0$.It should be noted that the vector field $X^{\overline{BRST}}$ which corresponds to the $\overline{BRST}$-transformations is not so easily obtained from $(2.8)_b$.But it is true under some hypotheses with respect to the polinomial $\mathcal{L}_{\alpha\beta\gamma}$.This means that the vector fields $X^{\overline{BRST}}_{\alpha\beta}$ have more fundamental significance than $X^{\overline{BRST}}$.

It follows from the theorem that the even vector fields $X^{0}_{\alpha\beta}$ form the Lie algebra.This algebra is generated by the group of transformations.This group makes the change of generators of the Grassmann algebra $\mathcal{Y}$ or the change of odd local coordinates on SM. It acts on SM in the following way

$$\begin{cases} \bar{c}^{\alpha} = \bar{c}'^{\beta} A^{\alpha}_{\beta} , \\ c^{\alpha} = B^{\alpha}_{\beta} c'^{\beta}, \end{cases} \tag{2.11}$$

where $\{\bar{c}'^{\beta}, c'^{\beta}\}$ are the new odd coordinates on SM and A^{α}_{β} ,B^{α}_{β} are the group parameters (in general, they are the kernels of the operators if we consider the $\bar{c}^{\alpha}$,c^{α} as the generators of the algebra $\mathcal{Y}$ which is realized on the spaces of the sections of some vector bundle.The S_Q is invariant under the transformations (2.11) if the following condition is satisfied

$$A^{\alpha}_{\beta}\ \mathcal{Y}_{\beta\gamma}\ B^{\gamma}_{\sigma} = \mathcal{Y}_{\alpha\sigma}, \tag{2.12}$$

or

$$B^{\alpha}_{\beta} = \mathcal{Y}^{-1}_{\alpha\gamma}(A^{-1})^{\gamma}_{\sigma}\ \mathcal{Y}_{\sigma\beta} \quad , \tag{2.13}$$

where $(A^{-1})^{\gamma}_{\sigma}$ is the kernel of the inverse operator.Let us denote SG the group which corresponds to the algebra of the theorem. Its parameters are $\mu=(A^{\alpha}_{\beta}, h_{\gamma\sigma}, \bar{h}_{\gamma\sigma})$ where A^{α}_{β} are the even parameters and $h_{\gamma\sigma}, \bar{h}_{\gamma\sigma}$ are the odd parameters.The group operation is defined by the expression

$$\mu\cdot\mu' = (A^{\alpha}_{\gamma}\ A'^{\gamma}_{\beta}, h_{\gamma\sigma}+h'_{\gamma\sigma}, \bar{h}_{\gamma\sigma}+\bar{h}'_{\gamma\sigma}). \tag{2.14}$$

SG acts on SM in the following way

$$\begin{aligned} \varphi'^{i} &= \varphi^{i} + h_{\alpha\beta}\ \mathcal{O}_{\alpha\beta\gamma}\mathcal{R}^{i}_{\gamma} + \bar{h}_{\alpha\beta}\mathcal{L}_{\alpha\beta\gamma}\mathcal{R}^{i}_{\gamma} \quad , \\ \bar{c}'^{\alpha} &= \bar{c}^{\beta}\ A^{\alpha}_{\beta} + h_{\alpha\beta}\ b^{\beta} + \bar{h}_{\gamma\sigma}\ \mathcal{Q}_{\gamma\sigma\alpha} \ , \\ c'^{\alpha} &= \mathcal{Y}^{-1}_{\alpha\gamma}(A^{-1})^{\gamma}_{\sigma}\mathcal{Y}_{\sigma\beta}\ c^{\beta} - h_{\gamma\sigma}\ \mathcal{B}_{\gamma\sigma\alpha} + \bar{h}_{\alpha\beta} b^{\beta} \ . \end{aligned} \tag{2.15}$$

It is natural that SG is the group of the supersymmetries of S_Q.

3. In this part we consider the case when the boson gauge field φ^{i} is the Yang-Mills field $A_{\mu}(x)$.Let T_{α} be the basis of the corresponding Lie algebra and

$$[T_{\alpha}, T_{\beta}] = f^{\gamma}_{\alpha\beta}\ T_{\gamma} \ . \tag{3.1}$$

Then $A_{\mu}(x) = A^{\alpha}_{\mu}(x)T_{\alpha}$ .It should be noted that in this part all indeces are discrete.The construction of the superspace SM allows us to analyze the algebraic and geometrical structure of the BRST-formalism,but it causes the appearance of the nonlocal objects as $\mathcal{Y}^{-1}_{\alpha\beta}$(indeed,if we take the gauge functional in the form

$\mathcal{G}_\alpha(x)=\partial_\mu A^\alpha_\mu(x)$ then $\mathcal{G}^{-1}_{\alpha\beta}(x,y)$ is the kernel of the operator which is inverse to the operator

$$\mathcal{G}_{\alpha\beta}(x,y) = \left[\partial^\mu \partial_\mu + \partial^\mu (f^\alpha_{\gamma\beta} A^\gamma_\mu)\right] \delta(x,y). \qquad (3.2)$$

But in case of Yang-Mills the effective lagrangian L_Q and BRST-transformations are the local polynomials on the fields and their derivatives. This observation leads us to the formal geometry which was introduced by Gelfand in [4].

Let SA denote the algebra of the polynomials on the commutative symbols $A^\alpha_\mu, A^\alpha_{\mu,\nu}, \ldots, A^\alpha_{\mu,\nu_1\ldots\nu_p}, b^\alpha, b^\alpha_\mu, \ldots, b^\alpha_{\mu_1\ldots\mu_p}$ and on the anticommutative symbols $\bar{c}^\alpha, \bar{c}^\alpha_\mu, \ldots, \bar{c}^\alpha_{\mu_1\ldots\mu_p}, c^\alpha, c^\alpha_\mu, \ldots, c^\alpha_{\mu_1\ldots\mu_p}$. All symbols are symmetric with respect to indeces $\mu_1 \ldots \mu_p$. It is obvious that SA is the superalgebra

$$SA = (SA)_0 + (SA)_1 \qquad (3.3)$$

where $(SA)_0$ is the subalgebra of the even polynomials and $(SA)_1$ is the subspace of the odd polynomials. In the usual way ([2]) one can define the ghost number of some polynom of SA. Let z be the general notation for the symbols $(A,b,\bar{c},c)$ where $z_{(0)}$ denotes one of the symbols $(A^\alpha_\mu, b^\alpha, \bar{c}^\alpha, c^\alpha)$ and $(a)_p = \mu_1 \ldots \mu_p$. The operator

$$X = \sum_{(a)_p=0}^{\infty} F_{(a)_p}[z] \frac{\partial}{\partial z_{(a)_p}}, \qquad (3.4)$$

is called the vector field on SA where we use the left derivatives with respect to all symbols and $F_{(a)}[z] \in SA$. Among all the vector fields one can distinguish the canonical even vector field

$$\partial_\mu = \sum_{(a)_p=0}^{\infty} z_{(a)_p,\mu} \frac{\partial}{\partial z_{(a)_p}} = A^\alpha_{\nu,\mu} \frac{\partial}{\partial A^\alpha_\nu} + \bar{c}^\alpha_\mu \frac{\partial}{\partial \bar{c}^\alpha} + \ldots \qquad (3.5)$$

It follows from the definition that

$$\partial_\mu [z_{(a)_p}] = z_{(a)_p,\mu}, \qquad \partial_{(a)_p} [z_{(0)}] = \partial_{\mu_1} \ldots \partial_{\mu_p} [z_{(0)}] = z_{(a)_p}. \qquad (3.6)$$

The commutator of the two vector fields is

$$[X,Y] = \sum_{(a)_p,(b)_q} \left\{ F_{(a)_p} \frac{\partial}{\partial z_{(a)_p}} (G_{(b)_q}) - (-1)^{p(X)p(Y)} G_{(a)_p} \frac{\partial}{\partial z_{(a)_p}} (F_{(b)_q}) \right\} \frac{\partial}{\partial z_{(b)_q}}, \qquad (3.7)$$

where $p(X), p(Y)$ are the parity of X and Y. Thus the space of all vector fields on SA is the Lie superalgebra. A differential 1-form on

SA is a formal finite sum

$$\omega = \sum \omega_{(a)_p}[z]\, dz_{(a)_p} \quad , \tag{3.8}$$

where $\omega_{(a)_p}[z] \in$ SA and $dz_{(a)_p}$ are the new symbols. In particular, $dA^{\alpha}_{\nu,\mu_1\dots\mu_p}$, $db^{\alpha}_{\mu_1\dots\mu_p}$ are the odd symbols and $d\bar{c}^{\alpha}_{\mu_1\dots\mu_p}$, $dc^{\alpha}_{\mu_1\dots\mu_p}$ are the even symbols. The values of the differential 1-form ω on the vector field X is

$$\omega(X) = \sum \omega_{(a)_p} F_{(a)_p} \quad . \tag{3.9}$$

In the usual way one can define the wedge product. A differential r-form is a finite sum

$$\omega = \sum \omega_{(a)_1\dots(a)_r} dz_{(a)_1} \dots dz_{(a)_r} \tag{3.10}$$

The differential r-form is called even if all terms in the expression (3.10) are the even polynomials with respect to the symbols $\bar{c}^{\alpha}_{\mu_1\dots\mu_p}$, $c^{\alpha}_{\mu_1\dots\mu_p}$, $dA^{\alpha}_{\nu,\mu_1\dots\mu_p}$ and $db^{\alpha}_{\mu_1\dots\mu_p}$. The exterior differential is the operator

$$d = \sum_{(a)_p} dz_{(a)_p} \frac{\partial}{\partial z_{(a)_p}} \quad , \qquad d^2 = 0 \quad . \tag{3.11}$$

The Faddeev-Popov Lagrangian is the even polynomial

$$L_Q = -\frac{1}{4} (F_{\mu\nu})^2 + \frac{1}{2} b^{\alpha} b^{\alpha} + b^{\alpha} \mathcal{Y}_{\alpha} - \bar{c}^{\alpha} X^B(\mathcal{Y}_{\alpha}), \tag{3.12}$$

on SA, where

$$F^{\alpha}_{\mu\nu} = A^{\alpha}_{\nu,\mu} - A^{\alpha}_{\mu,\nu} + f^{\alpha}_{\beta\gamma} A^{\beta}_{\mu} A^{\gamma}_{\nu} \ , \ L_Q = L_{cl} + L_{qu}, \tag{3.13}$$

$\mathcal{Y}_{\alpha}$ is the gauge polynomial on the symbols $A^{\alpha}_{\nu,\mu_1\dots\mu_p}$, and X^B is the odd vector field on SA which corresponds to the BRST-supersymmetry

$$X^B = (\nabla_{\mu} c^{\alpha}) \frac{\partial}{\partial A^{\alpha}_{\mu}} + \sum_{(a)_p} \partial_{(a)_p} (\nabla_{\mu} c^{\alpha}) \frac{\partial}{\partial A^{\alpha}_{\mu,(a)_p}} + b^{\alpha} \frac{\partial}{\partial \bar{c}^{\alpha}} + \\ + \sum_{(a)_p} \partial_{(a)_p} (b^{\alpha}) \frac{\partial}{\partial \bar{c}^{\alpha}_{(a)_p}} - \frac{1}{2} (f^{\alpha}_{\beta\gamma} c^{\beta} c^{\gamma}) \frac{\partial}{\partial c^{\alpha}} - \frac{1}{2} \sum_{(a)_p} \partial_{(a)_p} (f^{\alpha}_{\beta\gamma} c^{\beta} c^{\gamma}) \frac{\partial}{\partial c_{(a)_p}} \tag{3.14}$$

where $\nabla_{\mu} c^{\alpha} = c^{\alpha}_{\mu} + f^{\alpha}_{\beta\gamma} A^{\beta}_{\mu} c^{\gamma}$. It should be noted that

$$[X^B , \partial_\mu] = 0 , \qquad (3.15)$$

and

$$X^B(\nabla_\mu c^\alpha)=0, \; X^B (b^\alpha) =0, \; X^B(f^\alpha_{\beta\gamma} c^\beta c^\gamma)=0. \qquad (3.16)$$

It follow from (3.15),(3.16) that

$$[X^B ,X^B] =0 , \qquad X^B(L_Q) = 0 . \qquad (3.17)$$

Usually (2), the BRST-invariant generalization of L_{qu} search in the form

$$L_{qu}= s F , \qquad (3.18)$$

where F is some expression on the fields and their derivatives and s is the BRST-operator.In the frame of the formal geometry we can generalize the equation

$$L_{qu}= X^B(F) , \qquad (3.19)$$

where X^B is the odd vector field (3.14) and $F \in SA$.We can rewrite the equation (3.19) using the definition (3.8) of differential 1-form and the exterior differential (3.11) in the following way

$$L_{qu}= dF (X^B). \qquad (3.20)$$

PROPOSITION. The even polynomial $L_{qu} \in SA$ is BRST-invariant (i.e.that $X^B(L_{qu})=0$) if

$$L_{qu}= \omega (X^B), \qquad (3.21)$$

where ω is the even differential 1-form which satisfies the following condition

$$d \omega (X^B,X^B)=0. \qquad (3.22)$$

The proof follows from the equation

$$d \omega (X,Y)= X \omega (Y) + Y \omega (X) - \omega([X,Y]) \qquad (3.23)$$

It should be noted that in the case of the Faddeev-Popov Lagrangian (3.12) corresponding 1-form is

$$\omega = (y_\alpha + \tfrac{1}{2}b^\alpha)\, d\bar{c}^\alpha - \bar{c}^\alpha d y_\alpha - \tfrac{1}{2}\bar{c}^\alpha d b^\alpha,$$

and it is exact

$$\omega = d\left\{ (y_\alpha + \tfrac{1}{2}b^\alpha)\, \bar{c}^\alpha \right\}.$$

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Tartu State University
Laboratory of Applied Mathematics
Sector of Pure Mathematics
2 J.Liivi St., 202400 Tartu
USSR