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ON QUASIJET BUNDLES

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ABSTRACT. We discuss the Weil approach to the bundles of quasijets and describe the inclusion of the bundle of non-holonomic r-jets into the bundle of quasijets of order r. Applying this approach we rededuce a result by Dekrét characterizing non-holonomic r-jets among quasijets of order r.

1. PRELIMINARIES

We start from the concept of non holonomic r-jet, introduced by Ehresmann, [2] and investigated in works of Pradines, Kolář, Dekrét, Kureš, Virsik and others, [8], [3], [1], [7].

We follow the results of Dekrét from [1], namely the definition of quasijets with their basic properties and essentially use the result of Kolář and Mikulski from [4], giving the description of bundle functors defined on the category $\mathcal{M}f_m \times \mathcal{M}f$ from the point of view of the theory of Weil bundles. We use the standard notation from [5].

In the very beginning, we remind the basic concepts of non-holonomic r-jet and quasijet of order r. We also recall their basic properties and present the relation between them. We define the associated concept of (k,r)-quasivelocities and introduce the bundle functor of quasijets on $\mathcal{M}f_m \times \mathcal{M}f$.

Let M,N,P be manifolds. We recall that a non-holonomic r-jet is defined by induction as follows.

Definition 1. For $r = 1$, the set of non-holonomic 1-jets $\widetilde{J}^1(M,N)$ is the set of 1-jets $J^1(M,N)$ with their standard composition.

By induction, let $\alpha : \widetilde{J}^{r-1}(M,N) \to M$ denote the source projection and $\beta : \widetilde{J}^{r-1}(M,N) \to N$ the target projection of $(r-1)$-th order non-holonomic jets. Then

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X is said to be a non-holonomic r-jet with the source $x \in M$ and the target $y \in N$, if there is a local section $\sigma : M \to \widetilde{J}^{r-1}(M,N)$ such that $X = j^1_x\sigma$ and $\beta(\sigma(x)) = y$.

Let $Y = j^1_y\rho$ for a local section $\rho : N \to \widetilde{J}^{r-1}(N,P)$, $y = \beta(\sigma(x))$. The composition $Y \circ X$ of non-holonomic r-jets is defined by

$$Y \circ X = j^1_x(\rho(\beta(\sigma(u))) \circ_{r-1} \sigma(u))$$

where $\circ_{r-1}$ denotes the composition of non-holonomic $(r-1)$-jets and u is an element of M from a neighbourhood of x.

Now we are going to remind the concept of quasijet. For a manifold M, consider the r-times iterated tangent bundle T^rM. It is well-known that there are r structures of vector bundle on T^rM, namely $T^{r-i}p^i_M : T^rM \to T^{r-1}M$, where $p^i_M : T^iM \to T^{i-1}M$ denote the tangent bundle projection. The definition of quasijet of order r reads as follows

Definition 2. Let $x \in M$ and $y \in N$. A map $\varphi : (T^rM)_x \to (T^rN)_y$ is said to be a quasijet of order r with the source x and the target y, if it is a vector bundle morphism with respect to all vector bundle structures $(T^{r-k}p^k_M)_x$ and $(T^{r-k}p^k_N)_y$, $k = 1, \dots, r$. The set of all such quasijets is denoted by $QJ^r_x(M,N)_y$.

We need the coordinate description of quasijets. Let $x^i = x^i_0$ denote the coordinates on a manifold M and $x^i_1 = dx^i_0$ the additional coordinates on TM. Define the coordinates on T^rM by induction as follows. Let $x^i_{\varepsilon_1\dots\varepsilon_{r-1}}$ denote the coordinates on T^{r-1}, $\varepsilon_i \in \{0,1\}$ $\forall i \in \{1,\dots,r-1\}$. Then $x^i_{\varepsilon_1,\dots,\varepsilon_{r-1}0}$ denote the base coordinates on T^rM with respect to the tangent bundle projection $p^r_M : T^rM \to T^{r-1}M$, while $x^i_{\varepsilon_1\dots\varepsilon_{r-1}1} = dx^i_{\varepsilon_1\dots\varepsilon_{r-1}}$ denote the fiber ones.

By Dekrét, [1], every quasijet $\varphi \in QJ^r_x(M,N)$ is expressed in coordinates by $a^{p\gamma^1\dots\gamma^k}_{i_1\dots i_k}$ defined by the following equation

$$y^p_{\varepsilon_1\dots\varepsilon_r} = \sum_{(\gamma^1\dots\gamma^k)} a^{p\gamma^1\dots\gamma^k}_{i_1\dots i_k} x^{i_1}_{\gamma^1}\dots x^{i_k}_{\gamma^k} \tag{1}$$

where the sum is taken over all multiindices $\gamma^1,\dots,\gamma^k$ satisfying the following conditions
(i) $\gamma^1 + \dots + \gamma^k = \varepsilon = (\varepsilon_1,\dots,\varepsilon_r)$
(ii) $\deg\gamma^1 < \deg\gamma^2 \dots < \deg\gamma^k$, where $\deg\gamma$ denotes the number of the first unit component in γ.
(Here γ^i denotes the i-th multiindex, while γ_i denotes the i-th component in the multiindex γ).

In what follows, we interpret non-holonomic r-jets as quasijets of order r and prove the compatibility of their compositions. Every non-holonomic r-jet $X \in \widetilde{J}^r_x(M,N)_y$ determines a quasijet $\mu X \in QJ^r_x(M,N)_y$ as follows

Let $r = 1$ and $X = j^1_x f$. Then μX is defined as $T_x f$. By induction, we define $\mu X : T^r_x M \to T^r_y N$ for $X \in \widetilde{J}^r_x(M,N)_y$. Let $X = j^1_x\sigma$ for a local α-section $\sigma : M \to \widetilde{J}^r(M,N)$. Then $\sigma(u) \in \widetilde{J}^{r-1}_u(M,N)$ and $\mu(\sigma(u)) : T^{r-1}_u M \to T^{r-1}_{\beta(\sigma(u))}N$. We put $\mu X = T_x\mu(\sigma(u))$.

Proposition 3. *For a non-holonomic r-jet* $X \in \widetilde{J}^r_x(M,N)_y$, μX *is a quasujet. If* $Y \in \widetilde{J}^r_y(N,P)$, *then* $\mu(Y \circ X) = \mu(Y) \circ \mu(X)$.

Proof. We prove the assertion by induction. Let $X = j^1_x\sigma$ for a local α-section $\sigma : M \to \widetilde{J}^{r-1}(M,N)$. By induction, $\mu(\sigma(u)) : T^{r-1}_u M \to T^{r-1}_{\beta(\sigma(u))}N$ is a quasijet. Moreover, we have a map $\mu(\sigma) : T^{r-1}M \to T^{r-1}N$ defined by $\mu(\sigma)(z) = \mu(\sigma(p(z)))(z)$, where $p : T^{r-1}M \to M$ denotes the base projection. By the induction assumption, $\mu(\sigma) : T^{r-1}M \to T^{r-1}N$ is a vector bundle morphism with respect to all vector bundle structures $T^{r-1-i}p^i_M$ and $T^{r-1-i}p^i_N$. Then it is easy to see that $T\mu(\sigma) : p^r_M \to p^r_N$ is a vector bundle morphism as well as $T\mu(\sigma) : T^{r-i}p^i_M \to T^{r-i}p^i_N$ for $i = 1,\dots,r-1$. Thus $\mu X = T_x\mu(\sigma) : T^{r-i}_x p^i_M \to T^{r-i}_y p^i_N$ is a quasijet, which proves the first claim.

For the proof of the second assertion, consider local sections $\sigma : M \to \widetilde{J}^{r-1}(M,N)$ and $\rho : N \to \widetilde{J}^{r-1}(N,P)$ and define $\mu(\rho(\sigma)) : T^{r-1}_{\beta(\sigma(u))}N \to T^{r-1}P$ by

$$\mu(\rho(\sigma))(\mu(\sigma)(u)) = \mu(\rho \circ \beta(\sigma(u)))(\mu(\sigma(u)))\,.$$

We prove that $\mu(\rho) \circ \mu(\sigma)(u) = \mu(\rho(\sigma))(\mu(\sigma(u)))$. It holds

$$\begin{aligned}\mu(\rho) \circ \mu(\sigma)(u) &= \mu(\rho)(\mu(\sigma)(u)) = \mu(\rho)(\mu(\sigma(p(u)))(u)) \\ &= \mu(\rho \circ p(\mu(\sigma(p(u)))(u)))(\mu(\sigma(p(u)))(u)) \\ &= \mu(\rho \circ \beta(\sigma(u)))(\mu(\sigma)(u)) \\ &= \mu(\rho(\sigma))(\mu(\sigma(u)))\,.\end{aligned}$$

By induction, we have $\mu(\rho(\sigma))(\mu(\sigma(u))) = \mu((\rho \circ \beta(\sigma(u))) \circ \sigma(u))$ which implies $\mu((\rho \circ \beta(\sigma(u))) \circ \sigma(u)) = \mu(\rho) \circ \mu(\sigma)(u)$. Let $X = j^1_x\sigma$, $Y = j^1_{\beta(\sigma(x))}\rho$. Applying T to both sides of the last equations yields $\mu(Y \circ X) = \mu(Y) \circ \mu(X)$. This proves our claim. □

By Dekrét, [1], there is a bundle structure $QJ^r(M,N) \to M \times N$ on quasijets. Analogously to J^r, [5], we can consider QJ^r as the bundle functor on the category $\mathcal{M}f_m \times \mathcal{M}f \to \mathcal{FM}_m$, if we define $QJ^r(f,g)(X) = j^r_{\beta(X)}g \circ X \circ j^r_{f(\alpha(X))}f^{-1}$ for any local diffeomorphism $f : M \to \bar{M}$ and any smooth map $g : N \to \bar{N}$. The composition in the last expression denotes the composition of quasijets, where holonomic r-jets $j^r_{\beta(X)}g$ and $j^r_{f(\alpha(X))}f^{-1}$ are considered as quasijets.

Now we are going to define the bundle of (m,r)-quasivelocities. We put $QT^r_m N = QJ^r_0(\mathbb{R}^m, N)$ for a manifold N and $QT^r_m f = QJ^r_0(\mathrm{id}_{\mathbb{R}^m}, f)$ for a smooth map $f : N \to P$. Thus we have the functor $QT^r_m : \mathcal{M}f \to \mathcal{FM}$. It can be easily verified that, that QT^r_m is a product preserving functor and thus it is a Weil bundle T^A for $A = QT^r_m\mathbb{R}$. The situation is analogous to that for non-holonomic r-jets and non-holonomic (m,r)-velocities. Denote by $\mathbb{Q}^r_m$ the Weil algebra corresponding to the bundle of (m,r)-quasivelocities and $\widetilde{\mathbb{D}}^r_m$ the Weil algebra corresponding to the bundle of non-holonomic (m,r)- velocities.

2. Weil approach to quasijet bundles

We start this section from an important result of Kolář and Mikulski, [4], from which we gradually deduce the description of quasijet bundles from the point of view of the theory of Weil bundles. Applying this approach, we also describe the inclusion of non-holonomic jets into the bundle of quasivelocities.

Let F be a bundle functor defined on the product category $\mathcal{M}f_m \times \mathcal{M}f$. For a couple of manifolds $(M, N) \in \mathcal{M}f_m \times \mathcal{M}f$ we have two fibered manifold projections $a : F(M, N) \to M$ and $b : F(M, N) \to N$. For another couple of manifolds $(\bar{M}, \bar{N}) \in \mathcal{M}f_m \times \mathcal{M}f$, a local diffeomorphism $g : M \to \bar{M}$ and a smooth map $f : N \to \bar{N}$, we have a morphism $F(g, f) : F(M, N) \to F(\bar{M}, \bar{N})$. Kolář and Mikulski in [4] defined the associated bundle functor G^F on $\mathcal{M}f$ by $G^F(N) = F_0(\mathbb{R}^m, N)$, $G^F(f) = F_0(\mathrm{id}_{\mathbb{R}^m}, f)$. Moreover, they defined the action H^F of the jet group G^r_m on G^F by $H^F_N(j^r_0\varphi) = F_0(\varphi, \mathrm{id}_N)$ in the case F is a bundle functor of order r in the first factor. For every $j^r_0\varphi \in G^r_m$, $H^F(j^r_0\varphi)$ is a natural equivalence on G^F and thus $H^F : G^r_m \to \mathcal{NE}(G^F)$ is a group homomorphism of G^r_m into the group of all natural equivalences $\mathcal{NE}(G^F)$ on G^F.

Conversely, let G be a bundle functor defined on $\mathcal{M}f_m$ and $H : G^r_m \to \mathcal{NE}(G)$ be a group homomorphism. We remind the bundle functor (G, H) on $\mathcal{M}f_m \times \mathcal{M}f$ defined in [4]. We have $(G, H)(M, N) = P^rM[GN, H_N]$, the bundle associated to the frame bundle P^rM with the standard fiber GN and the action H_N of G^r_m on GN. For a local diffeomorphism $g : M \to \bar{M}$ and a smooth map $f : N \to \bar{N}$, we have $(G, H)(g, f) = P^rg[Gf]$. We have bundle projections $a : (G, H)(M, N) \to M$ and $b : (G, H)(M, N) \to N$.

Then the result of Kolář and Mikulski reads as follows

Proposition 4. (i) *For every bundle functor F defined on $\mathcal{M}f_m \times \mathcal{M}f$ of order r in the first factor it holds $F = (G^F, H^F)$.*
(ii) *For another bundle functor $\bar{F}$ of this kind, natural transformations $t : F \to \bar{F}$ are in a bijection with natural transformations $\tau : G^F \to G^{\bar{F}}$ satisfying the equivariancy condition*

$$H^{\bar{F}}_N(j^r_0\varphi) \circ \tau_N = \tau_N \circ H^F_N(j^r_0\varphi)$$

for any $j^r_0\varphi \in G^r_m$.
(iii) *A bundle functor F on $\mathcal{M}f_m \times \mathcal{M}f$ of order r in the first factor preserves products in the second factor if and only if $G^F = T^A$ for some Weil algebra A and H induces a homomorphism $G^r_m \to \mathrm{Aut}(A)$ of Lie groups.*

The well-known bundle functors satisfying the assumptions of Proposition 4 are the functors of holonomic jets J^r, non-holonomic jets $\tilde{J}^r$ and semiholonomic jets $\bar{J}^r$. It is easy to verify that the functor of quasijets QJ^r satisfies the assumptions of (iii) from Proposition 4 too. Then $G^{QJ^r} = QT^r_m = T^{\mathbb{Q}^r_m}$ for the Weil algebra $\mathbb{Q}^r_m$. The action of G^r_m on QT^r_m is defined by $H_N(j^r_0\varphi)(X) = X \circ (j^r_0\varphi)^{-1}$ for $X \in QT^r_mN$ and $j^r_0\varphi \in G^r_m$. The situation is analogous to J^r, $\tilde{J}^r$ and $\bar{J}^r$.

We are going to determine the Weil algebra $\mathbb{Q}^r_m = QT^r_m\mathbb{R} = QJ^r_0(\mathbb{R}^m, \mathbb{R})$. We come out from the coordinate expression of quasijets given by (1), using x^i for the canonical coordinates on $\mathbb{R}^m$ and y on $\mathbb{R}$. In what follows, we use multiindices γ formed by zeros and units, the number of which not exceed r. Denote by E_r the multiindex composed from r units. A multiindex γ is said to be contained in δ if $\gamma_j \leq \delta_j$ for any $j = 1, \dots, length(\gamma)$. Let us assign a polynomial $a_{i_1\dots i_k}^{\gamma^1\dots\gamma^k}\tau_{\gamma^1}^{(i_1)}\dots\tau_{\gamma^k}^{(i_k)}$ with variables $\tau_{\gamma^1}^{(i_1)},\dots,\tau_{\gamma^k}^{(i_k)}$ to a (m,r)-quasivelocity determined by coordinates $a_{i_1\dots i_k}^{\gamma^1\dots\gamma^k}$. Consider the Weil algebra $\mathbb{D}_k^r$ of polynomials of k variables of degree at most r. Then it holds

Proposition 5. *Let $\mathbb{D}_{m(2^r-1)}^r$ be generated by $\tau_\gamma^{(i)}$ for $i \in \{1,\dots,m\}$, $\gamma \subseteq E_r$. Then $\mathbb{Q}_m^r = \mathbb{D}_{m(2^r-1)}^r/I$ is the Weil algebra associated to the bundle of (m,r)-quasivelocities, where the ideal I is of the form $<\tau_\gamma^{(i)}\tau_\delta^{(j)}; \gamma+\delta \not\subseteq E_r>$. The multiplication is defined as follows. For $a = a_{i_1\dots i_k}^{\gamma^1\dots\gamma^k}\tau_{\gamma^1}^{(i_1)}\dots\tau_{\gamma^k}^{(i_k)}$ and $b = b_{j_1\dots j_l}^{\delta^1\dots\delta^l}\tau_{\delta^1}^{(j_1)}\dots\tau_{\delta^l}^{(j_l)}$, the element $c = ab$ satisfies*

$$c_{\iota_1\dots\iota_h}^{\varepsilon^1\dots\varepsilon^h} = a_{\iota_{i_1}\dots\iota_{i_l}}^{\varepsilon^{i_1}\dots\varepsilon^{i_l}} b_{\iota_{i_{l+1}}\dots\iota_{i_h}}^{\varepsilon^{i_{l+1}}\dots\varepsilon^{i_h}} \tag{2}$$

where the sum on the right-hand side of (2) is taken over all subsets $\{i_1,\dots,i_l\} \subseteq \{1,\dots,h\}$ including the empty one.

Proof. Let $a, b \in \mathbb{Q}_m^r = QT_m^r\mathbb{R}$ be any (m,r)-quasivelocities. Denote by $\mu : \mathbb{R}^2 \to \mathbb{R}$ the multiplication of reals. Then $ab = T^{\mathbb{Q}_m^r}\mu(a,b) = QT_m^r\mu(a,b) = j_{(\beta(a),\beta(b))}^r\mu(a,b)$. Since a, b can be considered as maps $T_0^r\mathbb{R}^m \to T^r\mathbb{R}$, fixing an element $x \in T_0^r\mathbb{R}^m$, we can evaluate $a(x)$ and $b(x)$. In coordinates, we can express x by x_γ^i for $i \in \{1,\dots,m\}$ and $a(x)$ and $b(x)$ as follows

$$\begin{aligned} a(x) &= \beta(a) + a_{i_1\dots i_k}^{\gamma^1\dots\gamma^k} x_{\gamma^1}^{i_1}\dots x_{\gamma^k}^{i_k} \\ b(x) &= \beta(b) + b_{j_1\dots j_l}^{\delta^1\dots\delta^l} x_{\delta^1}^{j_1}\dots x_{\delta^l}^{j_l} \end{aligned} \tag{3}$$

The element $j_{(\beta(a),\beta(b))}^r\mu$ can be considered as a quasijet satisfying $\mu_1^\varepsilon = \beta(b)$, $\mu_2^\varepsilon = \beta(a)$, $\mu_{12}^{\varepsilon\delta} = 1$, $\mu_{11}^{\varepsilon\delta} = \mu_{22}^{\varepsilon\delta} = 0$ for any multiindices $\varepsilon, \delta \subseteq E_r$ and $\mu_{i_1\dots i_l}^{\varepsilon^1\dots\varepsilon^l} = 0$ for $l > 2$. Thus $T^{\mathbb{Q}_m^r}\mu(a,b)(x) = \beta(b)a(x) + \beta(a)b(x) + a_{i_1\dots i_k}^{\gamma^1\dots\gamma^k} b_{j_1\dots j_l}^{\delta^1\dots\delta^l} x_{\gamma^1}^{i_1}\dots x_{\gamma^k}^{i_k} x_{\delta^1}^{j_1}\dots x_{\delta^l}^{j_l}$, where $\deg\gamma^1 < \dots \deg\gamma^k$ and $\deg\delta^1 < \dots < \deg\delta^l$. Comparing the coefficients by $x_{\varepsilon^1}^{\iota_1}\dots x_{\varepsilon^h}^{\iota_h}$ we obtain

$$c_{\iota_1\dots\iota_h}^{\varepsilon^1\dots\varepsilon^h} = \beta(b)a_{\iota_1\dots\iota_h}^{\varepsilon^1\dots\varepsilon^h} + \beta(a)b_{\iota_1\dots\iota_h}^{\varepsilon^1\dots\varepsilon^h} + a_{\iota_{i_1}\dots\iota_{i_l}}^{\varepsilon^{i_1}\dots\varepsilon^{i_l}} b_{\iota_{i_{l+1}}\dots\iota_{i_h}}^{\varepsilon^{i_{l+1}}\dots\varepsilon^{i_h}} \tag{4}$$

where the sum is taken over all proper subsets $\{i_1,\dots,i_l\} \subset \{1,\dots,h\}$. The coincidence of (4) with (2) proves our claim. □

Thus the functor QJ^r can be expressed as $(T^{\mathbb{Q}_m^r}, C)$, where $C : G_m^r \to \operatorname{Aut}\mathbb{Q}_m^r$ is defined by $C(j_0^r\varphi)(a) = a \circ j_0^r\varphi^{-1}$ for any $j_0^r\varphi \in G_m^r$ and $a \in \mathbb{Q}_m^r$.

Let us remind, that the Weil algebra $\widetilde{\mathbb{D}}^r_m$ of non-holonomic (m,r)-velocities is identified with $\underbrace{\mathbb{D}^1_m \otimes \ldots \otimes \mathbb{D}^1_m}$, [3]. Elements of $\widetilde{\mathbb{D}}^r_m$ are considered as polynomials $a_{i_1\ldots i_r} t_1^{(i_1)} \ldots t_r^{(i_r)}$ with variables $t_1^{(i_1)} \ldots t_r^{(i_r)}$ for $i_l \in \{0,1,\ldots,m\}$ and $t_j^{(0)} = 1$ for $j \in \{1,\ldots,r\}$.

The following assertion describes the canonical inclusion $i : \widetilde{\mathbb{D}}^r_m \to \mathbb{Q}^r_m$, from which we can deduce the inclusion $\widetilde{J}^r \to QJ^r$ from Proposition 4. Moreover, it determines non-holonomic (m,r)-velocities among (m,r)-quasivelocities by the fact that $A^{\gamma^1\ldots\gamma^k}_{i_1\ldots i_k} \tau^{(i_1)}_{\gamma^1} \ldots \tau^{(i_k)}_{\gamma^k}$ represents a non-holonomic (m,r)-velocity if and only if $A^{\gamma^1\ldots\gamma^k}_{i_1\ldots i_k}$ depend on $\gamma^1,\ldots,\gamma^k$ only up to $\deg\gamma^1,\ldots,\deg\gamma^k$.

Proposition 6. *Let $i : \widetilde{\mathbb{D}}^r_m \to \mathbb{Q}^r_m$ be a map defined by $i(a_{i_1\ldots i_r} t_1^{(i_1)} \ldots t_r^{(i_r)}) = A^{\gamma^1\ldots\gamma^k}_{j_1\ldots j_k} \tau^{(j_1)}_{\gamma^1} \ldots \tau^{(j_k)}_{\gamma^k}$ satisfying*

$$A^{\gamma^1\ldots\gamma^k}_{j_1\ldots j_k} = a_{j_l\delta_1^{\deg\gamma^l}\ldots j_l\delta_r^{\deg\gamma^l}} \tag{5}$$

Then i is an injective algebra homomorphism.

Proof. Let $a = a_{i_1\ldots i_r} t_1^{(i_1)} \ldots t_r^{(i_r)}$ and $b_{j_1\ldots j_r} t_1^{(j_1)} \ldots t_r^{(j_r)} \in \widetilde{\mathbb{D}}^r_m$. Then $c = ab$ satisfies $c = a_{i_1\ldots i_r} b_{j_1\ldots j_r} t_1^{(i_1+j_1)} \ldots t_r^{(i_r+j_r)}$, where $t_l^{(i_l)} = 0$ whenever $i_l > 1$.

Then $D = i(a)i(b)$ satisfies $D^{\alpha^1\ldots\alpha^k}_{\iota_1\ldots\iota_k} = \sum_{\mathcal{D}} A^{\beta^1\ldots\beta^h}_{i_1\ldots i_h} B^{\gamma^1\ldots\gamma^k}_{j_1\ldots j_{k-h}}$, where $A = i(a)$, $B = i(b)$ and $\mathcal{D}$ is the set of all decomposiotions of $\{\alpha^1,\ldots,\alpha^k\}$ onto $\{\beta^1,\ldots,\beta^h\}$ with complementary $\{\gamma^1,\ldots,\gamma^{k-h}\}$ and the bottom indices $i_1,\ldots,i_h$ as well as $j_1,\ldots,j_{k-h}$ correspond to the top multiindices. By the definition of i we have

$$D^{\alpha^1\ldots\alpha^k}_{\iota_1\ldots\iota_k} = \sum_{\mathcal{D}} a_{i_l\delta_1^{\deg\beta^l}\ldots i_l\delta_r^{\deg\beta^l}} b_{j_{\bar{l}}\delta_1^{\deg\gamma^{\bar{l}}}\ldots j_{\bar{l}}\delta_r^{\deg\gamma^{\bar{l}}}} \tag{6}$$

Further, $C = i(ab)$ satisfies

$$C^{\alpha^1\ldots\alpha^k}_{\iota_1\ldots\iota_k} = c_{\iota_l\delta_1^{\deg\alpha^l}\ldots\iota_l\delta_r^{\deg\alpha^l}} = \sum_{(j_1,\ldots,j_r)} a_{\iota_l\delta_1^{\deg\alpha^l}-j_1\ldots\iota_l\delta_r^{\deg\alpha^l}-j_r} b_{j_1\ldots j_r} \tag{7}$$

where $0 \le j_1 \le \iota_l\delta_1^{\deg\alpha^l},\ldots,0 \le j_r \le \iota_l\delta_r^{\deg\alpha^l}$.

The last equality follows from (2), the multiplication formula for (m,r)-quasivelocities. Obviously, (7) corresponds bijectively with decompositions $\{\alpha^1,\ldots\alpha^k\}$ and $\{\gamma^1,\ldots,\gamma^{k-h}\}$ in (6). This completes the proof. □

Proposition 7. *Let $\mu : \widetilde{J}^r \to QJ^r$ be the inclusion of non-holonomic r-jets into quasijets of order r from Proposition 3. Then the restriction $\widetilde{\mu}^r_m$ of μ to $\widetilde{T}^r_m\mathbb{R} = \widetilde{\mathbb{D}}^r_m$ coincides with $i : \widetilde{\mathbb{D}}^r_m \to \mathbb{Q}^r_m$ defined in Proposition 6.*

Proof. In general let $b_{i_1\ldots i_r}$ denote the coordinates of non-holonomic r-jets from $\widetilde{J}^r(\mathbb{R}^m,\mathbb{R})$, created by induction according to the definition of non-holonomic jets.

Let $\sigma : \mathbb{R}^m \to \widetilde{J}^r(\mathbb{R}^m, \mathbb{R})$ be a local section in the neighbourhood of $0 \in \mathbb{R}^m$. Then $\sigma(u)$ is expressed as $b_{i_1 \dots i_r}(u)$. Put $a_{i_1 \dots i_r} = b_{i_1 \dots i_r}(0) = \sigma(0)$. Further, assume $\mu(\sigma(u))$ has the coordinates $B^{\gamma^1 \dots \gamma^k}_{i_1 \dots i_k}(u)$ and put $A^{\gamma^1 \dots \gamma^k}_{i_1 \dots i_k} = B^{\gamma^1 \dots \gamma^k}_{i_1 \dots i_k}(0)$. As the assumption hypothesis we assume $B^{\gamma^1 \dots \gamma^k}_{i_1 \dots i_k}(u) = b_{i_l \delta_1^{\deg \gamma^l} \dots i_l \delta_r^{\deg \gamma^l}}(u)$ which implies the assertion for order r. To prove it for $r+1$, we have $j^1_v \sigma(u)$, in coordinates $(b_{i_1 \dots i_r}(v), b_{i_1 \dots i_r i_{r+1}}(v))$, where $b_{i_1 \dots i_r i_{r+1}}(v) = \frac{\partial b_{i_1 \dots i_r}(u)}{\partial u^{i_{r+1}}}|_v$. Further, $\mu(\sigma(u))$ considered as the map $T^r_u \mathbb{R}^m \to T^r_{\beta(\sigma(u))} \mathbb{R}$ is expressed by $y = B^{\gamma^1 \dots \gamma^k}_{i_1 \dots i_k}(u) u^{i_1}_{\gamma^1} \dots u^{i_k}_{\gamma^k}$. Then $T_v(\mu(\sigma(u)))$ satisfies $dy = \frac{\partial B^{\gamma^1 \dots \gamma^k}_{i_1 \dots i_k}(u)}{\partial u^{i_{k+1}}}|_v v^{i_1}_{\gamma^1} \dots v^{i_k}_{\gamma^k} v^{i_{k+1}}_{\gamma^{k+1}} + \sum_{l=1}^k B^{\gamma^1 \dots \gamma^k}_{i_1 \dots i_k}(v)$ $v^{i_1}_{\gamma^1} \dots v^{i_l}_{\gamma^l + e_{r+1}} \dots v^{i_k}_{\gamma^k}$, where e_{r+1} denotes the multiindex with just one unit on the $(r+1)$-st position. Setting $v = 0 \in \mathbb{R}^m$, comparing the components $x^{j_1}_{\alpha^1} \dots x^{j_l}_{\alpha^l}$ for $\alpha^h \subseteq E_{r+1}$ and taking into account Proposition 5 and Proposition 6, we prove our claim. □

Corollary 8. *Let $\mu : \widetilde{J}^r(M,N) \to QJ^r_m(M,N)$ be the inclusion from Proposition 3. The μ is the natural inclusion corresponding to $i : \widetilde{\mathbb{D}}^r_m \to \mathbb{Q}^r_m$.*

Proof. By Proposition 4 (i), every $X \in \widetilde{J}^r(M,N)$ is identified with $\{j^r_0 t_{\alpha(X)}, X \circ j^r_0 t_{\alpha(X)}\} \in P^r M[\widetilde{T}^r_m N, H^{\widetilde{J}^r}_N]$, where t_u is the translation, mapping 0 onto u. It follows from Proposition 4 (ii) and (iii) and Proposition 7 that $\{j^r_0 t_{\alpha(X)}, i(X \circ j^r_0 t_{\alpha(X)})\} \approx \{j^r_0 t_{\alpha(X)}, \mu(X \circ j^r_0 t_{\alpha(X)})\} = \{j^r_0 t_{\alpha(X)}, \mu(X) \circ \mu(t_{\alpha(X)})\} \approx \mu(X)$. □

Remark. We finish this section by a more geometrical description of the Weil algebra $\mathbb{Q}^2_m$ and $QJ^2_x(M,N)_y$. In general, let $A_1 = \mathbb{R} \times N_1$ and $A_2 = \mathbb{R} \times N_2$ be Weil algebras with nilpotent ideals N_1 and N_2. Their direct sum $A_1 \oplus A_2$ is defined as $\mathbb{R} \times N_1 \times N_2$, where we put $n_1 n_2 = 0$ for $n_1 \in N_1$ and $n_2 \in N_2$. By a direct evaluation, using Proposition 5, we obtain $\mathbb{Q}^2_m = \widetilde{\mathbb{D}}^2_m \oplus \mathbb{D}^1_m = (\mathbb{D}^1_m \otimes \mathbb{D}^1_m) \oplus \mathbb{D}^1_m$. In this way we find $QJ^2_x(M,N)_y = \widetilde{J}^2_x(M,N)_y \oplus J^1_x(M,N)_y$.

3. Quasijets and non-holonomic jets

In this section, we are going to apply the approach from Section 2 to rededuce a result by Dekrét in [1], giving the criterion how to recognize non-holonomic r-jets among quasijets of order r.

Let us recall the concept of the kernel injection, [1]. For a vector bundle $q : E \to M$ we have two structures of vector bundle on TE, namely $p : TE \to E$ and $Tq : TE \to TM$. Denote by $HE \to M$ (the so called heart of a vector bundle $E \to M$, [8], [6]) the vector bundle $Vp \cap VTq \to M$. The identification $VE \approx E \times_M E$ is well-known. The kernel injection $V^E_0 : E \approx HE \to TE$ is expressed by $V^E_0(x^i, y^p) = (x^i, 0, 0, y^p)$, [6].

Let us consider a vector bundle $T^{k-i} p^i_M : T^k M \to T^{k-1} M$ from Section 1. Denote by $V^{iM}_{0k} : T^{k-i} p^i_M \to T^{k-i+1} p^i_M$ the kernel injection on $T^k M$ with respect to the i-th vector bundle structure on $T^k M$. In Section 1, we defined the coordinates $x^p_{\varepsilon_1 \dots \varepsilon_k}$ on

T^kM. There is a Weil bundle structure on T^kM, namely $T^{\mathbb{D}^k}M$ corresponding to the Weil algebra $\mathbb{D}^k = \mathbb{D}\otimes\ldots\otimes\mathbb{D}$, where $\mathbb{D}$ denotes the algebra of dual numbers. Thus every element of T^kM with coordinates $x^p_{\varepsilon_1\ldots\varepsilon_k}$ can be represented by p polynomials of the form $x^p_{\varepsilon_1\ldots\varepsilon_k}\tau_1^{\varepsilon_1}\ldots\tau_k^{\varepsilon_k}$. It can be easily verified that

$$\text{(9)}\quad V^{iM}_{0k}(x^p_{\varepsilon_1\ldots\varepsilon_k}\tau_1^{\varepsilon_1}\ldots\tau_k^{\varepsilon_k}) = (1-\varepsilon_i)x^p_{\varepsilon_1\ldots\varepsilon_k}\tau_1^{\varepsilon_1}\ldots\tau_k^{\varepsilon_k} + \varepsilon_i(1-\varepsilon_i)x^p_{\varepsilon_1\ldots\varepsilon_k}\tau_1^{\varepsilon_1}\ldots\tau_k^{\varepsilon_k}\tau_{k+1}$$

The last formula is equivalent to $\tau_j \to (1-\delta^i_j)\tau_j + \delta^i_j\tau_j\tau_{k+1}$. By direct evaluation we obtain that $V^{i\mathbb{R}}_{0k} : \mathbb{D}^k \to \mathbb{D}^{k+1}$ is a homomorphism of Weil algebras and consquentely, $V^{iM}_{0k} : T^kM \to T^{k+1}M$ is a natural transformation. In the same way we obtain that $T^lV^{iM}_{0k} : T^{k+l}M \to T^{k+1+l}M$ is a natural transformation too. Denote by $\kappa_i : QJ^k \to QJ^{k-1}$ the projection of quasijet bundles induced by the projection $T^{k-i}p^i : T^k \to T^{k-1}$, [1]. Then the result by Dekrét reads

A quasijet $X \in QJ^k_x(M,N)_y$ represents a non-holonomic r-jet if and only if the following conditions are satisfied

$$\text{(10)}\quad \begin{aligned} &(T^{k-2}V^{1N}_{01})^{-1}\circ X\circ(T^{k-2}V^{1M}_{01}) = \kappa_2 X\\ &(T^{k-3}V^{1N}_{02})^{-1}\circ X\circ(T^{k-3}V^{1M}_{02}) = (T^{k-3}V^{2N}_{02})^{-1}\circ X\circ(T^{k-3}V^{2M}_{02}) = \kappa_3 X\\ &\vdots\\ &(V^{1N}_{0k-1})^{-1}\circ X\circ(V^{1M}_{0k-1}) = \;\ldots\ldots\; = (V^{k-1N}_{0k-1})^{-1}\circ X\circ(V^{k-1M}_{0k-1}) = \kappa_k X \end{aligned}$$

To deduce the result by our approach, denote by $(V^{jM,N}_{0i})^* : QJ^{i+1}(M,N) \to QJ^i(M,N) \hookrightarrow QJ^{i+1}(M,N)$ a map defined by $X \mapsto (V^{jN}_{0i})^{-1}\circ X\circ V^{jM}_{0i}$ for $X \in QJ^{i+1}(M,N)$. Analogously denote by $(T^lV^{jM,N}_{0i})^*: QJ^{i+l+1}(M,N) \to QJ^{i+l}(M,N) \hookrightarrow QJ^{i+l+1}(M,N)$ a map defined by $X \mapsto (T^lV^{jN}_{0i})^{-1}\circ X\circ(T^lV^{jM}_{0i})$ for $X \in QJ^{i+l+1}(M,N)$.

Proposition 9. *Let M,N be manifolds. Then $(T^{k-i-1}V^{jM,N}_{0i})^* : QJ^k(M,N) \to QJ^{k-1}(M,N)$ is a natural transformation for $i = 1,\ldots,k-1$ and $j = 1,\ldots,i$.*

Proof. By Proposition 4, it is sufficient to prove that $(T^{k-i-1}V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^* : \mathbb{Q}^k_m \to \mathbb{Q}^k_m$ is a homomorphism of Weil algebras equivariant in respect to the action of G^k_m on $\mathbb{Q}^k_m$. We prove this for $(V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^* : \mathbb{Q}^{i+1}_m \to \mathbb{Q}^{i+1}_m$ which proves our claim for $i = k-1$. We show, that this proof can be easily extended to other cases of i.

Let $Y_\varepsilon = Y_{\varepsilon_1\ldots\varepsilon_i 0}$ be the coordinates on $T^i\mathbb{R} = \mathbb{D}^i \hookrightarrow \mathbb{D}^{i+1} = T^{i+1}\mathbb{R}$ and $y_\delta = y_{\delta_1\ldots\delta_{i+1}}$ the coordinates on $T^{i+1}\mathbb{R} = \mathbb{D}^{i+1}$. Further, let $a^{\gamma^1\ldots\gamma^k}_{i_1\ldots i_k}$ be the coordinates on $QT^{i+1}_m\mathbb{R} = \mathbb{Q}^{i+1}_m$ and x^j_γ be the coordinates on $T^i\mathbb{R}^m = (\mathbb{D}^i)^m \hookrightarrow (\mathbb{D}^{i+1})^m = T^{i+1}\mathbb{R}^m$. Then the formula (9) implies

$$\text{(11)}\qquad Y_\varepsilon = (1-\varepsilon_{i+1})((1-\varepsilon_j)y_\varepsilon + \varepsilon_j y_{\varepsilon+e_{i+1}})$$

and the map $(V^{j\mathbb{R}^m\mathbb{R}}_{0i})^*$ satisfies

$$\text{(12)}\quad Y_\varepsilon = (1-\varepsilon_{i+1})((1-\varepsilon_j)a^{\gamma^1\ldots\gamma^k}_{i_1\ldots i_k}x^{i_1}_{\gamma^1}\ldots x^{i_k}_{\gamma^k} + \varepsilon_j\gamma^l_j a^{\gamma^1\ldots\gamma^l+e_{i+1}\ldots\gamma^k}_{i_1\ldots i_k}x^{i_1}_{\gamma^1}\ldots x^{i_k}_{\gamma^k})$$

for $\gamma^1+\dots+\gamma^k=\varepsilon$, $\deg\gamma^1<\dots<\deg\gamma^k$. Evaluating the coefficients by $x^{i_1}_{\gamma^1}\dots x^{i_k}_{\gamma^k}$, we obtain the coordinates $A^{\gamma^1\dots\gamma^k}_{i_1\dots i_k}$ on $\mathbb{Q}^i_m$ expressed by $a^{\delta^1\dots\delta^k}_{i_1\dots i_k}$ as follows

$$(13)\quad A^{\gamma^1\dots\gamma^k}_{i_1\dots i_k}=(1-\gamma^1_{i+1})\dots(1-\gamma^k_{i+1})((1-\sum_{l=1}^{k}\gamma^l_j)a^{\gamma^1\dots\gamma^k}_{i_1\dots i_k}+\gamma^l_j a^{\gamma^1\dots\gamma^l+ei+1\dots\gamma^k}_{i_1\dots i_k})$$

Let $a=a^{\gamma^1\dots\gamma^k}_{i_1\dots i_k}\tau^{(i_1)}_{\gamma^1}\dots\tau^{(i_k)}_{\gamma^k}\in\mathbb{Q}^{i+1}_m$, $b=b^{\delta^1\dots\delta^l}_{j_1\dots j_l}\tau^{(j_1)}_{\delta^1}\dots\tau^{(j_l)}_{\delta^l}\in\mathbb{Q}^{i+1}_m$ and $A=(V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^*(a)$, $B=(V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^*(b)$. Further, let $C=AB$ and $D=(V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^*(ab)$. Then we have $C^{\alpha^1\dots\alpha^k}_{i_1\dots i_k}=\sum_{\mathcal{D}}A^{\beta^1\dots\beta^h}_{j_1\dots j_h}B^{\gamma^1\dots\gamma^k}_{l_1\dots l_{k-h}}$, where $\mathcal{D}$ is the set of all decompositions of $\{\alpha^1,\dots,\alpha^k\}$ into $\{\beta^1,\dots,\beta^h\}$ with the complementary $\{\gamma^1,\dots,\gamma^{k-h}\}$ and the corresponding bottom indices. By (13) we have

$$C^{\alpha^1\dots\alpha^k}_{i_1\dots i_k}=\sum_{\mathcal{D}}[(1-\beta^1_{i+1})\dots(1-\beta^h_{i+1})((1-\sum_{l=1}^{h}\beta^l_j)a^{\beta^1\dots\beta^h}_{j_1\dots j_h}+\beta^l_j a^{\beta^1\dots\beta^l+e_{i+1}\dots\beta^h}_{j_1\dots j_h})]$$

$$(14)\qquad [(1-\gamma^1_{i+1})\dots(1-\gamma^{k-h}_{i+1})((1-\sum_{\bar{l}=1}^{k-h}\gamma^{\bar{l}_j})b^{\gamma^1\dots\gamma^{k-h}}_{l_1\dots l_{k-h}}+\gamma^{\bar{l}}_j b^{\gamma^1\dots\gamma^{\bar{l}}+e_{i+1}\dots\gamma^{k-h}}_{l_1\dots l_{k-h}})]$$

On the other hand

$$(15)\qquad D^{\alpha^1\dots\alpha^k}_{i_1\dots i_k}=(1-\alpha^1_{i+1})\dots(1-\alpha^k_{i+1})((1-\sum_{l=1}^{k}\alpha^l_j)\sum_{\mathcal{D}}a^{\beta^1\dots\beta^h}_{j_1\dots j_h}b^{\gamma^1\dots\gamma^{k-h}}_{l_1\dots l_{k-h}}+$$

$$\beta^l_j\sum_{\mathcal{D}}a^{\beta^1\dots\beta^l+e_{i+1}\dots\beta^h}_{j_1\dots j_h}b^{\gamma^1\dots\gamma^{k-h}}_{l_1\dots l_{k-h}}+\gamma^l_j\sum_{\mathcal{D}}a^{\beta^1\dots\beta^h}_{j_1\dots j_h}b^{\gamma^1\dots\gamma^l+e_{i+1}\dots\gamma^{k-h}}_{l_1\dots l_{k-h}}$$

It is easy to see that $C^{\alpha^1\dots\alpha^k}_{i_1\dots i_k}=D^{\alpha^1\dots\alpha^k}_{i_1\dots i_k}$ which follows that $(V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^*:\mathbb{Q}^i_m\to\mathbb{Q}^i_m$ is a homomorphism. The fact that $(T^{k-i-1}V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^*:\mathbb{Q}^k_m\to\mathbb{Q}^k_m$ is a homomorphism follows from (10), (12) and (13) remaining unchanged if we replace $(V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^*$ by $(T^lV^{j\mathbb{R}^m,\mathbb{R}}_{0i})^*$. The equivariancy of $(T^{k-i-1}V^{j\mathbb{R}^m,\mathbb{R}}_{0i})^*$ with respect to the action of G^k_m on $\mathbb{Q}^k_m$ follows from the fact that $T^{k-i-1}V^{j\mathbb{R}^m}_{0i}:\mathbb{D}^{k-1}\to\mathbb{D}^k$ is a natural transformation. This completes the proof. □

We state the following assertion, the proof of which is omitted since it is almost the same as that of Proposition 9, only technically easier.

Proposition 10. *The quasijet projection $\kappa_l: QJ^{k+1}\to QJ^k$ induced by the l-th vector bundle structure $T^{k+1-l}p^l: T^{k+1}\to T^k$ is a natural transformation.*

We shall need the coordinate expressions of homomorphisms $\kappa_{i+1}:\mathbb{Q}^{k+1}_m\to\mathbb{Q}^k_m\hookrightarrow\mathbb{Q}^{k+1}_m$. Let $a\in\mathbb{Q}^{k+1}_m$ and $A=\kappa_{i+1}(a)$. Further, let $a=a^{\gamma^1\dots\gamma^h}_{i_1\dots i_h}\tau^{(i_1)}_{\gamma^1}\dots\tau^{(i_h)}_{\gamma^h}$ and $A=A^{\delta^1\dots\delta^l}_{j_1\dots j_l}\tau^{(j_1)}_{\delta^1}\dots\tau^{(j_l)}_{\delta^l}$. Then it holds

$$(16)\qquad A^{\gamma^1\dots\gamma^h}_{i_1\dots i_h}=(1-\gamma^1_{i+1})\dots(1-\gamma^h_{i+1})a^{\gamma^1\dots\gamma^h}_{i_1\dots i_h}$$

If we compare (16) with (13), we have

$$a_{i_1\dots i_h}^{\gamma^1\dots\gamma^h} = (1 - \sum_{l=1}^{h} \gamma_j^l) a_{i_1\dots i_h}^{\gamma^1\dots\gamma^h} + \gamma_j^l a_{i_1\dots i_h}^{\gamma^1\dots\gamma^l + e_{i+1}\dots\gamma^h} \tag{17}$$

for all multiindices $\gamma^1, \dots, \gamma^h$ of order $k+1$ satisfying $\gamma^l \subseteq E_{k+1}$, $i+1 \notin \gamma^l$ for $l = 1, \dots, h$, $\deg\gamma^1 < \dots < \deg\gamma^h$.

By Proposition 6, a (m, r)-quasivelocity represents a non-holonomic (m, r)-velocity if and only if all $a_{i_1\dots i_h}^{\gamma^1\dots\gamma^h}$ depend on $\gamma^1 \dots \gamma^h$ only up to $\deg\gamma^1, \dots, \deg\gamma^h$. We prove the result of Dekrét if we show the equivalence of the last condition with (17).

Fix $\gamma^1, \dots, \gamma^h$ except of l and consider $\bar\gamma^l$ derived from γ^l by $\gamma_{i+1}^l = 0$ and $\bar\gamma^l = 1$. Further, denote by $e_{\deg\gamma^l}$ the multiindex containing the only unit at the $(\deg\gamma^l)$-th position. Clearly $\deg\gamma^l = \deg\bar\gamma^l$ and $a_{i_1\dots i_h}^{\gamma^1\dots\gamma^l\dots\gamma^h} = a_{i_1\dots i_h}^{\gamma^1\dots\bar\gamma^l\dots\gamma^h}$ is equivalent with (17), setting $j = \deg\gamma^l$. The condition $a_{i_1\dots i_h}^{\gamma^1\dots\gamma^l\dots\gamma^h} = a_{i_1\dots i_h}^{\gamma^1\dots e_{\deg\gamma^l}\dots\gamma^h}$ is equivalent with (17), which is obtained by setting $j = \deg\gamma^l$ and iterating the last step for all $i+1$ corresponding to units in the multiindex γ^l. This way we obtain the result by Dekrét.

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