

Lucie Kárná

Carcassonne – description of the game

In: Jan Brandts and Jan Chleboun and Sergey Korotov and Karel Segeth and Jakub Šístek and Tomáš Vejchodský (eds.): Applications of Mathematics 2012, In honor of the 60th birthday of Michal Křížek, Proceedings. Prague, May 2-5, 2012. Institute of Mathematics AS CR, Prague, 2012. pp. 107–116.

Persistent URL: <http://dml.cz/dmlcz/702897>

## Terms of use:

© Institute of Mathematics AS CR, 2012

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library*  
<http://dml.cz>

## CARCASSONNE — DESCRIPTION OF THE GAME

Lucie Kárná

CTU - Faculty of Transportation Sciences  
Na Florenci 25, Praha 1, Czech Republic  
karna@fd.cvut.cz

### Abstract

This article formalizes some aspects of the board game Carcassonne. Combinatorial problems related to the number of tile types are mentioned. Then the paper describes a game map using graph theory.

### 1. Introduction

Carcassonne is a tile-based board game for two to five players, designed by Klaus-Jürgen Wrede and published in 2000 by Hans im Glück Company in Germany<sup>1</sup>. It received two most prestigious European awards (Spiel des Jahres and Deutscher Spiele Preis) in 2001 and it was sold more than one million game sets. This game is so much popular because it is quite variable (thanks many available expansions), its rules are simple enough for pre-school children, and its strategy is intricate enough for adults.

The aim of the paper is to formalize some of rules and principles of the game. In the first chapter the rules of the game are shortly introduced. The second chapter describes principles of the terrain tiles construction. The third chapter demonstrates a graph-theory based method, providing description of the tile-covered area in any moment of the game.

### 2. Rules of the game

The basic game tile set consists of 72 square terrain tiles with picture of landscape part: green *meadows*, brown *cities* surrounded by city walls, *roads*, and *cloisters*. On the start of the game one of these tiles is lying on the table its face up. On each turn one player randomly draws one new terrain tile from a bag and places it adjacent to tiles that are already faced up. At least one edge of the new tile must fit tightly to the edge of some current placed tile, and roads must connect to roads, meadows to meadows, and cities to cities (see Fig. 1).

---

<sup>1</sup>In the Czech Republic this game is distributed by Albi.

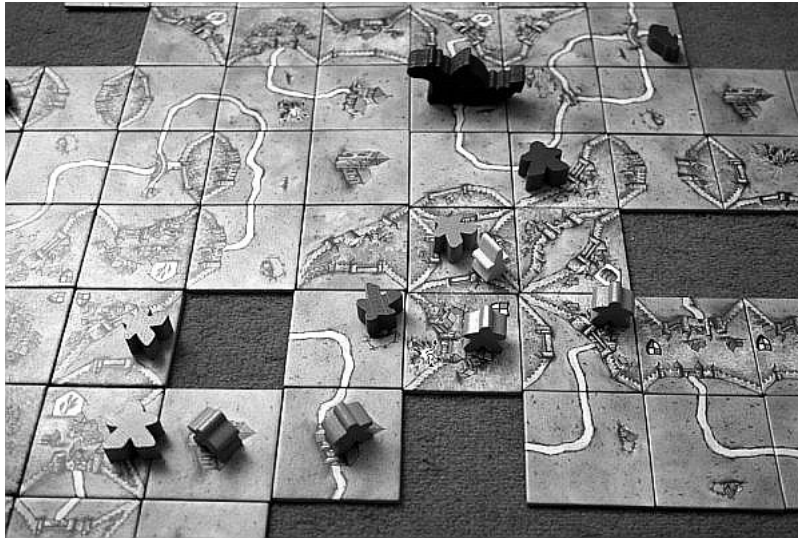


Figure 1: A part of the Carcassonne board after several turns. (Adopted from Wikimedia Commons, licensed under the Creative Commons. Author: Elentin.)

Once the tile is set, the player can occupy some feature on the new tile by his wooden figurine under simple rules. During the game any *completed* features are scored. A city is completed, when a city wall forms a closed cycle. Similarly, a road is completed when it is at both ends terminated by crossing, city wall or cloister. A cloister is completed when the tile with a cloister is fully surrounded with other tiles. A player, occupying completed feature with his figurine, becomes its *owner* and gains some amount of points, based on the principle “more is better”. After then all figurines from this feature are removed.

The game ends when the last tile has been placed. At that time, all incomplete features are scored, however for the cities their owner in this moment gains only half of the points. In addition, the farms are determined (a *farm* is a conected area of meadows), and scored. The player with the most points wins the game. For complete rules see [3].

### 3. Terrain tiles and its types

The tiles are essence of the game, hence their description is crucial for understanding of the game. A set of tiles with identical layout we call a *tile type*.

#### 3.1. Tile construction

Surveying a heap of diverse tiles in the box, we find that their construction follows certain rules:

- A city wall begins at one corner of the tile and ends in another corner, and it doesn't touch the tile edge on the way. On the one side of the city wall is a city terrain, on the other side is a meadow.

- A city wall doesn't intersect itself, another city wall or road.
- Every city area touches at least one edge of the tile. (It means that doesn't exist tiles like A or B on the Fig. 2. Such isolated city doesn't make sense in the game.)
- Inside of the tile it is at most one crossing or at most one cloister. There can be more than one city gate (every gate in another connected part of a city) together with one crossing.
- A road begins in a center of one tile edge and ends either on the center of another tile edge, or inside the tile in a crossing, a city gate or a cloister. It is at most one road end in the center of one tile edge. On both sides of the road is a meadow.
- A road doesn't intersect itself, another road or a city wall<sup>2</sup>.
- Every road segment has at least one end on the tile edge. (Similarly as for cities, the intra-tile roads – for example on the tile C on the Fig. 2 – are useless.)

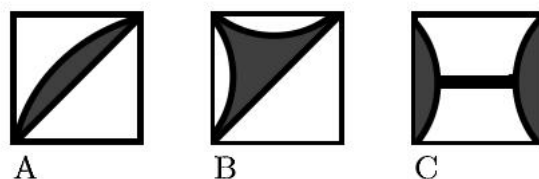


Figure 2: Examples of tiles, which doesn't meet "tile rules" mentioned in 3.1: isolated cities A and B, intra-tile road C. Dark grey area: city terrain, white area: meadow, thick line: a road.

### 3.2. How many tile types exist?

At first we focus only on the tile edges, the interior of the tile we will examine later. Tile edges determine possibility to locate a tile. A set of tiles with the same layout of edges we call a *group of tile types*.

According to the rules concerned in section 3.1, every tile edge is characterised by one of three "features": road (R), city (C), or meadow (M). Starting from fixed tile vertex (for example upper left corner), with fixed direction (for example clockwise), there are  $3^4 = 81$  possibilities how the edge values can be placed. However, the tiles can be rotated arbitrarily, and thus some possibilities are identical.

<sup>2</sup>In some expansion sets there exist tiles with bridge or tunnel. Although it looks interesting, it damages planarity of the game graph (and – as consequence – complicates definition of farms) without adding any new element to the game.

Enumerating all different options, we see, that there exist following alternatives: RRRR, RCCC, RRCC, RCRC, RRRC, RRCM, RRMC, RMRC plus alternatives obtained by the cyclic shift  $R \rightarrow C \rightarrow M \rightarrow R$ , it is  $8 \cdot 3 = 24$  possible groups of tile types.

In the game it is significant not only opportunity to locate tile to the specific place, but also the tile interior structure. For example tiles 2a and 1a+1a/A on the Fig. 5 can be located at the same place, but the first of them can relate to two cities, whereas the second one relates only to one city. The tiles 1a and 1c on the Fig. 3 differ only in the shape of the city, which can affect determination of farms.

**City tiles.** One tile can contain parts of multiple cities, up to four. A part of a single city on the tile (*city element*) can incide to one, two, three or four tile edges. All possible city elements are sketched on the Fig. 3.

In fact, in the basic game set occur only elements 1a, 2a, 2c, 3a and 4a – we denote these elements *ordinary city elements*. The element 1c shows up in one of expansion sets; the element 1b appears in another expansion set at the tile of the CCCM type in combination with element 2a, not solitary. Elements 1d and 2b don't occur in any of the known tile sets, neither alone nor in combination. Elements 1b and 1c we denote *extraordinary city elements*, elements 1d and 2b are *excluded city elements*.

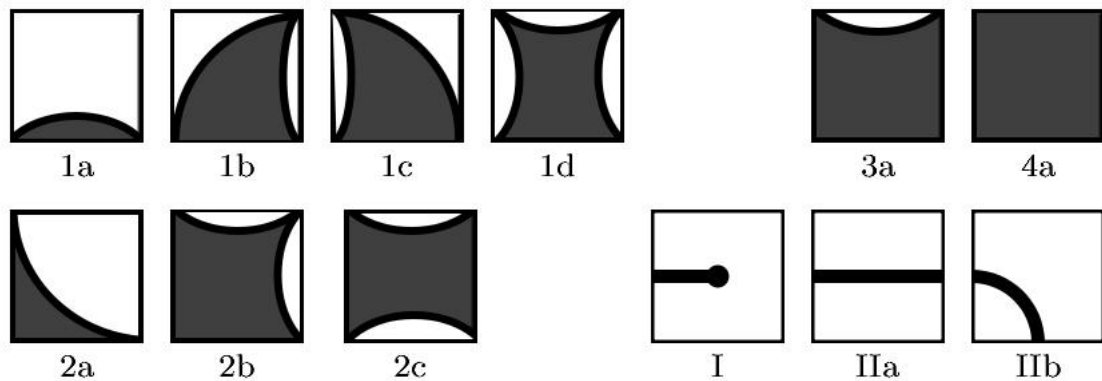


Figure 3: All possible shapes of city elements (1a – 4a) and road elements (I, IIa and IIb). Dark grey area is a city terrain, white area is a meadow, thick line is a road.

Combining these city elements, we obtain all types of tiles consisting only from city and meadow terrain and not containing any road. The combining procedure is demonstrated on the Fig. 4 for the group CCCM of tile types.



Figure 4: All terrain tiles from the group CCCM, consisting exclusively from ordinary city elements. Legend is the same as for Fig. 3.

Restricting to the ordinary city elements, we obtain these types of tiles:

- Group MMMM: a tile, containing only meadow and perhaps cloister inside, not any city or road.
- Group CMMM: element 1a.
- Group CCMM: element 2a and combination of elements 1a+1a.
- Group CMCM: element 2c and combination of elements 1a+1a.
- Group CCCM: combinations of the elements 1a+1a+1a, 1a+2c, 1a+2a, 2a+1a (different from the previous) and 3a.
- Group CCCC: combinations of elements 1a+1a+1a+1a, 1a+1a+2a, 1a+3a, 1a+2c+1a, 2a+2a and 4a.

There are 16 “city and meadow” tile types, made exclusively of ordinary city elements, and one “only meadow” type in total. However only 8 of them are contained in the basic game set (many of remaining types one can found in some expansion sets).

**Road tiles.** Because part of one road on the tile (*road element*) can incide to one or two tile edges, there are only three possible road elements – see Fig. 3.

Combining all possible road elements, we obtain all types of tiles not containing any city, namely:

- Group RMMM: road element I.
- Group RRMM: road element IIb and combination of the elements I+I.
- Group RMRM: road element IIa and combination of the elements I+I
- Group RRRM: combinations of the elements I+I+I and I+IIb (two possible configurations).
- Group RRRR: combinations of the road elements I+I+I+I or IIb+IIb.

Note that these 10 tile types can have several subtypes – in the tile centre can be either plain crossing or cloister<sup>3</sup>. Only 5 from these tile types are included in the basic game set.

**Complex tiles.** Tiles, containing both city and road elements, can be generated from “city and meadow” tile types named below by adding one, two and possibly three road elements. The basic game set contains 6 of them (for details see section 3.3 or Fig. 5).

### 3.3. Basic game tile set

The basic game set consists from these tile types: one piece of the tile I+I+I+I with plain crossing (group RRRR), three pieces of the tile 3a+I (group RCCC), five pieces of the tile 2a+IIb (group RRCC), three pieces of tile 1a+I+I+I (group RRRC), three pieces of the tile IIb+1a (group RRCM) and three pieces of its mirror image 1a+IIb (group RRMC), four pieces of the tile 1a+IIa (group RMRC), one piece of the tile 4a (group CCCC), five pieces of the tile 1a (group CMMM), two pieces of the tile 1a+1a and five pieces of the tile 2a (both from group CCMM), tiles 1a+1a and 2c (both from group CMCM), three pieces each, four pieces of tile 3a (group CCCM), four tiles with a cloister in the center (group MMMM), four pieces of the tile I+I+I with plain crossing (group MRRR), nine pieces of the tile IIb (group MMRR), eight pieces of the tile IIa (group MRMR), and two pieces of the tile I with a cloister in the center (group MMMR). In the basic game set doesn't occur any tiles from the groups RCRC, CCMR, CCRM, CRCM, MMRC, MMCR, and MCMR. You can see the sketch of all tile types from basic game set at Fig. 5. In the basic game set there is in total 72 tiles in 19 different types. Non-presence of some tile types (city + road + two meadows, two cities + road + meadow and RCRC group) seems to be intentional. Thanks to this, a player can prevent his opponent from completing his own city.

## 4. Map of the game

Observing the finished game, we can see a planar graph; edges of the graph are roads and city walls. In the following paragraphs we demonstrate one possible interpretation of this graph, which can be used for description of the game course.

In principle, we represent road as undirected path and city as interior of directed cycle. Since border of the tile-occupied area influences determination of farms, we have to reckon with them and perceive it as a edges of the graph as well. In the graph of the game we define several types of vertices and edges:

### Vertex types.

- *City vertex* represents an intersection of a citywall and tile corner. Vertex of this type incide with at least one city edge and can incide with city edges, city border edges or meadow border edges. Its degree is at least 2 and at most 8.

---

<sup>3</sup>In some expansion sets the tile centre can contain something else, for example volcano.

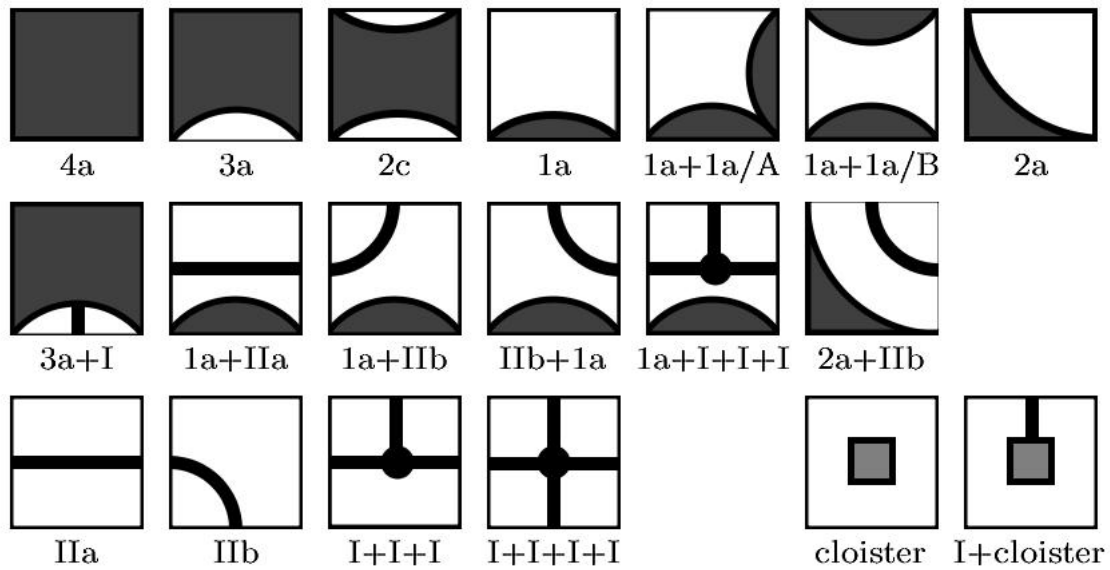


Figure 5: List of all tile types, contained in the basic game set. Legend for cities, roads and meadows is the same as for Fig. 3, grey square is a cloister.

- *City corner vertex* represents tile corner, which is surrounded by city terrain, and is located on the border of playfield. It's degree is 2 and incides with two city border edges. This type of vertex is temporary and can be cancelled or changed to a city vertex in the course of the game, when adjacent tile is added.
- *Road end vertex* represents definitive end of the road: crossing or cloister. Degree of the vertex can be any number from 0 to 4 (zero degree can have only a cloister) and incides only with road edges.
- *Connecting road vertex* represents intersection of a road and tile edge. This vertex has degree 2 or 3 and can incide with road edges and meadow border edges.
- *Gate vertex* represents intersection of the road and the city wall (city gate). Degree of the vertex is 3 or 4 and incides with two city edges and one or two road edges.
- *Meadow vertex* represents tile corner, which is surrounded by a meadow, and is located on the border of the game. Its degree is 2 and incides with two meadow border edges. Meadow vertex is temporary and can be cancelled in the course of the game, when adjacent tile is added.

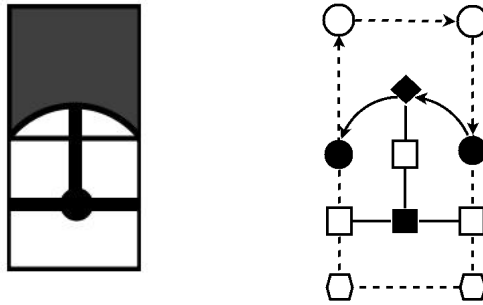


Figure 6: Example of the two connected tiles and corresponding graph representation. Legend for vertices: black circle = city vertex, white circle = city corner vertex, black square = road end vertex, white square = connecting road vertex, black diamond = gate vertex, white hexagon = meadow vertex; edges: full line = road edge, full arrow = city edge, dashed line = meadow border edge, dashed arrow = city border edge.

### Edge types.

- *City edge* represents a part of a city wall from one tile corner to another one, or from tile corner to the city gate. This edge is directed; on the right side of the edge is city terrain, on the left side is meadow. City edge can incide with city vertex or gate vertex.
- *City border edge* represents edge of a tile, occupied by a city terrain, where it isn't adjacent tile. It is directed, on the right side of the edge is city terrain, the left side is out of the tile. City border edge incides with city vertices or city corner vertices.
- *Road edge* is undirected and represents a part of a road from one tile edge to another tile edge, crossing, cloister or city gate. Road edge can incide with road end vertex, connecting road vertex or gate vertex.
- *Meadow border edge* represents edge of a tile (or its part from tile corner to road mouth), occupied by a meadow, where it isn't adjacent tile. It is undirected and can incide with meadow vertex, city vertex or connecting road vertex.

Both types of border edges are temporary and can be cancelled in the course of the game, when adjacent tile is added.

**Features.** The next step is to identify in the game graph particular features: roads, cities and farms. A *road* is undirected path, which all edges are road edges, all vertices except endpoints are connecting road vertex, and which cannot be extended according these rules. The road is *completed*, if both its endpoints are road end vertices or gate vertices, or if it forms a closed cycle (a “roundabout”). Otherwise it is *incompleted* road.

Because the game graph is planar, its faces can be identified. Excluding the outer face, the faces can be divided into two groups: cities and farms. The *city* is a face surrounded by directed cycle, which go round the face clockwise. If all edges of this cycle are city edges, then the city is *completed*, otherwise (at least one edge is a city border edge) it is *incompleted* city. If the face isn't city face nor outer face, we call it a *farm*.

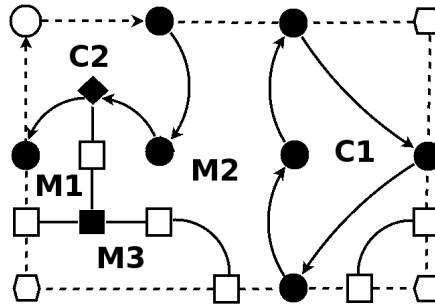


Figure 7: Example of the game graph with denoted features. Legend for vertices and edges is the same as in Fig. 6. The face C1 is a completed city, the face C2 is an incompleted city. The faces M1, M2 and M3 are farms. The road separating M1 and M2 is completed road, other two roads are incompleted.

**Adding a new tile.** Adding a tile to the game area can be interpreted as fusion of two graphs: graph of the current game area and graph of the new tile. The new added tile has to adjoin by at least one edge to edge of some current tile. Both of these edges must have identical terrain – either a city, or a meadow, or a road.

If these tile edges adjoin to a city terrain, they are in a game graph represented by a city border edges with opposite direction and their incident corners corresponds to city vertices or city corner vertices. Appending the tile to a game means, that both these edges evaporates, and head of one of these edges merges with tail of second edge and vice versa. If at least one of merging vertices is city vertex, resulting vertex will be city vertex. If both merging vertices are city corner vertices, they will be both cancelled and both edges inciding with them will be fused to one edge. (Note that both affected edges have congruent direction.)

If both tiles adjoin with meadow, they are represented by a meadow border edges and their incident corners correspond to city vertices or meadow vertices. Appending the tile to a game means, that both these edges disappear, and their corresponding edges merges together. If at least one of merging vertices is city vertex, resulting vertex will be city vertex. If both merging vertices are meadow vertices, they will be both cancelled and the remaining meadow border edges inciding with them will be fused.

The third case, a road, is similar to previous one. The only difference is, that both

adjacent tile edges correspond to two subsequent meadow border edges, connected by a connecting road vertex. Both these road vertices will be merged together; the two remaining vertices and inciding edges will be handled as in previous case of “meadow edges”.

When a player adds a tile to the game area, it can have one of more of following impacts on the features: increasing of completing of the city, fusion of two or more cities, elongating or completing of the road, joining of two roads, or fusion of two or more farms.

## 5. Conclusions

This simple and widespread board game is tightly related with several fields of mathematics – combinatorics, graph theory (and, of course, game theory).

Presented graph interpretation of the game is not the only reasonable approach. Author with nick Mr. Wang in his post [2] proves that it is impossible to construct a city using all 44 tiles from the basic tile set, containing a city. For this purpose he uses graph, which is in principle dual to our interpretation.

The description of the game can be expanded: game scoring can be modelled through graph labeling. For problems with property ownership the graph coloring looks to be usefull.

## References

- [1] Demel, J.: *Grafy a jejich aplikace*. Academia, Praha, 2002.
- [2] Mr. Wang: Carcassonne and Graph Theory. [online] <<http://mathlaoshi.com/2011/11/03/carcassonne-and-graph-theory/>>. Posted on 2011-11-3, [cit. 2011-12-10].
- [3] Carcassonne Central. [online] <<http://www.carcassonnecentral.com/>>. Updated 2010-5-22, [cit. 2011-12-10].