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USE OF A DIFFERENTIAL EVOLUTION ALGORITHM FOR THE OPTIMIZATION OF THE HEAT RADIATION INTENSITY

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Abstract: This article focuses on the heat radiation intensity optimization on the surface of an aluminium shell mould. The outer mould surface is heated by infrared heaters located above the mould and the inner mould surface is sprinkled with a special PVC powder. This is an economic way of producing artificial leathers in the automotive industry (e.g. the artificial leather on car dashboards). The article includes a description of a mathematical model that allows us to calculate the heat radiation intensity across the outer mould surface for every fixed location of the heaters. We also use this mathematical model for optimizing the locations of the heaters to generate uniform heat radiation intensity on the whole outer mould surface during the heating of the mould. In this way we obtain an even colour shade and material structure of the artificial leather. The problem of optimization is more complicated. Using gradient methods is not suitable because the minimized deviation function contains many local minima. A differential evolution algorithm is used during the process of optimization. The calculations were performed by a Matlab code written by the authors. The article contains a practical example including graphical outputs.

Keywords: heat radiation intensity, evolution optimization algorithm, mathematical model, experimental measurement, software implementation

MSC: 65K10, 78M50

1. Introduction

This article describes the calculation of radiation intensity on the whole mould surface for the fixed locations of infrared heaters above the mould and the process of heat radiation intensity optimization on the mould surface. The problem of optimization is rather complex (the used moulds often have very complicated surfaces, during the process of optimization possible collisions between one heater and another as well as collisions between a heater and the mould surface must be avoided). The minimized deviation function has many local minima. Using gradient methods for finding the global minimum is therefore unsuitable. Thus, we used an evolution optimization algorithm. A differential evolution algorithm *DE/rand/1/bin* (see details in [6]) is used to find suitable locations of the heaters over the mould to optimize the heat radiation intensity on the whole outer mould surface. The manufacturer needs to implement the optimization procedure on the production line (after its verification in the Matlab system). Therefore, we need to know the optimization process in every detail and to be able to perform own modifications of the programmed optimization algorithm. We do not use existing commercially available software tools.

In practice, an aluminium mould is heated by a set of infrared heaters located above the outer mould surface. It is necessary to ensure the same heat radiation intensity (within a given tolerance) on the whole outer mould surface by finding a suitable locations for the heaters. In this way the same colour and material structure of the artificial leather are assured. Moulds which very often have complicated shapes and which weigh from 100 to 300 kilograms are used. The infrared heaters have a tubular form and their length is about 20 centimeters. Every heater is equipped with a mirror located above the radiation tube which reflects heat radiation in a set direction (see Figure 1).

Figure 1: Infrared heater Ushio with heating power 2000 W.

2. Mathematical model of the heat radiation

In this chapter a mathematical model of the heat radiation produced by the infrared heaters on the outer mould surface is described. The heaters and the heated mould are represented in 3-dimensional Euclidean space E_3 using the Cartesian coordinate system (O, x_1, x_2, x_3) with basis vectors $e_1 = (1, 0, 0), e_2 = (0, 1, 0)$ and $e_3 = (0, 0, 1)$.

2.1. Representation of the heater

A heater is represented by a straight line segment of length d (see Figure 2). The location and orientation of a heater is defined by the following parameters:

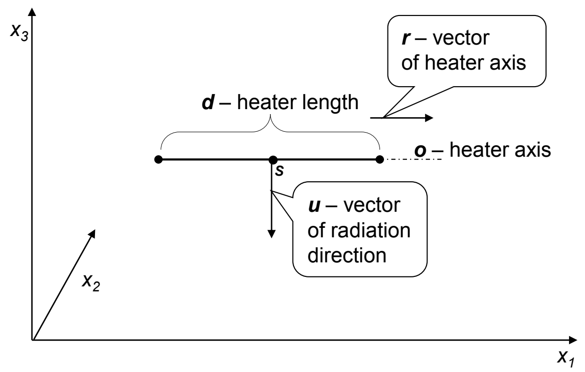

Figure 2: Schematic representation of the infrared heater.

(i) the coordinates of the heater centre $S = [s_1, s_2, s_3]$, (ii) the unit vector $u = (u_1, u_2, u_3)$ of the heat radiation direction, where component $u_3 < 0$ (i.e., the heater radiates "downward"), (iii) the vector of the heater axis $r = (r_1, r_2, r_3)$. Another way to determine the vector r is by using angle φ between the vertical projection of vector r onto the x_1x_2-plane and the positive part of axis x_1 (the vectors u and r are orthogonal, $0 \leq \varphi < \pi$). The location of each heater Z can be defined by the following 6 parameters

$$Z : (s_1, s_2, s_3, u_1, u_2, \varphi). \tag{1}$$

2.2. Representation of the mould

The outer mould surface P is described by elementary surfaces p_j, where $1 \leq j \leq N$. It holds that $P = \cup p_j$, where $1 \leq j \leq N$ and $int\ p_i \cap int\ p_j = \emptyset$ for $i \neq j$, $1 \leq i, j \leq N$. Each elementary surface p_j is described by the following parameters: (i) its centre of gravity $T_j = [t_1^j, t_2^j, t_3^j]$, (ii) the unit outer normal vector $v_j = (v_1^j, v_2^j, v_3^j)$ at the point T_j (we suppose v_j points "upwards"and therefore is defined through the first two components v_1^j and v_2^j), (iii) the area w_j of the elementary surface. Every elementary surface p_j thus can be defined by the following 6 parameters

$$p_j : (t_1^j, t_2^j, t_3^j, v_1^j, v_2^j, w_j). \tag{2}$$

2.3. Experimental measurement of the heater radiation intensity

We need to know the heat radiation intensity in the heater surroundings to calculate the total radiation intensity on the outer mould surface. The heater manufacturer does not provide the distribution function of the heat radiation intensity in the heater surroundings. We set up the experimental measurement of the heat radiation intensity as follows. The location of the heater is $Z : (0, 0, 0, 0, 0, 0)$ in accordance with relation (1), i.e., the centre S of the heater lies at the origin of the Cartesian coordinate system (O, x_1, x_2, x_3); the unit radiation vector has coordinates

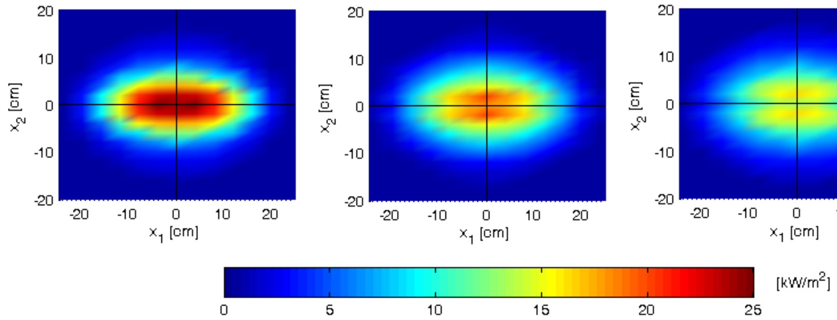

Figure 3: Heat radiation intensity in the planes at distances 9, 11 and 13 cm from the heater.

$u = (0, 0, -1)$ and the vector of the heater axis has coordinates $r = (1, 0, 0)$. We assume the heat radiation intensity across the elementary surface p_j is the same as at the centre of gravity T_j. The heat radiation intensity at T_j depends on the position of this point (determined by the first three parameter in the elementary surface p_j given by (2)) and on the direction of the outer normal vector v_j at point T_j (determined by the fourth and fifth parameters in the elementary surface p_j given by (2)). The heat radiation intensity I in the surroundings and below the heater was experimentally measured by a sensor at selected points $a = [a_1, a_2, a_3, a_4, a_5]$ (the first three parameters a_1, a_2, a_3 describe the position of the centre of gravity of a fictitious elementary surface and the fourth and fifth parameter describes the direction of the outer normal vector at the point $[a_1, a_2, a_3]$).

We use measured values $I(a)$ of heat radiation intensity at the selected points a and the linear interpolation function of five variables to calculate the heat radiation intensity $I(b)$ for the general point $b = [b_1, b_2, b_3, b_4, b_5]$ in the heater surroundings.

The measured heat radiation intensity, and its interpolated values in three parallel planes with x_1x_2-plane are shown in colour in Figure 3 in the case of 0^o deflection of the axis of the sensor (i.e., axis of the sensor is vertical). We use linear interpolation of a function of five variables. We assume that the point b holds $a_{j,i_j} \leq x_j^b \leq a_{j,i_j+1}$ for $1 \leq j \leq 5$. Let us denote $m_j = \frac{x_j^b - x_{j,i_j}}{a_{j,i_j+1} - a_{j,i_j}}$ for $1 \leq j \leq 5$. Then it holds for the interpolation value of radiation intensity $I(b)$ at the point b of heater Z

$$I(b) = I\left(x_1^b, x_2^b, x_3^b, x_4^b, x_5^b\right) = \sum_{k_1=i_1}^{i_1+1} \dots \sum_{k_5=i_5}^{i_5+1} I\left(a_{1,k_1}, a_{2,k_2}, a_{3,k_3}, a_{4,k_4}, a_{5,k_5}\right) \cdot \quad (3)$$

$$\cdot \prod_{l=1}^{5} H\left(l, k_l - i_l\right).$$

The interpolation formula is described in detail in [1], p. 148, and [3].

2.4. General case of the heater location

In this subsection we explain a transformation of the general case of a heater location with reference to the special heater position solved in Subsection 2.3. For a heater in a general position, we briefly describe the transformation of the previous Cartesian coordinate system (O, e_1, e_2, e_3) into a positively oriented Cartesian system $(S, r, n, -u)$, where S is the centre of the heater, r is the heater axis vector, and u is the direction vector of the heat radiation. The vector n is determined by the vector product of the vectors $-u$ and r (see more detail in [2], [7]) and is defined by the following relation

$$n = (-u) \times r = \left(- \begin{vmatrix} u_2 & u_3 \\ r_2 & r_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_3 \\ r_1 & r_3 \end{vmatrix}, - \begin{vmatrix} u_1 & u_2 \\ r_1 & r_2 \end{vmatrix} \right).$$

The vectors r, u and n are normalized to give the unit length. Then we can define an orthonormal transformation matrix

$$\mathbf{A} = \begin{pmatrix} r_1 & n_1 & -u_1 \\ r_2 & n_2 & -u_2 \\ r_3 & n_3 & -u_3 \end{pmatrix}.$$

Recall that for the elementary surface p_j, the respective triples T_j and v_j represent its centre of gravity and its outer normal vector in the Cartesian coordinate system (O, e_1, e_2, e_3). If S is the triple of parameters representing (in (O, e_1, e_2, e_3)) the centre of the heater that determines the coordinate system $(S, r, n, -u)$, then T_j and v_j are transformed as follows

$$\left(T_j^{'}\right)^T = \mathbf{A}^T \left(T_j - S\right)^T \quad \text{and} \quad \left(v_j^{'}\right)^T = \mathbf{A}^T v_j^T, \tag{4}$$

where $T_j^{'}$ and $v_j^{'}$ are the coordinates in $(S, r, n, -u)$. In this way, we transform the general case of the heater location to the measured case and we can calculate heat radiation intensity by using linear interpolation as described in the previous subsection (transformed point $T_j^{'}$ and vector $v_j^{'}$ correspond to point b in Subsection 2.3).

2.5. Calculation of the total heat radiation intensity

Now we describe the numerical computation procedure for the total heat radiation intensity on the mould surface. We denote by L_j the set of all heaters radiating on the jth elementary surface p_j $(1 \leq j \leq N)$ for the fixed locations of the heaters, and I_{jl} the heat radiation intensity of the lth heater on the p_j elementary surface. Then the total radiation intensity I_j on the elementary surface p_j is given by the following relation

$$I_j = \sum_{l \in L_j} I_{jl} \,. \tag{5}$$

The producer of artificial leathers recommends a constant value of the heat radiation intensity on the whole outer mould surface. Let us denote this constant value as I_{rec}. We can define function F, the deviation of the heat radiation intensity, by the relation

$$F = \frac{\sum_{j=1}^{N} |I_j - I_{rec}| w_j}{W} \tag{6}$$

and the deviation $\tilde{F}$ by the relation

$$\tilde{F} = \sqrt{\frac{1}{W} \cdot \sum_{j=1}^{N} (I_j - I_{rec})^2 \, w_j} \ , \tag{7}$$

where $W = \sum_{j=1}^{N} w_j$ and we highlight that w_j denotes the area of the elementary surface p_j. We need to find the locations of the heaters so that the value of deviation F (alternatively deviation $\tilde{F}$) is as small as possible.

3. Optimization of the heaters locations

Functions F and $\tilde{F}$ defined by (6) and (7) contain many local minima. Using gradient methods for finding global minima of the functions F and $\tilde{F}$ is not appropriate. If we used a gradient method, there would be a high probability that we would find only a local minimum of the function. Therefore, we use a differential evolution algorithm (more details in [6]) for finding an optimized minimum of function F (i.e., to optimize the locations of the heaters). The disadvantage of a differential evolution algorithm is its computational demandingness and slow convergence. The location of every heater is defined in accordance with the relation (1) by 6 parameters. Therefore $6M$ parameters are necessary to define the locations of all M heaters. One individual in the differential evolution algorithm represents one possible location of all $6M$ heaters. In the algorithm we successively construct populations of individuals. Every population includes NP individuals where every individual is a potential solution to our problem. We seek the individual $y_{min} \in C$ satisfying the condition

$$F(y_{min}) = min\{F(y); y \in C\}, \tag{8}$$

where $C \subset E_{6M}$ is the set we are searching for. Every element of C is formed by a set of $6M$ allowable parameters and this set defines just one constellation of the heaters above the mould. The identification of the individual y_{min} defined by (8) is not realistic in practice. But we are able to determine an optimized solution y_{opt}. The generated individuals are saved in the matrix $\mathbf{B}_{NP\times(6M+1)}$. Every row of this matrix represents one individual, y, and its evaluation, $F(y)$.

3.1. Differential evolution algorithm

Now we describe schematically the particular steps of the differential evolution algorithm named *DE/rand/1/bin* (for more details see [6] and [8]) which is applied to our problem.

We define a specimen which contains values ranges of each gene of the individual in the first step of the algorithm. Then we define an initial individual y_1 and randomly generate the initial generation of individuals. We create successively generations of individuals y and we are looking for an individual with the smallest value $F(y)$ (where function F is given by relation (6)) in the following steps of the algorithm. Note that four individuals y of a generation participate in the creation of individual y of the following generation. We describe the diagram of the algorithm.

Input: the initial individual y_1, population size NP, the number of used heaters M (dimension of the problem is $6M$), crossover probability CR, mutation factor f, the specified accuracy of the calculation ε.
Internal computation:
1. create an initial generation ($G = 0$) of NP individuals $y_i^G, 1 \leq i \leq NP$,
2.a) evaluate all the individuals y_i^G of the generation G (calculate $F(y_i^G)$ for every individual y_i^G), b) store the individuals y_i^G and their evaluations $F(y_i^G)$ into the matrix $\mathbf{B}$,
3. *repeat until* $min\{F(y_i^G); y_i^G \in \mathbf{B}\} < \varepsilon$
a) *for* $i := 1$ *step* 1 *to* NP *do*
(i) randomly select index $k_i \in \{1, 2, \ldots, 6M\}$,
(ii) randomly select indexes $r_1, r_2, r_3 \in \{1, 2, \ldots, NP\}$,
where $r_t \neq i$ for $1 \leq t \leq 3$ and
$r_1 \neq r_2, r_1 \neq r_3, r_2 \neq r_3$;
(iii) *for* $j := 1$ *step* 1 *to* $6M$ *do*
if $(rand(0,1) \leq CR$ *or* $j = k_i)$ *then*
$y_{i,j}^{trial} := y_{r_3,j}^G + f\left(y_{r_1,j}^G - y_{r_2,j}^G\right)$
else
$y_{i,j}^{trial} := y_{i,j}^G$
end if
end for (j)
(iv) *if* $F\left(y_i^{trial}\right) \leq F\left(y_i^G\right)$ *then* $y_i^{G+1} := y_i^{trial}$
else
$y_i^{G+1} := y_i^G$
end for(i),
b) store individuals y_i^{G+1} and their evaluations $F\left(y_i^{G+1}\right)$ $(1 \leq i \leq NP)$ of new generation $G + 1$ into the matrix $\mathbf{B}$, $G := G + 1$
end repeat.
Output:
the row of matrix $\mathbf{B}$ that contains corresponding value $min\{F(y_i^G); y_i^G \in \mathbf{B}\}$ represents the best found individual y_{opt}.

Note that function $rand(0, 1)$ randomly chooses a number from the interval $\langle 0, 1\rangle$. The notation $y_{i,j}^G$ means the jth component of an individual y_i^G in Gth generation. The individual y_{opt} is the final optimized solution that contains information about the location of every heater in the form (1).

4. Practical example

Now we describe a practical example of the heating of an aluminium shell mould. The volume of the mould is $0.8 \times 0.4 \times 0.15\,\mathrm{m}^3$, mould thickness is 8 mm (see Figure 5), the number of elementary surfaces, $N = 2,064$; the heat radiation intensity recommended by the producer of artificial leathers, $I_{rec} = 47\,\mathrm{kW/m}^2$. We use 16 infrared heaters (i.e., $M = 16$) of the same type (producer Philips, power 1,600 W, length 15 cm, width 4 cm). In the first step we calculate value $F(y_1)$ where the deviation of the heat radiation intensity F is defined by relation (6) and the initial individual y_1 corresponds to the following locations of the heaters. The centres of the heaters lie in the plane parallel to the x_1x_2-plane and at a distance of 10 cm from the centre of gravity T_j of the elementary surface p_j with the highest value $x_3^{T_j}$ $(1 \leq j \leq N)$. All the heaters have $r = (1, 0, 0)$ and $u = (0, 0, -1)$ (that is, all the heaters radiate downwards and they are parallel to the axis x_1). Then the deviation for this location of heaters is $F(y_1) = 20.74$.

We use the differential evolution algorithm described in subsection 3.1. to optimize the locations of the heaters. The parameters of the algorithm are as follows: population size $NP = 192$ (dimension of the problem is $6M = 96$), mutation factor $f = 0.98$ and crossover probability $CR = 0.60$. The heaters locations y_{tech} recommended by the producer technicians based on their experience in the production gives $F(y_{tech}) = 11.2204$. We obtain the optimized individual y_{opt} with value $F(y_{opt}) = 2.02$ after 4,000 generations of the differential evolution algorithm. The dependence of the deviation $F(y_{opt})$ on the number of generations is shown in Figure 4. Furthermore, Figure 5 shows a graphical representation of heat radiation on the mould surface corresponding to individual $F(y_{opt})$ (where the levels of radiation intensity in $\mathrm{kW/m}^2$ correspond to the shades of grey colouring).

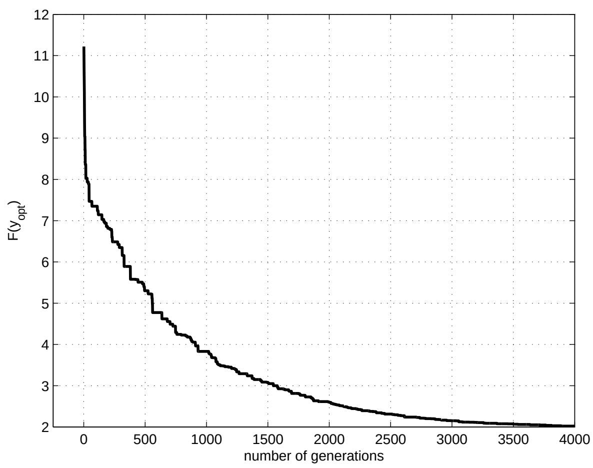

Figure 4: Dependence of $F(y_{opt})$ on the number of generations.

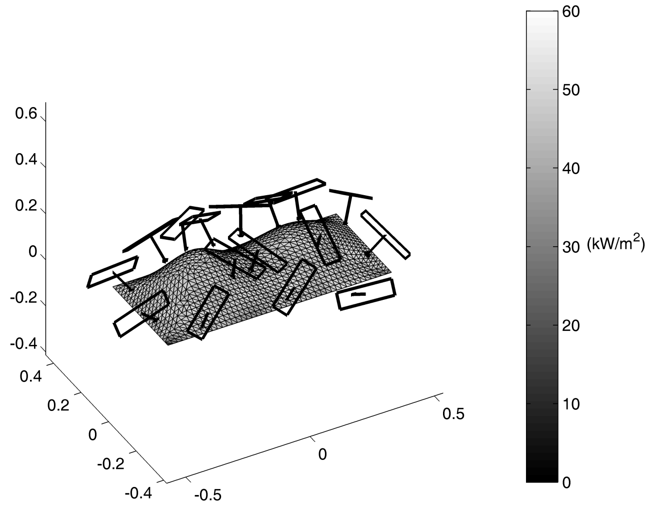

Figure 5: Heat radiation intensity (kW/m^2) on the mould surface and the locations of the heaters corresponding to the individual y_{opt}.

We made calculations on a PC computer with CPU: IntelCore i7-3770 CPU @3,4 GHz, RAM: 32 GB and GPU: GeForce GTX 460.

5. Conclusions

On the basis of practical calculations, we get a sufficiently exact solution for the optimized locations of heaters over the mould. We obtained more exact results using the differential evolution algorithm than using a genetic algorithm in numerical experiments (see [4], [5]). The temperature differences on the inner mould surface have to be maintained in the range of 3 °C during the mould heating process. The heat conductivity of the mould helps to unify different temperatures on the mould surface.

The locations of heaters determined on the basis of experience of technicians produces significantly worse results than the optimized locations. Generally, this approach is more time consuming (approximately two to three weeks depending on the mould size and the number of heaters). Furthermore, calculated optimization of the locations of heaters is more accurate and faster than optimization based on technicians experience.

The described method for manufacturing is an energy-efficient way of artificial leathers production. The given optimization process is advantageous for producer and induces virtually no additional cost.

Acknowledgements

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