# Jarník's note of the lecture course Punktmengen und reelle Funktionen by P. S. Aleksandrov (Göttingen 1928)

Mathematics in Göttingen

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# MATHEMATICS IN GÖTTINGEN

## University in Göttingen

The University in Göttingen, officially named Georg-August-Universität Göttingen but known informally as Georgia Augusta, was founded in 1734 by George II, King of Great Britain and Elector of Hannover, when he ordered his prime minister Gerlach Adolph von Münchhausen to establish, at Göttingen, a new German university having the four classic faculties of theology, law, philosophy and medicine. His main aim was practical: to improve education in Germany, especially the preparation of lawyers and doctors, state officers and teachers. He hoped, as well, to extend academic freedom in accordance with the then prevailing ideas of the European Enlightenment. The first lectures started in 1734; the official inauguration of the university, however, took place three years later.

Within fifty years of its founding, the university became one of the most important and influential German universities, teaching classical as well as modern science. During the 18th century it was renowned thanks mainly to its law faculty. However, it also enjoyed a lively spirit of inquiry providing a fruitful atmosphere for scientific study, exploration and research. Many students throughout Europe were attracted there by the lectures of Johann Stephan Pütter (1725–1807), one of the most prestigious professors of public law in Europe at that time, August Ludwig Schlözer (1735–1809), the historian, and Christian Gottlieb Heyne (1729–1812), the famous philologist. Those who studied there included Klemens Wenzel Nepomuk Lothar von Metternich (1773–1859), later the diplomat and prime minister of Austria, Karl Wilhelm von Humboldt (1767–1835), later the writer, philologist, organizer of higher education in Germany and professor at the University in Berlin, Napoleon Bonaparte (1769–1821), later French Emperor, Arthur Schopenhauer (1788–1860), later the outstanding German philosopher, Heinrich Heine (1797–1856), later the famous German poet, studied law or philosophy there at the end of 18th century and at the beginning of the 19th century. By 1812, it was acknowledged internationally as a modern European university of the first rank thanks to superior organization, lectures and lecturers as well as to its rich library containing more than 250 000 volumes. From 1810 up to the first half of the 1830's, the university was the Mecca for the study of public law in Germany, and almost every year law students made up more than half of the entire student body. Gustav von Hugo (1764–1844) and Rudolf von Jhering (1818–1892), the founders of the school of law and its most significant jurists, taught there in the 19th century; Otto von Bismarck (1815–1898), the first chancellor of the second German Empire, studied law there in 1833.

A significant role in the further development of the University in Göttingen was played by the political events of the late 18th and early 19th century: the French revolution, Napoleon's wars, political and social disturbances rocking the continent and the economic problems which followed, the Congress in

Vienna and the battle for freedom. These phenomena resulted in decreasing the number of students and professors in 1834. One of the most critical times came in 1837 when seven famous professors – Wilhelm Eduard Albrecht (1800–1876), the germanist, Friedrich Christoph Dahlmann (1785–1860), the historian, Georg Heinrich August Ewald (1803–1875), the orientalist, Georg Gottfried Gervinus (1805–1875), the historian, Wilhelm Eduard Weber (1804–1891), the physicist, Jakob Grimm (1785–1863), the philologist, and his brother Wilhelm Grimm (1786–1859), philologist – protested against the revocation of the liberal constitution of 1833 and further restriction of the academic prosperity and freedom by King Ernst Augustus I of Hannover. As a consequence of their protest, they were expelled from the university.

**Mathematics in Göttingen**

From the 1830's, the University in Göttingen reversed its main scientific and research trends. The exact and natural sciences, especially mathematics, took center stage, which ultimately resulted in Göttingen later becoming one of the most important world mathematical centers. From 1807 to 1855, Carl Friedrich Gauss (1777–1855), the most famous mathematician in the first half of the 19th century, taught and did research there. Later his successors and disciples, including Johann Peter Gustav Lejeune-Dirichlet (1805–1859), Bernhard Georg Friedrich Riemann (1826–1866), Rudolf Friedrich Alfred Clebsch (1833–1872), Hermann Amandus Schwarz (1843–1921), Heinrich Weber (1842–1913), as well as other outstanding mathematicians and their students made their studies and contributions to mathematics at Göttingen; thus, laying the groundwork for subsequent developments in mathematics at the beginning of the 20th century.

It should be mentioned that the years from 1866 up to the beginning of 1880's were not easy for Göttingen because, after the annexation of Hannover's Kingdom by Prussia, the university became one of the German regional schools. But, in 1885, the situation for mathematics changed dramatically when the Prussian minister of education called Felix Klein to the University in Göttingen. Ten years later, David Hilbert was invited to lecture there. By the end of the 19th century, their names and results attracted mathematicians from around the world to study, lecture or spend their scholarships in Göttingen making it the Mecca of mathematics at the beginning of the 20th century; a Mecca which, however, was temporarily interrupted by the First World War.

After the War, Richard Courant came to Göttingen as one of Klein's successors and he dedicated himself towards the advancement of the university and especially of mathematics there. In spite of post war crisis and inflation, he was able to find some state and private funding for the cultivation of mathematics and to help with the recovery of mathematics and physics in Germany. It was his urging that in 1922 mathematics along with the natural sciences formally separated from the other disciplines at the Faculty of Philosophy and a new mathematical center called *Mathematisches Institut der Universität* was created. The next year, he started the German collaboration with the International Education Board founded by John D. Rockefeller, Jr.,

which offered some fellowships to young scientists to study abroad. German mathematicians could finally travel out of Germany for study and other foreigners could come to work at Göttingen. International collaboration once again flourished.[1]

Courant also introduced many innovations in teaching, subjects matter, and the style of communication with students and colleagues. His new teaching methods brought early achievements:

*... Courant quite soon made an instructional innovation, the Anfängerpraktikum or "beginners' practice period," which was to have important results for the new "institute".*

*The Praktikum paralleled the calculus lectures. The students, often numbering as many as two hundred, received a mimeographed sheet of problems, some requiring inventive thinking as well as understanding of the material of the lectures. The professor regularly held a conference with a group of older students, discussing the problems and pointing out different methods of attack and various aspects of the solutions. The older students then went over the problems in the Praktikum with the beginning students and at the same time became personally acquainted with them. Solutions were written up and graded. Collusion was encouraged. Attendance was purely voluntary.*

*It is hard to realize today what a revolutionary innovation the Praktikum was in a German university at that time. Up until then, problems were never handed out except in applied courses, where they were usually not corrected. Textbooks were rarely utilized. Examinations were not given. The whole system was one of lecturing on the part of the professors and listening on the side of the students. The moment of truth did not arrive for several years, when the students had to take the state examination for teachers or the oral examination for the doctor's degree. For some, only then did it become clear that knowing mathematics is not like knowing the plot of a work of literature or the general outline of a historical period. The shock they experienced at this revelation not infrequently resulted in a nervous breakdown.*

*The Praktikum was a way of coping with a greatly increased number of students and a much lower level of ability and preparation It required a group of older students to supervise it and these became additional "assistants" with appropriate financial support from the government.*[2]

In the second decade of the 20th century, the University in Göttingen reached its academic peak: a high level of work prevailed not only in mathematics, but also in physics, chemistry, biology as well as in the social sciences and humanities. In Göttingen there again was a vibrant scientific atmosphere owing to the large and revitalized academic community, which included gifted students, distinguished visitors from all over the world, and guest professors who came to present papers, give seminars or hold regular lectures.

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[1] For more information see [Le], [PR] and [SS1].

[2] [R1], p. 99.

December 2, 1929, marked an important day for the mathematical community in Göttingen; the Mathematical Institute of the University in Göttingen formally opened its new residence. This building was a result of decades of work by Klein and by Courant and his contact with the Rockefeller Foundation and the German government. Mathematics obtained its own modern facilities which permitted a new interaction between students and teachers, German mathematicians and their foreign colleagues, as well as between mathematics and other sciences.[3]

In the 1930's, this fruitful and inspiring atmosphere came to its final end. Following the end of the academic year 1931–1932, the consequences of the worldwide depression began to be felt in Germany. Economic restrictions were adopted and Faculties were ordered to dismiss most of the younger assistants. To make matters worse, the university early on became a focal point of the Nazi ideology.

*Brown shirts and swastikas suddenly began to appear in mathematics lecture halls. The wearers of these were not members of the "in group" but several of them were good mathematicians. Werner Weber, one of the most active of the pro-Nazi students, had been a Privatdozent since 1931. The 20-year-old Oswald Teichmüller was extremely gifted. The Nazi sympathies of these and others came as a complete shock to Courant and the young mathematicians of the "in group."*[4]

On April 7, 1933, the government announced a series of laws – the Reichsgesetze – for the "restoration of the professional civil service". Their main intent was to remove Jewish professors, professors married to Jews or people who sympathized with Jews, as well as any academic freedom or left-leaning opinions that might still remain in political, social and intellectual life. As a result of these laws many "non-Aryans" were expelled from the universities or simply fled abroad before the summons came. This, later called the "great purge", included academics as Max Born, Victor Goldschmidt, James Franck, Eugene Wigner, Leó Szilárd, Edward Teller, Emmy Noether, Richard Courant, Otto Neugebauer, Hermann Weyl, Edmund Landau, Carl Ludwig Siegel, Felix Bernstein, Paul Bernays, etc. The legacy and effort of the last one hundred years was destroyed.[5]

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[3] The list of visitors looks like a "who's who" of the world of mathematics. There can be found: B. van der Waerden (from Netherlands), G. M. H. Köthe and O. Taussky (from Austria), K. Shoda and T. Takagi (from Japan), C. Chevalley and A. Weil (from France), P. S. Aleksandrov, A. O. Gel'fond, A. N. Kolmogorov, O. Yu. Schmidt, P. S. Urysohn (from the Soviet Union), Chiungtze C. Tsen (from China, known as Tseng Chiung-chih or Zeng Jiongzhi), K. Kuratowski (from Poland), G. D. Birkhoff, S. Lefschetz, S. MacLane, N. Wiener (from the USA), E. Artin, R. D. Brauer, H. Hasse, J. von Neumann, C. L. Siegel (from Germany). For more information on the development of the Mathematical Institute in Göttingen see for example [Gr], [Ne], [Pr], [R1], [R2], [R3], [Ro1], [Ro2], [Te] and [To2].

[4] [R1], pp. 137–138.

[5] For more information on the situation in Germany in the 1930's see for example [Ep], [NS], [R1], [R2], [PR] [Sch], [S], [SS1], [SS2] and [Te].

Although in 1945, after the Second World War, the University was reopened under the British control and quickly began to develop its facilities, the mathematics department never attained the unique reputation and the prestige it had had before the war.

### Mathematical circle

As we have already mentioned, the mathematical circle at the University in Göttingen dated its origins to the early 18th century. It started as a very small community concentrated around one regular professor's chair and one other professor named only for his person. Only a few students chose mathematics as their main area of study at that time and only a minority those continued on towards their doctorate. In 1848, a second regular chair was created and from that time, two or three professors (two ordinary and one "special") and some private docents lectured and collaborated together. At the beginning of the 20th century the number of students of mathematics increased and the ministry of education officially decided to establish a third chair for pure mathematics and a fourth chair for applied mathematics. After the First World War, the community consisted of four ordinary professors, three or more extraordinary professors, some guest professors, several private docents, lecturers, senior and junior assistants, about two hundreds undergraduate and graduate students and visitors. The growth of the faculty is described in the following tables:[6]

**First Department of Mathematics**

| | |
|---|---|
| 1807–1855 | Carl Friedrich Gauss |
| 1856–1859 | Johann Peter Gustav Lejeune-Dirichlet |
| 1860–1866 | Bernhard Georg Friedrich Riemann |
| 1868–1872 | Rudolf Friedrich Alfred Clebsch |
| 1874–1875 | Lazarus Fuchs |
| 1875–1892 | Hermann Amandus Schwarz |
| 1892–1895 | Heinrich Weber |
| 1895–1930 | **David Hilbert** |
| 1930–1933 | **Hermann Weyl** |
| 1934–1945 | Helmut Hasse |

**Second Department of Mathematics**

| | |
|---|---|
| 1848–1883 | Moritz Abraham Stern |
| 1886–1913 | Felix Klein |
| 1913–1918 | Constantin Carathéodory |
| 1918–1919 | Erich Hecke |
| 1920–1934 | **Richard Courant** |
| 1934–1954 | Theodor Kaluza |

[6] The names of professors who were Aleksandrov's colleagues and contemporaries in Göttingen are written in bold.

**Third Department of Mathematics**

| | |
|---|---|
| 1902–1909 | Hermann Minkowski |
| 1909–1934 | **Edmund Landau** |
| 1934–1936 | Erhard Tornier |
| 1936–1937 | Rolf Nevanlinna |
| 1938–1940 | Carl Ludwig Siegel |

**Fourth Department of Mathematics – Applied Mathematics**

| | |
|---|---|
| 1904–1924 | **Carl David Tolmé Runge** |
| 1925–1948 | **Gustav Herglotz** |

**Other Professors of Mathematics**[7]

| | |
|---|---|
| 1802–1832 | Bernard Friedrich Thibaut |
| 1817–1879 | Georg Karl Justus Ulrich |
| 1857–1859 | Bernhard Riemann |
| 1860–1897 | Ernst Schering |

**Extraordinary Professors of Mathematics**

| | |
|---|---|
| 1907–1933 | **Felix Bernstein** |
| 1919–1920 | Richard Courant |
| 1919–1933 | **Paul Bernays** |
| 1919–1933 | **Emmy Noether** |
| 1927–1936 | **Otto Neugebauer** |

From 1797 up to 1945, eighty four mathematicians obtained their *Habilitation* in mathematics, applied mathematics or mathematical physics at the University in Göttingen. Some of them continued their careers as private docents, extraordinary professors or ordinary professors there; others lectured at German universities as well as at European mathematical institutions or later moved on to positions in the USA. During 1923–1931, when Aleksandrov studied and lectured in Göttingen, among his young colleagues, who finished their *Habilitation* thesis, we can find the following mathematicians:

[7] In the 19th century, there was no position of extraordinary professor at the University in Göttingen.

| **Name** | **Habil.** | **Docent** |
|---|---|---|
| H. Behmann | 1921 | 1921–1935 |
| H. Kneser | 1922 | 1922–1925 |
| A. Ostrowski | 1923 | 1923–1927 |
| A. Walther | 1924 | 1924–1928 |
| E. Bessel-Hagen | 1925 | 1925–1927 |
| K. Grandjot | 1925 | 1925–1929 |
| O. Neugebauer | 1927 | 1927–1932 |
| B. van der Waerden | 1927 | 1927–1928 |
| H. Lewy | 1927 | 1927–1933 |
| W. Cauer | 1928 | 1928–1935 |
| U. Wegner | 1929 | 1929–1931 |
| W. Prager | 1927 | 1929–1934 |
| K. Friedrichs | 1929 | 1929–1930 |
| S. Cohn-Vossen | 1929 | 1929–1930 |
| W. Weber | 1931 | 1931–1935 |

It should be mentioned that A. Ostrowski obtained his *Habilitation* in Hamburg in 1923, W. Prager in Darmstadt in 1927, K. Friedrichs in Aachen in 1929.[8]

During the 1920's and at the beginning of 1930's many visiting professors spent some time in Göttingen in order to lecture or to collaborate with others there (for example Emil Artin (from Hamburg), Reinhold Baer (from Freiburg), Ruth Moufang (from Frankfurt), Richard von Mises (from Berlin), John von Neumann (from Berlin), George Polya (from Zurich), Oswald Veblen (from Princeton)). At the same time, some of the future outstanding mathematicians studied in Göttingen or worked there as Courant's, Hilbert's or Landau's assistants (for example Herbert Busemann, Max Deuring, Saunders MacLane, Gerhard Gentzen, Olga Taussky).

In their memoirs, several distinguished mathematicians described the unusually fruitful, open and friendly atmosphere that existed in the mathematical circle at Göttingen and stressed Courant's decisive role in the development of international contacts among young mathematicians. In [R1], for instance we can find:

*Friedrichs remembers the seminar as being impressively well attended.*

*"Siegel and Artin came, also Kneser. There was present every assistant who was in Göttingen at that time. They all had to come and participate. Such a group of people, who knew everything about everything – it was very exciting to me."*

*"When you say 'they had to come and participate,' do you mean that Courant required them to come, or that it was customary for everybody to come to the seminars?"*

*Friedrichs laughed.*

[8] For more information about the academic staff in Göttingen see [BN], [E] and [Sch].

*"That question cannot be answered. Such a notion did not exist for Courant. He did not ask the people as a requirement as Klein might have done. No. He would say, 'It's very important that you help us with this, we need your help'; and he would manage somehow that everybody who had something to contribute did attend. That's the way he always operated."*[9]

In Göttingen, informal personal contacts and discussions played a very important role in the development of mathematical ideas. In [R1], we can read:

*Much of the social life in Göttingen depended on the parties which the professors gave at various time during the year. These were characteristic. Landau's parties were intellectual tests, to which Hilbert never came. Games were played, there were winners and losers. "Hilbert didn't like the premises," Courant explained to me. Herglotz, naturally, gave no parties. Emmy Noether was famous for her "children's parties", to which Hilbert did come. The Weyls hosted a tea dance on a Saturday afternoon – very elegant and formal with many pretty girls present. At the Courant's house there was an unending succession of musical evenings, to which some students were always invited.*[10]

**Aleksandrov and his activities in Göttingen**

As we described in the previous chapter, Aleksandrov studied and then regularly lectured at the University in Göttingen mostly every summer semester from 1923 up 1932. His lectures on topology became famous and many students attended them attracted by his deep mathematical knowledge, his interest in pure abstract mathematics, insightful concepts as well as his perfect German. Quickly, he became a natural leader of Russian mathematicians there; at the same time, he also maintained a close working relationship with a number of mathematicians throughout the world.[11]

Saunders MacLane, one of these outstanding mathematicians, recalled his Göttingen experience from 1931 to 1933 in these words:

*Emmy Noether also was an active host for a visit by Russian topologist Paul Alexandroff. He gave (in excellent German) lectures on the elementary concepts of algebraic topology, as subsequently published in Springer booklet of the title (1932). Evidently Noether and Alexandroff talked about mathematics a great deal. It is clear that Noether's enthusiasm for using the appropriate abstract algebraic concepts had a major influence on the development of algebraic topology at that time. Specifically, up until 1926 or later, Alexandroff, Hopf, and others had measured the homological connectivity of topological spaces in terms of numbers: the Betti numbers and torsion coefficients.*[12]

In the book on Courant's life and work, the experience with Russian visitors in Göttingen before the World War Second are described as follows:

[9] This English quotation is taken from [R1], pp. 91–92.
[10] [R1], p. 128.
[11] For more information see [A], [G] and [R3].
[12] This English quotation is taken from [BS], pp. 71–72.

*Since 1923 Alexandroff had returned each year, either alone or accompanied by countrymen. From 1926 through 1930 Courant always arranged for him to give regular lectures. Once the Russian mathematician taught three courses in topology, each for a quite different audience of mathematicians. The summer* [1926] *that Courant was trying to keep up some semblance of attendance at Wiener's lectures, Alexandroff's were crowded.*

*"Alexandroff and the other Russian visitors were very important, very influential," I was told by Herbert Busemann, who came to Göttingen in 1925 to study mathematics after "wasting" – as he said – several years of his life in business to please his father, one of the directors of Krupp. "The Russians filled a gap because they were familiar with certain more abstract tendencies which were not well represented in Göttingen. Courant, as probably many have told you, was rather reactionary in his mathematical outlook. He didn't see the importance of the modern things."*

*The subject matter of Alexandroff's work, unlike that of Wiener, was far removed from Cournat's own – very pure and very abstract. Courant, however, was extremely enthusiastic about it, as he was always to be about the work of the many other Russians whom Alexandroff introduced into Göttingen over the years. They were all "of the first rank", he wrote somewhat later. The blind Pontryagin, who had always to be accompanied, was "absolutely leading spirit in topology." Gelfond "had astonished the world" with his proof of the transcendence of certain famous numbers; the work on prime numbers by Schnirelman, who walked barefoot through the street of Göttingen, "likewise." Lusternik "had the most highly original thoughts" concerning topology and analysis. Kolmogoroff was "an absolute master" in a great variety of fields.*

*To Alexandroff this enthusiasm for the work and the success of other mathematicians was impressive.*

*"There is very seldom the combination of really great scientist and great human presonality," he said me when I talked to him in Moscow in 1971. "And the emotional nature of Courant, his unselfish relation to other men, his interest in other human beings, his lack of egoism in relation to the world – he is absolutely exceptional. I do not know of any other man who had this kind of personality, this unselfish position in the science, and this joy – do you understand me? – in the success of other younger men around him. Courant was himself quite a young man at that time, but he was like that even then; and I think that really explains him and his success."*

*The Göttingen summer of 1926 had a special significance for Alexandroff, because it was then that he met Heinz Hopf, with whom he soon established a professional collaboration and a friendship that was to last until the end of Hopf's life. ...*

*Even in those long ago days, Alexandroff wore extremely thick lenses and was already quite bald. When I visited him in Moscow, he still looked very much as he had in snapshots I had seen of him from the 1920's. On a table in his crowded, rather Germanic living room there was a photograph of Urysohn, who, swimming with Alexandroff on the coast of Brittany in 1924, had been dashed against the rocks and killed at the age of twenty-six. I had recently*

*left Courant, tired and depressed, in America. Hopf had just died in Zurich. Alexandroff was recuperating from an almost fatal illness which had prevented him from attending Hopf's funeral. We talked for quite a time about that long ago summer 1926, when he had first met Hopf, and the others summers which he had spent in Göttingen.*

*"It was a beautiful time in my life," he told me as I prepared to leave, "and I cannot forget it yet."*[13]

Russian mathematicians, inspired by the organization and research activities in the Göttingen mathematical institute, tried to establish a similar school of mathematics in Moscow at the end of the 1920's. Their efforts, however, were not very successful owing to political and economic problems connected with the Soviet regime, as well as political trials and personal conflicts and controversies. Later, the time was not favourable for theoretical or abstract mathematics; the Second World War focused mathematical research first of all on applications.[14]

**Aleksandrov's recollections of Göttingen**

Aleksandrov considered the summers which he spent in Göttingen the most beautiful period in his life, the most decisive and creative years for his scientific works. In his article *In Memory of Emmy Noether*, he described the very lovely and fruitful atmosphere in the mathematical community in Göttingen and his professional as well as personal contacts with Noether:

*... Emmy Noether had close ties with Moscow. These ties began in 1923 when the late Pavel Samuelovich Urysohn and I first arrived in Göttingen and immediately fell in with the mathematical circle of which Noether was the head. We were immediately struck with the fundamental traits of the Noether school: the mathematical enthusiasm of its leader, which she conveyed to all*

[13] This English quotation is taken from [R1], pp. 106–107.

[14] More information about the birth and development of the Russian mathematical school in the first half of the 20th century can be found in [ZD] and in Russian language articles П. С. Александров, Б. В. Гнеденко, В. В. Степанов, *Развитие математики в московском университете в* XX*в. (до 1940 г.)*, Историко-математические исследования [P. S. Aleksandrov, B. V. Gnedenko, V. V. Stepanov, Development of mathematics at the Moscow University in the 20th century (up 1940), Istoriko-Matematicheskie Issledovanija] 1948, No. 1, pp. 9–42; П. С. Александров, *Математика в московском университете в первой половине* XX *века*, Историко-математические исследования [P. S. Aleksandrov, Mathematics at the Moscow University in the first half of the 20th century, Istoriko-Matematicheskie Issledovanija] 1955, No. 8, pp. 9–54; П. С. Александров, *Математическая жизнь в СССР*, Успехи математических наук [P. S. Aleksandrov, Mathematical life in the USSR, Uspekhi Matematicheskikh Nauk] 34(1979), pp. 219–249, 35(1980), pp. 241–278; Л. А. Люстерник, *Молодость московской математической школы*, Успехи матемаческих наук [L. A. Lyusternik, Youth years of mathematical school in Moscow, Uspekhi Matematicheskikh Nauk] 22(1967), No. 1, pp. 137–161, No. 2, pp. 195–253, No. 4, pp. 147–185, 25(1970), No. 4, pp. 188–196; Д. Е. Меньшов, *Воспоминания о молодых годах и о возникновении московской школы теории функций*, Историко-Математические Исследования [D. E. Menshov, Memories on the youth years and development of the Moscow school of function theory, Istoriko-matematicheskie issledovanija] 1983, No. 27, pp. 312–333. The political situation and personal conflicts in the Moscow mathematical circle is briefly written in [Gr] and [Lo].

*her students, her deep confidence in the importance and the mathematical productiveness of her ideas – a confidence which was far from universally shared even in Göttingen – and the unusual straightforwardness and sincerity of the relations between Noether and her students. At the time, this school consisted almost entirely of young students from Göttingen. The period when it would become international in its composition and be recognized as the primary center of algebraic thought in the world was still in the future. ...*

*My mathematical and personal friendship with Emmy Noether began in 1923 and did not cease up to the date of her death. In his eulogy, Weyl referred to this as follows: "She held a rather close friendship with Alexandroff in Moscow. I believe that her mode of thinking has not been without influence upon Alexandroff's investigations." I am glad to be able now to confirm Weyl's supposition. Noether's influence both on my own research and on that of other Moscow topologists was great and touched on the very essence of our work. In particular, my theory of continuous decompositions of topological spaces was created in large part under the influence of my conversations with her in December and January of 1925–1926 when we were in Holland together. It was also at this time that Noether's first ideas on the set-theoretic foundations of group theory were developing. She lectured on these in the summer of 1926. Although she returned to them several times later, these ideas were not developed further in their initial form, probably because of the difficulty of axiomatizing the concept of a group by taking the coset decomposition as the basic notion. But the idea of set-theoretic analysis of the concept of a group turned out to be fruitful, as shown by the subsequent work by Ore, Kurosh, and others.*

*Later years brought a strengthening and deepening of Noether's interest in topology. In the summers of 1926 and 1927 she attended courses given by myself and Hopf in Göttingen. She quickly found her way in what was for her a completely new field and made constant comments, some of which were deep and subtle. On first learning the systematic construction of combinatorial topology from our lectures, she immediately observed that it would be useful to consider directly the groups of chains and cycles of a given polyhedron. She proposed that instead of the usual definition of Betti numbers and torsion coefficients one should immediately define the Betti group as the factor group of the group of cycles modulo the subgroup of bounding cycles. This observation now seems self-evident. But in those years (1925–1928) this was a completely new point of view whose reception by many eminent topologists was far from sympathetic. Hopf and I immediately saw things Noether's way, but for some time in this respect we were a minority. Of course now it would scarcely occur to anyone to construct combinatorial topology without using abelian group theory as the foundation – all the more reason to note that the idea was originally Noether's. She immediately observed how simple and transparent the proof of the Euler-Poincaré formula became using the Betti group concept systematically. These remarks inspired Hopf to completely reprove the well-known fixed point theorem which Lefschetz had proved for manifolds and Hopf*

*had generalized to arbitrary polyhedra. Hopf's paper Eine Verallgemeinerung der Euler-Poincaréschen Formel, published in the Göttinger Nachrichten in 1928, bears the stamp of Noether's remarks.*

*Noether spent the winter of 1928–1929 in Moscow. She gave a course in abstract algebra at Moscow University and conducted a seminar in algebraic geometry at the Communist Academy. She quickly made contacts with most Moscow mathematicians, in particular with L. S. Pontryagin and O. Ju. Schmidt. The strong algebraic note in Pontryagin's work undoubtedly gained a great deal in its development from his contacts with Noether. She quickly fell in with our Moscow ways, both in mathematics and in everyday life. She lived in a modest room in the KSU dormitory at the Crimean Bridge and generally walked to the University. She was very interested in our way of life, and in particular in the life of Soviet youth, especially the students.*

*In the winter of 1928–1929, as usual, I traveled back and forth to Smolensk, and I gave lectures on algebra at the Pedagogical Institute there. Inspired by my constant conversations with Noether, I oriented my lectures that year around her concepts. Among my audience there, A. G. Kurosh immediately singled himself out, finding the Noetherian theory I was expounding much to his taste. And it was thus through my teaching that Emmy Noether acquired a student who has since – as everyone knows – grown to the status of an independent mathematician, whose work since that time has been carried on in the circle of ideas created by her.*

*In the spring of 1929 she left Moscow for Göttingen with the firm intention of returning the following summer. Several times she was close to the accomplishment of this intention, closest of all in the last year of her life. After being driven out of Germany, she seriously considered coming to Moscow for good, and I corresponded with her on that topic. She understood perfectly that nowhere else would she find the same opportunities to create a splendid new mathematical school that would replace the one that had been taken away from her in Göttingen. And I was already negotiating with the Commissariat of Education about obtaining a chair for her at Moscow University. However, as usual, the Commissariat was slow to make its decision and did not give me a final answer. Meanwhile, time was passing, and Emmy Noether, deprived of even that modest stipend which she had formerly received, could not wait and had to accept the invitation of the women's college at Bryn Mawr. ...*

*In her home, which is to say in the garret where she lived in Göttingen (Frieländerweg 57), she was an enthusiastic party giver. People of every scientific stature, from Hilbert, Landau, Brauer, and Weyl to quite young students, met there and felt at ease, which was hardly the case in many other European scientific salons. These "idle evenings" were arranged for the most diverse reasons – in the summer of 1927, for instance, because of the frequent visits of her student van der Waerden from Holland. Noether's parties, and likewise her excursions into the country, were a bright and unforgettable feature of the Göttingen mathematical life of the entire decade from 1923 to 1932.*

*Many lively mathematical discussions were carried on at these parties, but there was also a lot of just good fun and joking, and sometimes good Rheinwine and other delicacies.*[15]

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