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INFINITESIMAL DEFORMATIONS OF SURFACES IN E^3 *)

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The Killing vector field associated to a differentiable deformation of a surface in E^3 cannot be normal even locally, the mean curvature being not equal to zero. We shall see that, for a compact surface, it cannot be too "near" to the normal vector field.

Let $S \to E^3$ be a surface of class C^3 in the Euclidean 3-space E^3 described by the point $r = r(u)$; denote by $n(u)$ the normal unit vector to S at the point $r(u)$. In a suitable domain of S, we may write

$$r = r(u^1, u^2)\,; \tag{1}$$

let $g_{ij}(b_{ij})$ be the first (second) fundamental tensor. If x^i is any vector field on S, define $x^{i;j} = g^{jk}x^i{}_{;k}$. The following result due to K. YANO is well known: *Let S be compact. Then for each vector field x^i on S, we have*

$$\int_S (Kg_{ij}x^ix^j + x^{i;j}x_{j;i} - x^i{}_{;i}x^j{}_{;j})\,\mathrm{d}\sigma = 0\,. \tag{2}$$

This assertion remains valid for any S if

$$x^i{}_{;j}x^j - x^j{}_{;j}x^i = 0 \tag{3}$$

outside a compact subset of S.

Let $S \times R \to E^3$ be a one-parametric family of isometric surfaces in E^3; suppose that $S(0)$ is the surface S. Let us denote by w the vector field

$$w = \frac{\partial r(u, t)}{\partial t} \tag{4}$$

in E^3; let

$$V = v^ir_i + vn \tag{5}$$

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be the restriction of the field (5) to the surface S. We have

(6) $$v_{i;j} + v_{j;i} = 2vb_{ij}\,.$$

Each vector field (5) on S satisfying (6) is called the *Killing vector field*. For each Killing vector field, we have

(7) $$v^i_{\ ;i} = vH\,.$$

For a Killing vector field (5), the integral formula (2) reduces to

(8) $$\int_S (Kg_{ij}v^iv^j + 2vb_{ij}v^{i;j} - v^{i;j}v_{i;j} - v^2H^2)\,\mathrm{d}\sigma = 0\,.$$

We are now in the position to prove

Theorem 1. *Let* (5) *be a Killing vector field on the surface* (which is not necessary to be compact) S, *and suppose*: (a) $V = 0$ *outside some compact subset* $C \subset S$, (b) *the Gauss curvature of* S *is non-positive inside a domain* $D \supset C$, (c) *we have*

(9) $$b_{ij}v^{i;j} \geqq 0$$

on S. *If* $v \leqq 0$ *on* S, *then*

(10) $$v^i_{\ ;j} = 0$$

on S *and* (i) $V_T \equiv v^ir_i = 0$ *at each point with* $K < 0$, (ii) $V = V_T$ *at each point with* $H \neq 0$.

Theorem 2. *Let* S *be a compact surface with non-vanishing mean curvature. Let* (5) *be a Killing vector field on* S *such that*

(11) $$b^2 + H^2Kl \geqq 0 \quad on \quad S\,,$$

where

(12) $$b = b_{ij}v^{i;j}\,, \quad l = g_{ij}v^iv^j\,.$$

Further, let

(13) $$v \geqq \frac{b + \sqrt{(b^2 + H^2Kl)}}{H^2} \quad or \quad v \leqq \frac{b - \sqrt{(b^2 + H^2Kl)}}{H^2} \quad on \quad S\,.$$

Then we have (10) *and*

(14) $$v^2 = \frac{Kl}{H^2}\,.$$

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Резюме

ИНФИНИТЕЗИМАЛЬНОЕ ИЗГИБАНИЕ ПОВЕРХНОСТЕЙ В E^3

АЛОИС ШВЕЦ (Alois Švec), Прага

Пусть $r = r(u; t)$ – однопараметрическая система $S(t)$ поверхностей в E^3. Пусть

$$V = v^i \frac{\partial r(u; 0)}{\partial u^i} + v \,.\, n(u; 0)$$

является ограничением векторного поля $w = \partial r(u; t)/\partial t$ на поверхность $S(0)$; $n(u; 0)$ означает единичное нормальное векторное поле поверхности $S(0)$. Если $S(t)$ – система изометрических поверхностей, то выполнено (6), где b_{ij} – второй тензор поверхности $S(0)$. Далее, справедлива интегральная формула (8), на основании которой высказаны две теоремы.