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SUBMANIFOLDS OF KLEIN SPACES

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It is my feeling that the theory of partial differential equations developed in the last years especially at Harvard is of great importance for the differential geometry. Because of that Chap. 1 of this paper is a report on some results due to H. GOLDSMITH and others; see [1]. In the second part I apply this theory, and I show that to solve the equivalence problem for submanifolds in Klein spaces only a finite process is needed. I restrict myself to the linear case, the non-linear case might be treated in a similar way.

1. LINEAR HOMOGENEOUS SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

1.1. Involution. Let V and W be vector spaces of finite dimension; $\dim V = n$. Denote by $S^m V^*$ the symmetric tensor product of m copies of V^* , the symmetric product of $v_1^*, v_2^* \in V^*$ denote by $v_1^* \circ v_2^*$. For $v \in V$ introduce the homomorphisms

$$(1.1) \quad \delta_v : W \otimes S^{m+1} V^* \rightarrow W \otimes S^m V^*$$

satisfying

$$(1.2) \quad \delta_v(w \otimes f) = w \otimes f_v \quad \text{for } w \in W, f \in S^{m+1} V^* ;$$

f_v is the derivative of f with respect to v . If v_1, \dots, v_n is a basis of V and v^1, \dots, v^n the dual basis of V^* , define the homomorphism

$$(1.3) \quad \delta \equiv \delta_{m+1,j} : W \otimes S^{m+1} V^* \otimes \wedge^j V^* \rightarrow W \otimes S^m V^* \otimes \wedge^{j+1} V^*$$

by means of the relation

$$(1.4) \quad \delta(\xi \otimes v^{i_1} \wedge \dots \wedge v^{i_j}) = \sum_{i=1}^n \delta_{v_i} \xi \otimes v^i \wedge v^{i_1} \wedge \dots \wedge v^{i_j}$$

for $\xi \in W \otimes S^{m+1} V^*$. This definition does not depend on the choice of the basis.

Theorem 1.1. *We have*

$$(1.5) \quad \delta^2 = 0, \quad \text{i.e.,} \quad \delta_{m,j+1} \delta_{m+1,j} = 0$$

for each m and j . The sequence

$$(1.6) \quad 0 \rightarrow W \otimes S^m V^* \xrightarrow{\delta} W \otimes S^{m-1} V^* \otimes V^* \xrightarrow{\delta} W \otimes S^{m-2} V^* \otimes \Lambda^2 V^* \xrightarrow{\delta} \dots \\ \dots \xrightarrow{\delta} W \otimes S^{m-n} V^* \otimes \Lambda^m V^* \rightarrow 0$$

is exact; we set $S^l V^* = 0$ for $l < 0$.

For each $m \geq k$ be given a subspace $g^m \subset W \otimes S^m V^*$. The sequence $\{g^m\}$ is consistent if

$$(1.7) \quad \delta_v(g^m) \subset g^{m-1}$$

for each $m > k$ and $v \in V$. If the sequence $\{g^m\}$ is consistent we have

$$(1.8) \quad \delta(g^m \otimes \Lambda^j V^*) \subset g^{m-1} \otimes \Lambda^{j+1} V^*,$$

and we may consider the sequences

$$(1.9) \quad 0 \rightarrow g^m \xrightarrow{\delta} g^{m-1} \otimes V^* \xrightarrow{\delta} g^{m-2} \otimes \Lambda^2 V^* \xrightarrow{\delta} \dots \\ \dots \xrightarrow{\delta} g^k \otimes \Lambda^{m-k} V^* \xrightarrow{\delta} W \otimes S^{k-1} V^* \otimes \Lambda^{m-k+1} V^*$$

for $m \geq k$. Denote by $H^{m-j,j} \equiv H^{m-j,j}(g^k)$ the cohomology of the sequence (1.9) on the $(j+1)$ -th place:

$$(1.10) \quad H^{m-j,j} = \text{Ker } \delta_{m-j,j} / \text{Im } \delta_{m-j+1,j-1}.$$

This is the so-called *Spencer cohomology*. A consistent sequence $\{g^m\}$ is called *involutionary* if all the sequences (1.9) are exact. $\{g^m\}$ is called *r-acyclic* if

$$(1.11) \quad H^{m,j} = 0 \quad \text{for } m \geq k, \quad 0 \leq j \leq r.$$

Let $V_1 \subset V$ be a subspace. Define

$$(1.12) \quad (g^m)_{V_1} = \{\xi \in g^m \mid \delta_v \xi = 0 \text{ for all } v \in V_1\}.$$

The sequence $\{g^m\}$ being consistent, we have

$$(1.13) \quad \delta_v((g^m)_{V_1}) \subset (g^{m-1})_{V_1} \quad \text{for all } v \in V_1.$$

Let v_1, \dots, v_n be a basis of V ; denote by $\{v_r, v_{r+1}, \dots, v_n\}$ the space spanned by v_r, \dots, v_n . The basis v_1, \dots, v_n is *regular* with respect to $\{g^m\}$ if the mappings

$$(1.14) \quad \begin{aligned} \delta_{v_n} &: g^{m+1} \rightarrow g^m, \\ \delta_{v_{n-1}} &: (g^{m+1})_{(v_n)} \rightarrow (g^m)_{(v_n)}, \\ \delta_{v_{n-2}} &: (g^{m+1})_{(v_{n-1}, v_n)} \rightarrow (g^m)_{(v_{n-1}, v_n)}, \\ &\dots \dots \dots \\ \delta_{v_1} &: (g^{m+1})_{(v_2, \dots, v_n)} \rightarrow (g^m)_{(v_2, \dots, v_n)} \end{aligned}$$

are onto for all m 's.

Theorem 1.2. *The consistent sequence $\{g^m\}$ is involutive if and only if there is a regular basis with respect to it.*

Be given a space $g^k \subset W \otimes S^k V^*$. Its first *prolongation* is defined as

$$(1.15) \quad pg^k = (g^k \otimes V^*) \cap (W \otimes S^{k+1} V^*);$$

put $p^l g^k = p(p^{l-1} g^k)$ for $l > 1$. The sequence $\{g^m\}$ with $g^m = p^{m-k} g^k$ for $m \geq k$ is consistent. The space g^k is called *involutive (r-acyclic)* if the corresponding sequence $\{g^m\} = \{p^{m-k} g^k\}$ is involutive (r-acyclic).

Theorem 1.3. *Be given a space $g^k \subset W \otimes S^k V^*$. Let there exist a basis v_1, \dots, v_n of the space V such that the maps (1.14) are onto for $m = k$ and $g^{k+1} = pg^k$. Then g^k is involutive and v_1, \dots, v_n is regular with respect to $\{g^m\} = \{p^{m-k} g^k\}$.*

Theorem 1.4. *Let, for each $m \geq 1$, be given a space $g^m \subset W \otimes S^m V^*$. If $pg^m \supset g^{m+1}$ for each $m \geq 1$, there is a number m_0 such that*

$$(1.16) \quad pg^m = g^{m+1} \quad \text{for } m \geq m_0$$

and the space g^{m_0} is involutive.

1.2. Differential equations. Let X be a differentiable manifold of class C^∞ , $\dim X = n$. Denote by $T = T(X)$ its tangent bundle, T^* be its cotangent bundle. E being a vector bundle over X , E_x is the fibre over $x \in X$ and \mathcal{E} is the sheaf of germs of C^∞ sections of E .

Be given a C^∞ vector bundle E over X with the projection $\pi : E \rightarrow X$; let $x_0 \in X$ be a fixed point and $s_1, s_2 \in \mathcal{E}$ sections defined in a neighborhood of x_0 . Let $U \subset X$ be a coordinate neighborhood of x_0 such that s_1, s_2 are defined in it and we may write $\pi^{-1}(U) = U \times V^m$, V^m being an m -dimensional vector space. Choosing a basis v_1, \dots, v_m in V^m , the section s_τ ; $\tau = 1, 2$; is given (in U) by $y^\alpha = f_\tau^\alpha(x^1, \dots, x^n)$; $\alpha = 1, \dots, m$; x^1, \dots, x^n being the local coordinates in U . We say that s_1 and s_2 belong to the same k -jet at x_0 , and we write $j_{x_0}^k(s_1) = j_{x_0}^k(s_2)$, if all the partial derivatives at x_0 of the functions f_1^α up to the order k are equal to the corresponding derivatives of f_2^α .

Denote by $J^k(E)$ the set of all k -jets of sections of E . This set has a natural structure of a vector bundle over X . The bundle $J^0(E)$ is obviously equal to E . Let $\pi : J^k(E) \rightarrow J^l(E)$, $k > l$, be the natural projection. Denote by $\mathcal{J}^k(E)$ the sheaf of the germs of the sections of $J^k(E)$. The mapping $j^k : \mathcal{E} \rightarrow \mathcal{J}^k(E)$ be defined as follows: $s \in \mathcal{E}$ being defined on $U \subset X$, set $[j^k(s)](x) = j_x^k(s)$ for $x \in U$.

There is just one bundle morphism $\varepsilon : S^k T^* \otimes E \rightarrow J^k(E)$ characterized by the following property. Let $x \in X$; $t_1, \dots, t_k \in T_x^*$; $e \in E_x$. Choose functions f_1, \dots, f_k on a neighborhood of x such that $(df_\sigma)_x = t_\sigma$ and $f_\sigma(x) = 0$ for $\sigma = 1, \dots, k$; further, choose $s \in \mathcal{E}$ such that $s(x) = e$. Now, we have

$$\varepsilon(t_1 \circ \dots \circ t_k \otimes e) = j_x^k(f_1 \dots f_k s).$$

Theorem 1.5. *The sequence*

$$(1.17) \quad 0 \rightarrow S^k T^* \otimes E \xrightarrow{\varepsilon} J^k(E) \xrightarrow{\pi} J^{k-1}(E) \rightarrow 0$$

is exact.

Let E, F, G be vector bundles over X ; $f: E \rightarrow F$ be a bundle morphism and $\bar{f}: \mathcal{E} \rightarrow \mathcal{F}$ the corresponding sheaf morphism.

Theorem 1.6. *To a given bundle morphism $f: E \rightarrow F$ there is a unique bundle morphism $j^l(f): J^l(E) \rightarrow J^l(F)$ such that the diagram*

$$(1.18) \quad \begin{array}{ccc} \mathcal{E} & \xrightarrow{\bar{f}} & \mathcal{F} \\ j^l \downarrow & & \downarrow j^l \\ \mathcal{J}^l(E) & \xrightarrow{j^l(f)} & \mathcal{J}^l(F) \end{array}$$

is commutative.

Theorem 1.7. *There is a unique bundle monomorphism*

$$p^l(\text{id}^k): J^{k+l}(E) \rightarrow J^l(J^k(E))$$

such that the diagram

$$(1.19) \quad \begin{array}{ccc} \mathcal{E} & \xrightarrow{j^k} & \mathcal{J}^k(E) \\ j^{k+l} \downarrow & & \downarrow j^l \\ \mathcal{J}^{k+l}(E) & \xrightarrow{p^l(\text{id}^k)} & \mathcal{J}^l(J^k(E)) \end{array}$$

is commutative.

Considering (1.19) and (1.18) with $G = J^k(E)$, we get the following corollary: Given a bundle morphism $f: J^k(E) \rightarrow F$ there is a unique bundle morphism $p^l(f): J^{k+l}(E) \rightarrow J^l(F)$ such that the diagram

$$(1.20) \quad \begin{array}{ccc} \mathcal{E} & \xrightarrow{\bar{f}j^k} & \mathcal{F} \\ j^{k+l} \downarrow & & \downarrow j^l \\ \mathcal{J}^{k+l}(E) & \xrightarrow{p^l(f)} & \mathcal{J}^l(F) \end{array}$$

is commutative; here, $p^l(f) = j^l(f) p^l(\text{id}^k)$.

Considering $F = J^k(E)$ and $f = \text{id}$, we get $p^l(\text{id}) = p^l(\text{id}^k)$, thus explaining why we have denoted by $p^l(\text{id}^k)$ the morphism of Theorem 1.7. The morphism $p^l(f)$ is the l -th prolongation of $f: J^k(E) \rightarrow F$.

The differential equation (of order k) is a C^∞ sub-bundle $R^k \subset J^k(E)$. The l -th ($l \geq 0$) prolongation of R^k is the subset

$$(1.21) \quad p^l R^k = R^{k+l} = J^l(R^k) \cap J^{k+l}(E).$$

Obviously, $J^l(R^k) \subset J^l(J^k(E))$ and $J^{k+l}(E)$ is identified with its image in the monomorphism $p^l(\text{id}^k)$ of Theorem 1.7. Define $R^{k-l} = J^{k-l}(E)$ for $1 \leq l \leq k$. R^{k+l} need not to be a sub-bundle of $J^{k+l}(E)$ as $\dim R_x^{k+l}$ is not constant in general.

Theorem 1.8. *To each differential equation R^k on E there is a bundle F and a bundle morphism $f: J^k(E) \rightarrow F$ such that $R^k = \text{Ker } f$. Then $R^{k+l} = \text{Ker } p^l(f)$.*

1.3. The formal complete integrability. The equation $R^k \subset J^k(E)$ is *formally completely integrable* if, for each $l \geq 0$, R^{k+l} is a vector bundle and

$$(1.22) \quad R^{k+l+1} \rightarrow R^{k+l} \rightarrow 0$$

is exact.

One of the main results is to obtain criteria for the formal complete integrability of a given differential equation.

Let $R^k \subset J^k(E)$ be an equation. For each $l \geq 0$, define $g^{k+l} \subset S^{k+l}T^* \otimes E$ by the exact sequence

$$(1.23) \quad 0 \rightarrow g^{k+l} \xrightarrow{\epsilon} R^{k+l} \xrightarrow{\pi} R^{k+l+1};$$

further, set $g^{k-l} = S^{k-l}T^* \otimes E$ for $1 \leq l \leq k$. g^k is the symbol of R^k . In general, g^{k+l} is not a bundle.

Theorem 1.9. *Let R^k be an equation of order k and g^{k+l} the corresponding system of spaces. Then $g_x^{k+l} = p^l g_x^k$ for each $l \geq 0$ and $x \in X$, and the sequence*

$$(1.24) \quad 0 \rightarrow g^{k+l+1} \xrightarrow{\delta} T^* \otimes g^{k+l} \xrightarrow{\delta} \wedge^2 T^* \otimes g^{k+l-1}$$

is exact for $l \geq 0$; i.e., $\{g^{k+l}\}$ is 1-acyclic.

Theorem 1.10. *The manifold X being connected, there is an integer $k_0 > k$ (depending only on $k, n = \dim X$ and $\dim E$) such that $H^{k_0+m,j} = 0$ for each $m \geq 0, j \geq 0$.*

Theorem 1.11. *If R^{k+l} is a vector bundle (for an $l \geq 0$) and $\pi: R^{k+l} \rightarrow R^{k+l-1}$ is an epimorphism, there is a bundle morphism (the so-called curvature of R^{k+l})*

$$(1.25) \quad \kappa = \kappa(R^{k+l}): R^{k+l} \rightarrow H^{k+l-1,2}$$

such that the sequence

$$(1.26) \quad R^{k+l+1} \xrightarrow{\pi} R^{k+l} \xrightarrow{\alpha} H^{k+l-1,2}$$

is exact.

Theorem 1.12. *If g^k and g^{k+1} are vector bundles and g^k is 2-acyclic, g^{k+1} is a vector bundle for any $l \geq 0$.*

Combining the preceding two theorems, we get

Theorem 1.13. *Let $R^k \subset J^k(E)$ be a differential equation and suppose: (1) R^{k+1} is a vector bundle, (2) $\pi : R^{k+1} \rightarrow R^k$ is an epimorphism, (3) g^k is a 2-acyclic system, (4) g^k and/or g^{k+2} is a vector bundle. Then R^k is formally completely integrable.*

Finally, combining this theorem with Theorem 1.10, we get

Theorem 1.14. *If X is connected and $R^k \subset J^k(E)$ a differential equation, there is a number $k_0 > k$ (depending only on n, k and $\dim E$) such that if R^{k+l+1} is a vector bundle and $\pi : R^{k+l+1} \rightarrow R^{k+l}$ is an epimorphism for $0 \leq l \leq k_0 - k$, then R^k is formally completely integrable.*

A (local) section $s \in \mathcal{E}$ is called a *solution* of R^k if $j^k s \in \mathcal{J}^k(E)$ is contained in R^k .

Theorem 1.15. *Let X be an analytic manifold, E an analytic vector bundle and $R^k \subset J^k(E)$ an analytic sub-bundle. Let R^k be a formally completely integrable equation. Be given a point $u \in R^{k+1}$, let $\pi(u) = x \in X$. Then there is a neighborhood $U \subset X$ of x and an analytic section $s : U \rightarrow E$ which is a solution of R^k and $j_x^{k+1}(s) = u$.*

2. EQUIVALENCE OF SUBMANIFOLDS

2.1. Initial conditions of differential equations. Let X be a C^∞ differentiable manifold and E a vector bundle over it. Suppose that all the manifolds and maps considered are of the class C^∞ . Let X_1 be a submanifold of X . Let E_1 be the restriction of E to X_1 , i.e., $E_1 = E | X_1$. Define the bundle morphisms

$$(2.1) \quad \eta^k : J^k(E) | X_1 \rightarrow J^k(E_1)$$

as follows. Let $u \in J^k(E) | X_1$, i.e., $x = \pi(u) \in X_1$. Then there is a neighborhood $U \subset X$ of x and a section $s : U \rightarrow E$ such that $u = j_x^k(s)$. We set $\eta^k(u) = j_x^k(s | X_1 \cap U)$; $\eta^k(u)$ obviously does not depend on s .

Be given a formally completely integrable equation $R^k \subset J^k(E)$. For each $m \geq k$ define a system of spaces $S_{(m)}^m \subset J^m(E_1)$ as

$$(2.2) \quad S_{(m)}^m = \eta^m(R^m | X_1);$$

of course, R^m is the $(m - k)$ -th prolongation of R^k . Although $S_{(m)}^m$ is not always a differential equation, we may define its prolongation $S_{(m)}^{m+p}$ in a natural way. The diagram

$$(2.3) \quad \begin{array}{ccc} J^{m+1}(E) | X_1 & \xrightarrow{\eta^{m+1}} & J^{m+1}(E_1) \\ \pi \downarrow & & \downarrow \pi \\ J^m(E) | X_1 & \xrightarrow{\eta^m} & J^m(E_1) \end{array}$$

being commutative, we get

$$(2.4) \quad \pi(S_{(m+1)}^{m+1}) = S_{(m)}^m$$

for each $m \geq k$ because of the formal complete integrability of R^k . Indeed, let $u \in S_{(m)}^m$. Then there exists a $v \in R^m$ such that $\eta^m(v) = u$. Because of the complete integrability there is a $w \in R^{m+1}$ such that $\pi(w) = v$, from the commutativity of the diagram (2.3) it follows $\pi(\eta^{m+1}(w)) = u$, and the mapping $\pi : S_{(m+1)}^{m+1} \rightarrow S_{(m)}^m$ is onto. Further, to each $t \in S_{(m+1)}^{m+1}$ there is a $t_1 \in R^{m+1}$ such that $\eta^{m+1}(t_1) = t$, hence $\pi(t) = \eta^m(\pi(t_1)) \in S_{(m)}^m$ because of $\pi(t_1) \in R^m$. Thus we get (2.4). It is easy to see that

$$(2.5) \quad S_{(m+1)}^{m+1} \subset S_{(m)}^{m+1} = pS_{(m)}^m$$

for each $m \geq k$. The system of subspaces $g_{(m)}^{m+p+1}$ be defined as in (1.23) by the exact sequence

$$(2.6) \quad 0 \rightarrow g_{(m)}^{m+p+1} \xrightarrow{\varepsilon} S_{(m)}^{m+p+1} \xrightarrow{\pi} S_{(m)}^{m+p}.$$

Let us choose a fixed point $y \in X$. For each $m > k$ consider the space

$$(2.7) \quad h^m = g_{(m)}^m(y);$$

from (2.5) we get $h^{m+1} \subset ph^m$ for each $m \geq k$. Applying Theorem 1.4, we get the existence of an integer $k_0 \geq k$ such that

$$(2.8) \quad h^{m+1} = ph^m \quad \text{for each } m \geq k_0.$$

Thus we obtain

Theorem 2.1. *Let $R^k \subset J^k(E)$ be a formally completely integrable equation. Let $X_1 \subset X$ be a submanifold, $y \in X_1$ a fixed point and $S_{(m)}^m$ be defined by (2.2). Then there is a $k_0 \geq k$ such that*

$$(2.9) \quad S_{(m+1)}^{m+1}(y) = S_{(m)}^{m+1}(y) \quad \text{for each } m \geq k_0.$$

The formal solution of the equation R^k at the point $y \in X$ is a sequence $\{s^q\}$ with

$s^q \in R^q(y)$ and $\pi(s^{q+1}) = s^q$ for $q = 1, 2, \dots$. The formal initial condition at $y \in X_1 \subset X$ with respect to the submanifold X_1 is a sequence $\{r^q\}$ with $r^q \in J^q(E_1)_y$ and $\pi(r^{q+1}) = r^q$ for $q = 1, 2, \dots$. The formal solution $\{s^q\}$ goes through the formal initial condition $\{r^q\}$ if $\eta^q(s^q) = r^q$ for each q . The formal initial condition $\{r^q\}$ is m -admissible if $r^{m+p} \in S_{(m)}^{m+p}(y)$ for each $p \geq 0$.

It is easy to prove the following

Theorem 2.2. *Be given the situation described in Theorem 2.1. There is a $k_0 \geq k$ such that through each m -admissible, $m \geq k_0$, formal initial condition at y with respect to X_1 there goes a formal solution of R^k .*

2.2. Equivalence of submanifolds. Let X be a vector space. Consider the trivial vector bundle $E = X \times X$ over it with the projection $\pi = \text{pr}_1 : X \times X \rightarrow X$, pr_1 being the projection on the first factor. Be given a formally completely integrable equation $R^k \subset J^k(E)$ and a submanifold $X_1 \subset X$. Let $s \in \mathcal{E}$ be a local section of E over $U \subset X$, i.e., a mapping $s(x) = (x, \hat{s}(x))$ for $x \in U$ determining thus the mapping $\hat{s} : U \rightarrow X$. Denote by \mathcal{S} the set of all local mappings $\hat{s} : U \rightarrow X$ such that the corresponding section $s(x) = (x, \hat{s}(x))$, $x \in U$, is a solution of R^k . Further, be given a mapping $\tilde{f} : X_1 \rightarrow X$. \tilde{f} is called the deformation of order m of X_1 with respect to \mathcal{S} if, for each $x_1 \in X_1$, there is a $\hat{s}_{x_1} \in \mathcal{S}$ (defined in the neighborhood of x_1) such that $j_{x_1}^m(\tilde{f}) = j_{x_1}^m(\hat{s}_{x_1} | X_1)$. The mapping $\tilde{f} : X_1 \rightarrow X$ determines the section $f : X_1 \rightarrow E_1 \equiv E | X_1$ given by $f(x_1) = (x_1, \tilde{f}(x_1))$. It is easy to see that f is the deformation of order m if and only if \tilde{f} is a solution of the (generalized) equation $S_{(m)}^m$ constructed above.

The mapping \tilde{f} is called the formal equivalence at $y \in X_1$ with respect to \mathcal{S} if there is a sequence of local maps $\{\hat{s}^q : U^q \rightarrow X\}$ of the neighbourhoods U^q of y such that, for $q = 1, 2, \dots$, $j_y^q(\hat{s}^{q+1}) = j_y^q(\hat{s}^q)$, $\hat{s}^q \in \mathcal{S}$, $j_y^q(\tilde{f}) = j_y^q(\hat{s}^q | X_1)$. We get from Theorem 2.2

Theorem 2.3. *Be given a vector space X and a set \mathcal{S} of local maps on it which are the solutions of an equation R^k (in the way described above). Be given a submanifold $X_1 \subset X$ and $y \in X_1$. Then there is an integer $k_0 \geq k$ with the following property: $\tilde{f} : X_1 \rightarrow X$ being the deformation of order $m \geq k_0$, \tilde{f} is the formal equivalence at y .*

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