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SUMMARIES OF ARTICLES PUBLISHED IN THIS ISSUE

(Publication of these summaries is permitted)

S. MRÓWKA, Kalamazoo: *Characterization of classes of functions by Lebesgue sets*. Czech. Math. J. *19 (94)*, (1969), 738—744. (Original paper.)

The Lebesgue sets of a function f defined on a set X are the sets $L_a(f) = X \cap \{p; f(p) \leqq a\}$, $L^a(f) = X \cap \{p; f(p) \geqq a\}$, $a \in R^1$. If $\mathfrak{F}$ is a class of functions on X (which is always supposed to contain constants) then $\Lambda(\mathfrak{F})$ means the system of all $L^a(f)$, $L_a(f)$ for $f \in \mathfrak{F}$ and $a \in R^1$. The paper deals with the relations between $\Lambda(\mathfrak{F})$ and $\mathfrak{F}$ for some $\mathfrak{F}$.

KAREL KARTÁK, JAN MAŘÍK, Praha: *On representations of some Perron integrable functions*. Czech. Math. J. *19 (94)*, (1969), 745—749. (Original paper.)

Some theorems are given concerning the possibility of representation of a Perron integrable function f in the form $f = l + n$, where l is Lebesgue integrable and n has a primitive function in some sense.

STANISLAV TOMÁŠEK, Liberec: *On a certain class of Λ-structures*, I. Czech. Math. J. *20 (95)*, (1970), 1—18. (Original paper.)

The aim of this paper is a systematic discussion of Λ-structures $(E(X), \lambda)$ over a separated uniform space X where λ stands for appropriate locally convex topologies on $E(X)$ studied by D. A. Raikov. The concept of a Λ-structure is used here in the same sense as it had been introduced by M. Katětov. Since any uniform space X may be isomorphically embedded in $(E(X), \lambda)$, such Λ-structure represents a topological overstructure on X and therefore suggests a new approach to certain questions of classical analysis and of general topology. As an illustration of this idea some equivalent theorems of Ascoli's type have been established. A dual characterization of the completion of a uniform space by extremal points presents a further application in this line. At last, it is shown that the classical Banach-Stone theorem is a special case of an assertion on Λ-structures.

STANISLAV TOMÁŠEK, Liberec: *On a certain class of Λ-structures*, II. Czech. Math. J. *20 (95)*, (1970), 19—33. (Original paper.)

In this article completions of Λ-structures are characterized under topologies generated by the spaces of uniformly continuous functions on a uniform space. A dual characterization of pseudocompact spaces representing a solution of a problem of V. Pták and elementary proofs of the main theorems on compactness in locally convex spaces are stated as straightforward consequences of the preceding results. Finally $\mathscr{DB}$ and $\mathscr{DM}$-spaces are introduced generalizing the Λ-structures of the considered kind as well as the normed and the $\mathscr{DF}$-spaces of A. Grothendieck respectively. The following diverse modifications of extension theorems for separately weakly compact bilinear functions in the last terms include the corresponding results of A. Grothendieck and the Extension Theorem of V. Pták.

R. M. GOEL, Patiala: *On a class of functions with starshaped images.* Czech. Math. J. *20 (95)*, (1970), 34—38. (Original paper.)

Wu Zwao-jen proved the following theorem: Let S be the class of functions $f(z) = z + \sum_2^\infty b_k z^k$. If $f(z) \in S$ then any section $f_n(z) = z + b_2 z^2 + \ldots + b_n z^n$ $(n \geqq 2)$ of $f(z)$ is starshaped in $|z| < \frac{1}{2}$ for $n \neq 3, 4, 5$. In this paper it is proved that this theorem holds for $n = 4,5$.

MILAN TVRDÝ, Praha: *The normal form and the stability of solutions of a system of differential equations in the complex domain.* Czech. Math. J. *20 (95)*, (1970), 39—73. (Original paper.)

In the paper the problem of the stability of the trivial solution of a complex system of autonomous d fferential equations with holomorphic right-hand sides in some critical cases is dealt with. The case of several purely imaginary roots (of the characteristic equation of the given system) is solved in Sec. 4,2 under some additional assumptions and the case of several zero roots is solved in Sec. 4,3 completely. To prove these theorems, the concept of the pseudonormal form of the given system has been introduced and a general assertion concerning the transformation of the given system to its pseudonormal form has been proved.

LADISLAV BICAN, Praha: *On splitting mixed abelian groups.* Czech. Math. J. *20 (95)*, (1970), 74—80. (Original paper.)

Let H be a splitting mixed group and G an extension or a subgroup of H. Sufficient conditions for splittingness of G are given. Further, let H and G have the same meaning as above, let G_t denote the maximal torsion subgroup of G. Sufficient conditions for splittingness of G and for the isomorphism $G/G_t \cong H/H_t$ are given.

BOHDAN ZELINKA, Liberec: *Centrally symmetric Hasse diagrams of finite modular lattices.* Czech. Math. J. *20 (95)*, (1970), 81—83. (Original paper.)

A centrally symmetric graph is a non-directed graph such that for any of its vertices there exists exactly one vertex with the maximal distance from it. In the paper the theorem is proved that any finite modular lattice whose Hasse diagram is centrally symmetric is a Boolean algebra and its Hasse diagram is a graph of an n-dimensional cube for some n.

STANISLAV JÍLOVEC, Praha: *On the consistency of estimates.* Czech. Math. J. *20 (95)*, (1970), 84—92. (Original paper.)

The relations between four types of consistent estimates are studied. Especially, it is proved that the existence of a (wide sense) consistent estimate implies the existence of a (wide sense) superconsistent estimate. It is also shown that the existence of a wide sense (super)consistent estimate does not guarantee the existence of a (super)consistent estimate.

John W. Heidel, Knoxville: *The existence of oscillatory solutions for a nonlinear odd order differential equation.* Czech. Math. J. *20 (95)*, (1970), 93—97. (Original paper.)

Sufficient conditions are found for the equation $y^{(n)} + q(t)\,y^{\gamma} = 0$ (n, p, r, being odd positive integer, $n \geqq 3$, $\gamma = p/r$) to have oscillatory solutions.

Zdeněk Frolík, Praha: *A separation theorem and applications to Borel sets.* Czech. Math. J. *20 (95)*, (1970), 98—108. (Original paper.)

The aim of this paper is to make a further little step in clarifying possible limits of separable Descriptive Theory in the class of all completely regular spaces. In this paper B-spaces are introduced and studied. These spaces form a generalization of classical Borel spaces. In section 1 the definitions and basic properties of previous generalizations are given, and two corollaries of the theory of B-spaces to classical theory are described. In section 2 two general separation lemmas are proved, and applied to a separation theorem for countable collection of analytic sets. In section 3 B-spaces are introduced and studied.

Václav Chvátal, Waterloo: *A remark on a problem of Harary.* Czech. Math. J. *20 (95)*, (1970), 109—111. (Original paper.)

The paper contains some partial results concerning a problem of Theory of graphs posed by Harary on the Conference at Smolenice 1963.

Vlastimil Pták, Praha: *Extension of sequentially continuous functionals in inductive limits of Banach spaces.* Czech. Math. J. *20 (95)*, (1970), 112—121. (Original paper.)

The present paper is devoted to the open mapping theorem in spaces of distributions. In a recent paper, we have introduced the notion of orthogonality for subspaces of inductive limits of sequences of Fréchet spaces which is very useful in formulating sufficient conditions for openness of linear mappings. In a forthcoming note we intend to discuss important situation in which the notion occurs quite naturally. In order to obtain conditions which are both necessary and sufficient we shall need a slightly weaker notion which we propose to call semiorthogonality. In the present paper we discuss this notion in the case of an inductive limit of a sequence of Banach spaces.

Jan Kučera, Praha: *On the accessibility of control system $\dot{x} \in Q(x)$.* Czech. Math. J. *20 (95)*, (1970), 122—129. (Original paper.)

In this paper we present an algebraic condition under which the set of all points which are accessible from a fixed point ω at a constant time along solutions of a given system $\dot{x} \in Q(x)$, $x(0) = \omega$ is a closed manifold whose dimension depends only on algebraic properties of ω. At the same time we present an explicit formula for this manifold.

Josef Vala, Brno: *Über einige spezielle Kongruenzenpaare.* Czech. Math. J. *20 (95)*, (1970), 140—148. (Original paper.)

In the three dimensional projective space P_3 let a couple P of line congruences $\Gamma_1(p_1)$, $\Gamma_2(p_2)$ and the correspondence C: $p_1 \to p_2$ be given. The corresponding lines p_1, p_2 let be skew. The properties of some special couples of congruences Γ_1, Γ_2 are found. The couple P is studied especially in the case when C is a projective deformation.

Bohuslav Diviš, Praha: *Über Gitterpunkte in mehrdimensionalen Ellipsoiden, I, II.* Czech. Math. J. *20 (95)*, (1970), 130—139, 149—159. (Originalartikel.)

Es sei $\sigma \geqq 2$ ganz; weiter seien $\alpha_j > 0$, $r_j \geqq 1$, r_j ganz ($j = 1, 2, \ldots, \sigma$), $r = r_1 + r_2 + \ldots + r_\sigma$). Es sei $Q(u)$ die quadratische Form der Gestalt $Q(u) = \sum_{j=1}^{\sigma} \alpha_j(u_{1,j}^2 + u_{2,j}^2 + \ldots + u_{r_j,j}^2)$. Bezeichnen wir mit $A_Q(x)$ die Anzahl der Gitterpunkte im abgeschlossenen Ellipsoid $Q(u) \leqq x$. Mit $V_Q(x)$ bezeichnen wir den Inhalt dieses Ellipsoides und setzen $P_Q(x) = A_Q(x) - V_Q(x)$. Für jede gegebene Form Q bedeutete $f(Q)$ die untere Grenze derjenigen ω, für welche $P_Q(x) = O(x^\omega)$ ist. Mit $\beta = \beta(\alpha_2|\alpha_1, \alpha_3|\alpha_1, \ldots, \alpha_\sigma|\alpha_1)$ bezeichnen wir die obere Grenze derjenigen ω, für welche das System der Ungleichungen $|\alpha_j/\alpha_1 - p_j/q| < 1/q^\omega$ ($j = 2, 3, \ldots, \sigma$) unendlich viele Lösungen mit ganzzahligen σ-Tupeln $\{p_{2n}, p_{3n}, \ldots, p_{\sigma,n}; q_n\}_{n=1}^{\infty}$, $q_n \to \infty$ hat. In der ersten Arbeit beweisen wir hauptsächlich folgendes: Es seien $r_j \geqq 2\beta(\beta - 1)$, $j = 1, 2, \ldots, \sigma$. Dann ist $f(Q) = (r/2) - 1 - 1/(\beta - 1)$. Für $\beta = +\infty$ setzen wir $1/(\beta - 1) = 0$, $2\beta/(\beta - 1) = 2$. In der zweiten Arbeit beweisen wir folgendes: Es sei $3 \leqq \tau \leqq \sigma$, τ ganz, $\beta = \beta(\alpha_2/\alpha_1, \ldots, \alpha_{\tau-1}/\alpha_1)$, $r_j \geqq 2\beta(\beta - 1)$ ($j = 1, 2, \ldots, \tau - 1$), $r_j \geqq 4$ ($j = \tau, \ldots, \sigma$). Dann gilt für fast alle positive Wertsysteme $(\alpha_\tau, \ldots, \alpha_\sigma)$: $f(Q) \leqq (r/2) - 1 - 1/(\beta - 1) - (\sigma + 1 - \tau)$.

Jan Kučera, Praha: *On accessibility of bilinear systems.* Czech. Math. J. *20 (95)*, (1970), 160—168. (Original paper.)

In this paper the author presents an explicit formula for solutions of a bilinear system (*) $\dot{x} = (\sum_{i=1}^{\alpha} A_i u_i) x + \sum_{j=1}^{\beta} b_j v_j$ where A_i, $i = 1, 2, \ldots, \alpha$ are n-by-n matrices and b_j, $j = 1, 2, \ldots, \beta$, are vectors (both independent on time), and $w = (u, v) = (u_1, \ldots, u_\alpha, v_1, \ldots, v_\beta)$ ranges the set W of all vector-functions which are measurable on $[0, \infty)$ and have values in the interval $[-1, 1]^{\alpha+\beta} \subset E_{\alpha+\beta}$. Further, the author constructs an involutive distribution V on E_n and shows that the set of all points accessible along solutions of (*) which fulfil the initial condition $x(0) = \omega$, is just the maximal integral manifold of V which passes through ω.