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ON A THEOREM ON THE CHANGE OF BASE POINT IN A PATH CONNECTED SPACE

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In [1], PUPPE has proved by elementary means the following

Theorem 1. *If $A \subset X$, $B \subset Y$ are cofibrations, where A and B are closed subsets of X, Y respectively, then also $(X \times B) \cup (A \times Y) \subset X \times Y$ is a cofibration.*

There is no doubt that this general theorem will soon be part of any introduction to homotopy theory. The object of this note is to remark that it leads to an immediate proof of the following basic.

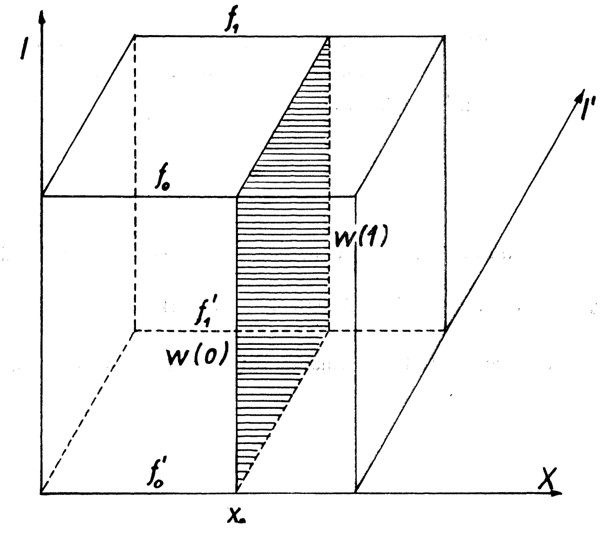

Fig. 1.

Theorem 2. (cf. [2] pp. 380–382). *Let X be a space with a non-degenerate base point x_0, and assume that $\{x_0\}$ is closed in X. Let w be a path in the space Y. Then there is a bijection depending only on $[w]$, homotopy class of w*

$$h_{[w]} : [X, x_0; Y, w(1)] \to [X, x_0; Y, w(0)]\,,$$

where a representative of $h_{[w]}[f_1]$ is obtained by deforming f_1 so that the image of x_0 moves along w from $w(1)$ to $w(0)$.

$[X, x_0; Y, w(i)]$ is the set of homotopy classes of maps $(X, x_0) \to (Y, w(i))$.

Proof. Recall (i) x_0 non-degenerate means $\{x_0\} \subset X$ is a cofibration, (ii) $\dot{I} \subset I$ is a cofibration (obvious) and $\dot{I}$ is closed in I.

Apply Theorem 1: $(x_0 \times I) \cup (X \times \dot{I}) \subset X \times I$ is a cofibration, i.e. each map $\{(x_0 \times I) \cup (X \times \dot{I})\} \times I' \to Y$ can be extended to a map $(X \times I) \times I' \to Y$ with a given map $(X \times I) \times 1 \to Y$ coinciding with the first map where defined; or (cf. fig.) a map to Y from the bottom – top – back faces and the section through x_0 can be extended to the whole cube.

Then the proof follows quickly:

A homotopy $f_1 \simeq f_1'$ in $(X, x_0; Y, w(1))$ is a map to Y of the back face, a map f_0 corresponding to f_1 is obtained by filling in the top face, and similarly for the bottom face; a homotopy of a path in Y rel $(w(0), w(1))$ is a map to Y of the section through x_0. But then the Theorem of Puppe says that there is map from the front face to Y, i.e. $f_0 \simeq f_0'$ rel x_0. So $h_{[w]}$ is well-defined. Interchanging the role of the indices 0 and 1, shows that its inverse is also well-defined.

Remark. The condition that $\{x_0\}$ is closed in X is necessary since Theorem 1 is false whithout it as a simple counter-example in [1] shows. This is not a severe restriction since already all points of a T_1-space are submitted to it.

References

[1] *D. Puppe,* Bemerkungen über die Erweiterung von Homotopien, Arch. der Math. (1967), 81–88.

[2] *E. Spanier,* Algebraic Topology, McGraw-Hill, New York, 1966.

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