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ON A GROUP OF HOLOMORPHIC TRANSFORMATIONS IN \mathcal{C}^2

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0. Consider the space \mathcal{C}^2 with the complex coordinates (x, y) and let V be the layer of real hypersurfaces given by

$$(0.1) \quad i(y - \bar{y}) + (x - \bar{x})^2 = r, \quad r \in \mathcal{R}.$$

Each hypersurface of V has a non-degenerate Levi form at each its point. The Lie group

$$(0.2) \quad X = \alpha x + \beta, \quad Y = 2i\alpha(\beta - \bar{\beta})x + i\alpha(\alpha - \bar{\alpha})x^2 + \alpha\bar{\alpha}y + \gamma;$$

$$\alpha, \beta, \gamma \in \mathcal{C};$$

of the biholomorphic mappings of \mathcal{C}^2 preserves V , the hypersurface (0.1) with the parameter r being transformed into the hypersurface (0.1) with the parameter

$$(0.3) \quad r' = \frac{1}{\alpha\bar{\alpha}} \{r + i(\bar{y} - \gamma) - (\beta - \bar{\beta})^2\}.$$

Obviously, $\dim_{\mathcal{R}} G = 6$. We are going to prove the following

Theorem. *Let V be a layer of real hypersurfaces in \mathcal{C}^2 such that each hypersurface of V has a non-degenerate Levi form. Let G be a Lie group of biholomorphic transformations of \mathcal{C}^2 which is transitive on \mathcal{C}^2 and preserves the layer V . Then $4 \leq \dim G \leq 6$. In the case $\dim G = 6$ there are, in \mathcal{C}^2 , holomorphic coordinates (x, y) such that G is given by (0.2) and V by (0.1).*

1. Be given a differentiable manifold M^{2n} and an almost complex structure J over it; all manifolds and maps are supposed to be of class C^∞ . The torsion of J is defined as the vector 2-form $[J, J]$ given by

$$(1.1) \quad \frac{1}{2}[J, J](u, v) = [Ju, Jv] - J[Ju, v] - J[u, Jv] - [u, v].$$

On M^{2n} , let us choose vector fields $v_\alpha, v_{n+\alpha}$; $\alpha = 1, \dots, n$; such that

$$(1.2) \quad Jv_\alpha = v_{n+\alpha}, \quad Jv_{n+\alpha} = -v_\alpha; \quad \alpha = 1, \dots, n;$$

and write

$$(1.3) \quad \begin{aligned} [v_\alpha, v_\beta] &= a_{\alpha\beta}^\gamma v_\gamma + a_{\alpha\beta}^{n+\gamma} v_{n+\gamma}, \\ [v_\alpha, v_{n+\beta}] &= a_{\alpha, n+\beta}^\gamma v_\gamma + a_{\alpha, n+\beta}^{n+\gamma} v_{n+\gamma}, \\ [v_{n+\alpha}, v_{n+\beta}] &= a_{n+\alpha, n+\beta}^\gamma v_\gamma + a_{n+\alpha, n+\beta}^{n+\gamma} v_{n+\gamma}. \end{aligned}$$

For

$$(1.4) \quad u = x^\alpha v_\alpha - x^{n+\alpha} v_{n+\alpha}, \quad v = y^\alpha v_\alpha - y^{n+\alpha} v_{n+\alpha},$$

we obtain

$$(1.5) \quad \begin{aligned} \frac{1}{2}[J, J](u, v) &= \\ &= (a_{n+\alpha, n+\beta}^\gamma - a_{\alpha\beta}^\gamma + a_{\alpha, n+\beta}^{n+\gamma} - a_{\beta, n+\alpha}^{n+\gamma}) \cdot \\ &\cdot \{(x^\alpha y^\beta - x^{n+\alpha} y^{n+\beta}) v_\gamma + (x^\alpha y^{n+\beta} + x^{n+\alpha} y^\beta) v_{n+\gamma}\} + \\ &+ (a_{\alpha\beta}^{n+\gamma} - a_{n+\alpha, n+\beta}^{n+\gamma} + a_{\alpha, n+\beta}^\gamma - a_{\beta, n+\alpha}^\gamma) \cdot \\ &\cdot \{(x^\alpha y^{n+\beta} + x^{n+\alpha} y^\beta) v_\gamma - (x^\alpha y^\beta - x^{n+\alpha} y^{n+\beta}) v_{n+\gamma}\}. \end{aligned}$$

The condition $[J, J] = 0$ is thus equivalent to

$$(1.6) \quad \begin{aligned} a_{n+\alpha, n+\beta}^\gamma - a_{\alpha\beta}^\gamma + a_{\alpha, n+\beta}^{n+\gamma} - a_{\beta, n+\alpha}^{n+\gamma} &= 0, \\ a_{\alpha\beta}^{n+\gamma} - a_{n+\alpha, n+\beta}^{n+\gamma} + a_{\alpha, n+\beta}^\gamma - a_{\beta, n+\alpha}^\gamma &= 0; \quad \alpha, \beta, \gamma = 1, \dots, n. \end{aligned}$$

The following result is classic: Be given a manifold M^{2n} , the almost complex structure J over M^{2n} be given by means of the vector fields $v_\alpha, v_{n+\alpha}$ and (1.2); the structure J is complex if and only if (1.6).

2. Consider a manifold M^4 , a complex structure J over M^4 , and let V be a layer of hypersurfaces in M^4 . At each point $m \in M^4$, let us choose vectors $v_1, \dots, v_4 \in T_m(M^4)$ such that: (i) v_1, v_2, v_3 are tangent to the hypersurface of V going through m , (ii) $Jv_1 = v_3, Jv_2 = v_4$. (iii) the vector fields v_1, \dots, v_4 are of class C^∞ . The vector fields w_1, \dots, w_4 satisfying (i)–(iii) as well, there are real-valued functions $\alpha, \beta, \gamma, \varphi, \delta$ on M^4 such that

$$(2.1) \quad \begin{aligned} v_1 &= \alpha w_1 - \beta w_3, \quad v_2 = \gamma w_1 + \varphi w_2 - \delta w_3, \\ v_3 &= \beta w_1 + \alpha w_3, \quad v_4 = \delta w_1 + \gamma w_3 + \varphi w_4; \quad (\alpha^2 + \beta^2) \varphi \neq 0. \end{aligned}$$

from $B_2 = b_2 = 1$, we get

$$(2.9) \quad \varphi = \alpha^2 + \beta^2.$$

Further,

$$\begin{aligned} [v_1, v_4] &= [\alpha w_1 - \beta w_3, \delta w_1 + \gamma w_3 + \varphi w_4] = \\ &= (\cdot)w_1 + (\cdot)w_3 + (\cdot)w_4 + (\alpha\gamma + \beta\delta + \alpha\varphi C_2 - \beta\varphi F_2) w_2 = \\ &= (\cdot)w_1 + (\cdot)w_3 + (\cdot)w_4 + c_2\varphi w_2 \\ [v_3, v_4] &= [\beta w_1 + \alpha w_3, \delta w_1 + \gamma w_3 + \varphi w_4] = \\ &= (\cdot)w_1 + (\cdot)w_3 + (\cdot)w_4 + (\beta\gamma - \alpha\delta + \beta\varphi C_2 + \alpha\varphi F_2) w_2 = \\ &= (\cdot)w_1 + (\cdot)w_3 + (\cdot)w_4 + f_2\varphi w_2, \end{aligned}$$

i.e.,

$$(2.10) \quad \begin{aligned} \alpha\gamma + \beta\delta + \alpha\varphi C_2 - \beta\varphi F_2 &= \varphi c_2, \\ \beta\gamma - \alpha\delta + \beta\varphi C_2 + \alpha\varphi F_2 &= \varphi f_2. \end{aligned}$$

The frames of B_G may be chosen in such a way that (2.8) and

$$(2.11) \quad c_2 = f_2 = 0,$$

and we get

$$(2.12) \quad \gamma = \delta = 0.$$

Further,

$$\begin{aligned} [v_2, v_4] &= [\varphi w_2, \varphi w_4] = \\ &= \varphi w_2\varphi \cdot w_4 - \varphi w_4\varphi \cdot w_2 + \varphi^2(E_1w_1 + E_2w_2 + E_3w_3 + E_4w_4) = \\ &= e_1(\alpha w_1 - \beta w_3) + e_2\varphi w_2 + e_3(\beta w_1 + \alpha w_3) + e_4\varphi w_4, \end{aligned}$$

i.e.,

$$(2.13) \quad \varphi^2 E_1 = \alpha e_1 + \beta e_3, \quad \varphi^2 E_3 = -\beta e_1 + \alpha e_3$$

and

$$(2.14) \quad \varphi^2(E_1w_1 + E_3w_3) = e_1v_1 + e_3v_3.$$

The direction of the vector $e_1v_1 + e_3v_3$ is thus invariant. Suppose that $e_1v_1 + e_3v_3 \neq 0$. The frames $(v_1, \dots, v_4) \in B_G$ may be chosen in such a way that

$$(2.15) \quad e_1 = 1, \quad e_3 = 0.$$

This means

$$(2.16) \quad \alpha = 1, \quad \beta = 0, \quad \varphi = 1,$$

and the induced structure B_G is reduced to the $\{e\}$ -structure $B_{\{e\}}$. Denote by $G(V)$ the group of biholomorphic transformations of \mathcal{C}^2 preserving V . Obviously, $G(V)$ preserves the induced G -structure B_G and the reduced structure $B_{\{e\}}$. In our case $\dim G(V) \leq 4$.

If $\dim G(V) > 4$, we should have

$$(2.17) \quad e_1 = e_3 = 0,$$

i.e.,

$$(2.18) \quad \begin{aligned} [v_1, v_2] &= a_1 v_1 + a_2 v_2 + a_3 v_3, \\ [v_1, v_3] &= b_1 v_1 + v_2 + b_3 v_3, \\ [v_1, v_4] &= c_1 v_1 + c_3 v_3 + a_2 v_4, \\ [v_2, v_3] &= d_1 v_1 + d_2 v_2 + d_3 v_3, \\ [v_2, v_4] &= e_2 v_2 + e_4 v_4, \\ [v_3, v_4] &= f_1 v_1 + f_3 v_3 - d_2 v_4, \end{aligned}$$

$$(2.19) \quad f_1 - a_1 + c_3 - d_3 = 0, \quad a_3 - f_3 + c_1 - d_1 = 0;$$

the admissible transformations of the frames are given by

$$(2.20) \quad \begin{aligned} v_1 &= \alpha w_1 - \beta w_3, & v_2 &= \varphi w_2, \\ v_3 &= \beta w_1 + \alpha w_3, & v_4 &= \varphi w_4; \quad \varphi = \alpha^2 + \beta^2. \end{aligned}$$

From the Jacobi identities

$$(2.21) \quad [v_i, [v_j, v_k]] + [v_j, [v_k, v_i]] + [v_k, [v_i, v_j]] = 0; \quad i, j, k = 1, 2, 3, 4;$$

it follows

$$(2.22) \quad \begin{aligned} v_1 d_1 - v_2 b_1 + v_3 a_1 + (d_2 + b_1) a_1 + (d_3 - a_1) b_1 - (b_3 + a_2) d_1 &= 0, \\ v_1 d_2 + v_3 a_2 + (d_2 + b_1) a_2 + d_3 - a_1 - (b_3 + a_2) d_2 &= 0, \\ v_1 d_3 - v_2 b_3 + v_3 a_3 + (d_2 + b_1) a_3 + (d_3 - a_1) b_3 - (b_3 + a_2) d_3 &= 0, \\ -v_2 c_1 + v_4 a_1 + (e_2 + c_1) a_1 + (e_4 - a_1) c_1 - c_3 d_1 - a_3 f_1 &= 0, \\ v_1 e_2 + v_4 a_2 + (e_2 + c_1) a_2 - c_3 d_2 - 2a_2 e_2 &= 0, \\ -v_2 c_3 + v_4 a_3 + (e_2 + c_1) a_3 + (e_4 - a_1) c_3 - c_3 d_3 - a_3 f_3 &= 0, \\ v_1 e_4 - v_4 a_2 + (e_4 - a_1) a_2 - 2a_2 e_4 + a_3 d_2 &= 0, \\ v_1 f_1 - v_3 c_1 + v_4 a_1 + (f_3 + c_1) b_1 - (d_2 + a_1) c_1 - (a_2 + a_3) f_1 &= 0, \\ v_4 a_2 + f_3 + c_1 - a_2 e_2 &= 0, \\ v_1 f_3 - v_3 c_3 + v_4 a_3 + (f_3 + c_1) b_3 - (d_2 + a_1) c_3 - (a_2 + a_3) f_3 &= 0, \\ -v_1 d_2 - v_3 a_2 - (d_2 + a_1) a_2 + (a_2 + a_3) d_2 - a_2 e_4 &= 0, \\ v_2 f_1 + v_4 d_1 - f_1 a_1 + (f_3 + e_2) d_1 - (e_4 + d_3) f_1 - d_1 c_1 &= 0, \\ -v_3 e_2 + v_4 d_2 - f_1 a_2 + (f_3 + e_2) d_2 - 2d_2 e_2 &= 0, \\ v_2 f_3 + v_4 d_3 - f_1 a_3 + (f_3 + e_2) d_3 - (e_4 + d_3) f_3 - d_1 c_3 &= 0, \\ -v_2 d_2 - v_3 e_4 - 2d_2 e_4 + (e_4 + d_3) d_2 - d_1 a_2 &= 0. \end{aligned}$$

From (2.18), the analogous equations for $[w_i, w_j]$ and from (2.20), we get

$$(2.23) \quad \begin{aligned} -\varphi w_2 \alpha + \alpha \varphi A_1 + \beta \varphi D_1 &= \alpha a_1 + \beta a_3, \\ \varphi w_2 \alpha - \beta \varphi A_3 + \alpha \varphi D_3 &= -\beta d_1 + \alpha d_3, \\ \varphi w_2 \beta + \alpha \varphi A_3 + \beta \varphi D_3 &= -\beta a_1 + \alpha a_3, \\ \varphi w_2 \beta - \beta \varphi A_1 + \alpha \varphi D_1 &= \alpha d_1 + \beta d_3, \\ -\varphi w_4 \alpha + \alpha \varphi C_1 - \beta \varphi F_1 &= \alpha c_1 + \beta c_3, \\ -\varphi w_4 \alpha + \beta \varphi C_3 + \alpha \varphi F_3 &= -\beta f_1 + \alpha f_3, \\ \varphi w_4 \beta + \alpha \varphi C_3 - \beta \varphi F_3 &= -\beta c_1 + \alpha c_3, \\ -\varphi w_4 \beta + \beta \varphi C_1 + \alpha \varphi F_1 &= \alpha f_1 + \beta f_3, \end{aligned}$$

$$(2.24) \quad \begin{aligned} \alpha w_1 \varphi - \beta w_3 \varphi + \alpha \varphi A_2 + \beta \varphi D_2 &= \varphi a_2, \\ -\beta w_1 \varphi - \alpha w_3 \varphi - \beta \varphi A_2 + \alpha \varphi D_2 &= \varphi d_2, \end{aligned}$$

$$(2.25) \quad w_2 \varphi + \varphi E_4 = e_4, \quad -w_4 \varphi + \varphi E_2 = e_2,$$

$$(2.26) \quad \begin{aligned} \alpha w_1 \beta - \beta w_3 \beta - \beta w_1 \alpha - \alpha w_3 \alpha + \varphi B_1 &= \alpha b_1 + \beta b_3, \\ \alpha w_1 \alpha - \beta w_3 \alpha + \beta w_1 \beta + \alpha w_3 \beta + \varphi B_3 &= -\beta b_1 + \alpha b_3. \end{aligned}$$

From (2.23_{1,2}) + (2.23_{3,4}) and (2.23_{5,6}) + (2.23_{7,8}), we get

$$(2.27) \quad \begin{aligned} \alpha \varphi(A_1 + D_3) + \beta \varphi(D_1 - A_3) &= \alpha(a_1 + d_3) - \beta(d_1 - a_3), \\ \alpha \varphi(D_1 - A_3) - \beta \varphi(A_1 + D_3) &= \alpha(d_1 - a_3) + \beta(a_1 + d_3), \end{aligned}$$

$$(2.28) \quad \begin{aligned} \alpha \varphi(C_1 - F_3) - \beta \varphi(F_1 + C_3) &= \alpha(c_1 - f_3) + \beta(f_1 + c_3), \\ \alpha \varphi(F_1 + C_3) + \beta \varphi(C_1 - F_3) &= \alpha(f_1 + c_3) - \beta(c_1 - f_3). \end{aligned}$$

The equations (2.28) are the consequence of (2.27) because of (2.19). From (2.27),

$$(2.29) \quad \varphi\{(A_1 + D_3)^2 + (D_1 - A_3)^2\} = (a_1 + d_3)^2 + (d_1 - a_3)^2.$$

Suppose

$$(2.30) \quad (a_1 + d_3)^2 + (d_1 - a_3)^2 \neq 0.$$

Thus we may choose the frames of B_G in such a way that

$$(2.31) \quad (a_1 + d_3)^2 + (d_1 - a_3)^2 = 1,$$

i.e.,

$$(2.32) \quad \varphi = \alpha^2 + \beta^2 = 1.$$

We have $\dim G(V) \leq 5$ because the system (2.23)–(2.26) is, in the best case, completely integrable.

Suppose

$$(2.33) \quad d_3 = -a_1, \quad d_1 = a_3;$$

from (2.19), we obtain

$$(2.34) \quad f_1 = -c_3, \quad f_3 = c_1.$$

From (2.23_{1,3}) + (2.25₁) and (2.23_{5,7}) + (2.25₂),

$$(2.35) \quad \varphi(2A_1 + E_4) = 2a_1 + e_4, \quad \varphi(E_2 - 2C_1) = e_2 - 2c_1.$$

Again, $2a_1 + e_4 \neq 0$ or $e_2 - 2c_1 \neq 0$ implies $\dim G(V) \leq 5$.

3. Finally, suppose (2.18) with

$$(3.1) \quad d_3 = -a_1, \quad d_1 = a_3, \quad f_1 = -c_3, \quad f_3 = c_1, \quad e_4 = -2a_1, \quad e_2 = 2c_1.$$

The equations (2.25) reduce to

$$(3.2) \quad w_2\varphi = 2\varphi A_1 - 2a_1, \quad w_4\varphi = 2\varphi C_1 - 2c_1.$$

Consider the system

$$(3.3) \quad w_2\varphi = -2a_1, \quad w_4\varphi = -2c_1.$$

Then $w_4w_2\varphi = -2w_4a_1$, $w_2w_4\varphi = -2w_2c_1$, and we get

$$[w_2, w_4]\varphi = -2w_2c_1 + 2w_4a_1 = 2c_1w_2\varphi - 2a_1w_4\varphi = -4c_1a_1 + 4a_1c_1 = 0$$

by means of (2.18₅). The integrability condition of the system (3.3) is $w_2c_1 - w_4a_1 = 0$, i.e., $v_2c_1 - v_4a_1 = 0$. This equation being satisfied because of (2.22₁₄), the system (3.3) is completely integrable. It follows the possibility to choose the frames of B_G such that $A_1 = C_1 = 0$. Let us suppose

$$(3.4) \quad a_1 = c_1 = 0$$

and

$$(3.5) \quad w_2\varphi = w_4\varphi = 0.$$

From (2.23_{3,7}),

$$(3.6) \quad \varphi w_2\beta + \alpha\varphi A_3 = \alpha a_3, \quad \varphi w_4\beta + \alpha\varphi C_3 = \alpha c_3.$$

Consider the system

$$(3.7) \quad v_2\beta = \alpha a_3, \quad v_4\beta = \alpha c_3.$$

From (3.5) and (3.7), $v_2\alpha = -\beta a_3$, $v_4\alpha = -\beta c_3$, from (3.7) and (2.18₅), $v_2v_4\beta = -\beta a_3c_3 + \alpha v_2c_3$, $v_4v_2\beta = -\beta c_3a_3 + \alpha v_4a_3$. The integrability condition of (3.7) $0 = [v_2, v_4]\beta = \alpha(v_2c_3 - v_4a_3)$ is now satisfied because of (2.22₆). The system (3.7) being integrable, we are in the position to choose the frames in such a way that $A_3 = C_3 = 0$. Suppose

$$(3.8) \quad a_3 = c_3 = 0$$

and, consequently,

$$(3.9) \quad w_2\alpha = w_2\beta = w_4\alpha = w_4\beta = 0.$$

The condition $\dim G(V) > 5$ for a layer V implies the existence of sections (v_1, \dots, v_4) of B_G such that

$$(3.10) \quad \begin{aligned} [v_1, v_2] &= a_2v_2, & [v_2, v_3] &= d_2v_2, \\ [v_1, v_3] &= b_1v_1 + v_2 + b_3v_3, & [v_2, v_4] &= 0, \\ [v_1, v_4] &= a_2v_4, & [v_3, v_4] &= d_2v_4. \end{aligned}$$

The equations (2.22) reduce to

$$(3.11) \quad \begin{aligned} v_2b_1 = v_2b_3 = 0, & \quad v_2a_2 = v_4a_2 = 0, & \quad v_2d_2 = v_4d_2 = 0, \\ v_1d_2 + v_3a_2 = 0, & \quad a_2d_2 + a_2b_1 - b_3d_2 = 0. \end{aligned}$$

The equations (2.24) may be written as

$$(3.12) \quad v_1\varphi + \alpha\varphi A_2 + \beta\varphi D_2 = \varphi a_2, \quad -v_3\varphi - \beta\varphi A_2 + \alpha\varphi D_2 = \varphi d_2,$$

the equation (2.26) as

$$(3.13) \quad v_1\beta - v_3\alpha + \varphi B_1 = \alpha b_1 + \beta b_3, \quad v_1\alpha + v_3\beta + \varphi B_3 = -\beta b_1 + \alpha b_3.$$

The integrability condition of (3.12) is

$$(3.14) \quad a_2d_2 = \varphi A_2 D_2.$$

The condition $a_2d_2 \neq 0$ implies $\dim G(V) \leq 5$. Suppose

$$(3.15) \quad a_2 = 0,$$

the case $d_2 = 0$ being symmetric. Because of (3.11), the system $v_1\varphi = 0$, $v_3\varphi = -\varphi d_2$ is integrable, and we may choose the frames of B_G in such a way that

$$(3.16) \quad a_2 = d_2 = 0$$

which implies

$$(3.17) \quad v_1\varphi = v_3\varphi = 0.$$

Then $\alpha v_1 \alpha + \beta v_1 \beta = \alpha v_3 \alpha + \beta v_3 \beta = 0$, and we get

$$(3.18) \quad v_1 \alpha = \alpha \beta B_1 - \beta^2 B_3 - \beta b_1, \quad v_3 \alpha = \beta^2 B_1 + \alpha \beta B_3 - \beta b_3$$

from (3.13). The integrability condition of (3.18) is

$$(3.19) \quad v_1 b_3 - v_3 b_1 - b_1^2 - b_3^2 = \varphi(w_1 B_3 - w_3 B_1 - B_1^2 - B_3^2).$$

The condition $v_1 b_3 - v_3 b_1 - b_1^2 - b_3^2 \neq 0$ implies $\dim G(V) \leq 5$. Let us suppose

$$(3.20) \quad v_1 b_3 - v_3 b_1 - b_1^2 - b_3^2 = 0.$$

The system $v_1 \alpha = -\beta b_1$, $v_3 \alpha = -\beta b_3$ being integrable, there are sections (v_1, \dots, v_4) satisfying

$$(3.21) \quad b_1 = b_3 = 0,$$

and we have

$$(3.22) \quad v_1 \alpha = v_1 \beta = v_3 \alpha = v_3 \beta = 0.$$

4. The condition $\dim G(V) > 5$ implies $\dim G(V) = 6$ and the existence of a section (v_1, \dots, v_4) of B_G such that

$$(4.1) \quad [v_1, v_3] = v_2, \quad [v_1, v_2] = [v_1, v_4] = [v_2, v_3] = [v_2, v_4] = [v_3, v_4] = 0.$$

Consider the layer $V(0.1)$. It is easy to check that the real vector fields

$$(4.2) \quad v_1 = i \frac{\partial}{\partial x} + 2(\bar{x} - x) \frac{\partial}{\partial y} - i \frac{\partial}{\partial \bar{x}} + 2(x - \bar{x}) \frac{\partial}{\partial \bar{y}}, \quad v_2 = 4 \left(\frac{\partial}{\partial y} + \frac{\partial}{\partial \bar{y}} \right),$$

$$v_3 = -\frac{\partial}{\partial x} + 2i(\bar{x} - x) \frac{\partial}{\partial y} - \frac{\partial}{\partial \bar{x}} - 2i(x - \bar{x}) \frac{\partial}{\partial \bar{y}}, \quad v_4 = 4i \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial \bar{y}} \right)$$

over \mathcal{C}^2 satisfy the following conditions: (i) $v_3 = Jv_1$, $v_4 = Jv_2$, (ii) at each point $z \in \mathcal{C}^2$, v_1, v_2, v_3 are tangent to the hypersurface of V going through z , (iii) v_1, \dots, v_4 satisfy (4.1). The Lie group (0.2) preserving V , we have obtained an example of a layer satisfying the conditions of our Theorem. It remains to show that any two layers satisfying these conditions are biholomorphically equivalent. Consider the complex manifold M^4 and its layer V such that in its corresponding structure B_G there is a section (v_1, \dots, v_4) satisfying (4.1). Let N^4 be another complex manifold with a layer W of hypersurfaces such that in the associated structure B'_G there is a section (w_1, \dots, w_4) such that

$$(4.3) \quad [w_1, w_3] = w_2,$$

$$[w_1, w_2] = [w_1, w_4] = [w_2, w_3] = [w_2, w_4] = [w_3, w_4] = 0.$$

On $M^4 \times N^4$, consider the vector fields v_i^* , w_i^* defined by the relations $d\pi_1(v_i^*) = v_i$, $d\pi_2(v_i^*) = 0$, $d\pi_1(w_i^*) = 0$, $d\pi_2(w_i^*) = w_i$; $\pi_1 : M^4 \times N^4 \rightarrow M^4$, $\pi_2 : M^4 \times N^4 \rightarrow N^4$ being the natural projections. Let $\alpha, \beta \in \mathcal{R}$, $\varphi = \alpha^2 + \beta^2 \neq 0$. On $M^4 \times N^4$, consider the distribution D such that its space $D_{(m,n)}^4 \subset T_{(m,n)}(M^4 \times N^4)$ is spanned by the vectors

$$\begin{aligned} V_1 &= v_1^* + \alpha w_1^* - \beta w_3^*, & V_2 &= v_2^* + \varphi w_2^*, & V_3 &= v_3^* + \beta w_1^* + \alpha w_3^*, \\ & & & & & V_4 &= v_4^* + \varphi w_4^*. \end{aligned}$$

Because of

$$[V_1, V_3] = V_2, \quad [V_1, V_2] = [V_1, V_4] = [V_2, V_3] = [V_2, V_4] = [V_3, V_4] = 0,$$

the distribution D is integrable and its integral manifold represents a (local) biholomorphic map $M^4 \rightarrow N^4$ transforming V into W .

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