

## Summaries of articles published in this issue

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## SUMMARIES OF ARTICLES PUBLISHED IN THIS ISSUE

(Publication of these summaries is permitted)

BOHDAN ZELINKA, Liberec: *Intersection graphs of finite Abelian groups*. Czech. Math. J. 25 (100), (1975), 171—174. (Original paper.)

If  $A$  is an Abelian group, then the intersection graph of  $A$  is an undirected graph whose vertices are all proper subgroups of  $A$  and in which two vertices are joined by an edge if and only if the corresponding subgroups have a non-trivial intersection (i.e. an intersection containing more than one element). In the paper some theorems on this topic are proved.

BOHDAN ZELINKA, Liberec: *Tolerance in algebraic structures, II*. Czech. Math. J. 25 (100), (1975), 175—178. (Original paper.)

A tolerance is a reflexive and symmetric relation on a set. If  $\mathfrak{A} = \langle A, \mathcal{F} \rangle$  is an algebra and  $\xi$  is a tolerance on  $A$ , we say that  $\xi$  is compatible with  $\mathfrak{A}$ , if and only if for any  $n$ -ary operation  $f \in \mathcal{F}$ , where  $n$  is a positive integer, and for any elements  $x_1, \dots, x_n, y_1, \dots, y_n$  of  $A$  such that  $(x_i, y_i) \in \xi$  for  $i = 1, \dots, n$  we have  $(f(x_1, \dots, x_n), f(y_1, \dots, y_n)) \in \xi$ . This paper continues the study of tolerances compatible with algebras which was begun in a previous paper of the author in this Journal.

М. Б. КУДАЕВ, Нальчик: *О поведении решений систем дифференциальных уравнений второго порядка*. Czech. Math. J. 25 (100), (1975), 179—189. (Оригинальная статья.)

В этой заметке исследуется поведение (ограниченность, стремление к нулю, неограниченность) решений системы  $n$  уравнений (\*)  $y'' + A_1(t)y' + A_2(t)y = 0$ . Основные результаты заключены в леммах 1—3 в виде оценок сверху и снизу для норм решений (\*) и их производных на  $I_T = [t_0, T]$ ,  $T \leq +\infty$ , где последние предполагаются существующими. Из этих оценок получены некоторые признаки поведения  $y(t)$  и  $y'(t)$  на  $I_\infty$ . В конце указано на возможные обобщения для нелинейных уравнений с отклоняющимся аргументом.

MANFRED MÜLLER, Berlin und JINDŘICH NEČAS, Praha: *Über die Regularität der schwachen Lösungen von Randwertaufgaben für quasilineare elliptische Differentialgleichungen höherer Ordnung*. Czech. Math. J. 25 (100), (1975), 227—239. (Originalartikel.)

In vorliegenden Artikel werden für die Dimension  $N \geq 3$  Abschätzungen der Art bewiesen:  $\int_{\Omega} [(1 + \sum_{|i| \leq k} |D^i u|)^{m-2} \sum_{|j| \leq k+1} |D^j u|^2] dx < \infty$ , mit  $1/q = 1/2 + (N-2)(m-2)/2Nm$ , für die schwache Lösung der Differentialgleichung  $\sum_{|i| \leq k} (-1)^{|i|} D^i a_i(x, D^j u) = \sum_{|i| \leq k} (-1)^{|i|} D^i f_i$ . Gewöhnliche Bedingungen wie zum Beispiel

$$\gamma_1 (1 + \sum_{|i| \leq k} |\xi_i|)^{m-2} \sum_{|j| \leq k} \lambda_j^2 \leq \sum_{|i|, |j| \leq k} (\partial a_i / \partial \xi_j) \lambda_i \lambda_j$$

sind vorausgesetzt.

CHARLES SWARTZ, Las Cruces: *Translation invariant linear operators and generalized functions*. Czech. Math. J. 25 (100), (1975), 202–213. (Original paper.)

The general problem of representing translation invariant linear maps between test spaces or generalized function spaces as convolution operators is considered. Three different types of mappings are considered: maps between test spaces, maps between generalized function spaces, and maps from  $\mathcal{D}$  into a certain class of generalized function space.

FRANTIŠEK MACHALA, Olomouc: *Projektive Abbildungen projektiver Räume mit Homomorphismus*. Czech. Math. J. 25 (100), (1975), 214–226. (Originalartikel.)

In vorliegender Arbeit wird eine Verallgemeinerung des Struktursatzes der projektiven Geometrie auf projektive Räume mit Homomorphismus durchgeführt. Der Beitrag besteht aus drei Teilen. Im ersten Teil werden freie Moduln beliebiger Dimension über einem Stellenring betrachtet und eine halblinare bzw. teilweise halblinare Abbildung der Moduln  $M, N$  über Stellenringen  $R, Q$  eingeführt. Jede teilweise halblinare Abbildung der Moduln  $M, N$  kann in genau eine halblinare Abbildung derjenigen Moduln erweitert werden. Im zweiten Teil definiert man mit Hilfe des Moduln  $M$  einen projektiven Raum  $P(M)$  mit Homomorphismus. Weiter ist eine projektive Abbildung der projektiven Räume  $P(M), P(N)$  erklärt und es wird gezeigt, dass jede teilweise halblinare Abbildung der Moduln  $M, N$  eine projektive Abbildung der Räume  $P(M), P(N)$  induziert. Im dritten Teil wird der Struktursatz für projektive Räume mit Homomorphismus bewiesen: Jede projektive Abbildung der Räume  $P(M), P(N)$  wird durch eine teilweise halblinare Abbildung der Moduln  $M, N$  induziert.

JÁN JAKUBÍK, Košice: *Unoriented graphs of modular lattices*. Czech. Math. J. 25 (100), (1975), 240–246. (Original paper.)

In this paper it is proved that if the unoriented graph  $G(L_1)$  of a locally finite modular lattice  $L_1$  is isomorphic with the unoriented graph  $G(L_2)$  of a locally finite lattice  $L_2$ , then the lattice  $L_2$  is modular.

J. R. WALL, Auburn: *Green's relations for stochastic matrices*. Czech. Math. J. 25 (100), (1975), 247–260. (Original paper.)

Green's relations on the semigroup of all square stochastic matrices of a given order are characterized for regular elements. This is equivalent to solving certain systems of stochastic matrix equations. The same is done in the doubly stochastic case. New proofs of the characterizations of the maximal subgroups of these semigroups are obtained, as well as several conditions equivalent to regularity.

D. J. HARTFIEL and C. J. MAXSON, College Station: *A matrix characterization of the maximal groups in  $\beta_X$* . Czech. Math. J. 25 (100), (1975), 274–278. (Original paper.)

This note gives a matrix characterization of the maximal groups of  $\beta_X$ , the semigroup of relations on the finite set  $X$ .

KAR-PING SHUM, Hong-Kong: *On compressed ideals in topological semi-groups*. Czech. Math. J. 25 (100), (1975), 261—273. (Original paper.)

The aim of this paper is to study the compressed ideals in compact semi-groups. The author first shows that a compressed ideal of a semigroup  $S$  is in fact a completely semi-prime ideal of  $S$ , and thus the Thierrin radical of an ideal  $A$  in a commutative semigroup is the algebraic radical of  $A$ . If  $B$  is a compressed ideal of  $S$ , then for any element  $a$  of  $S$ , the set  $\{s \in S \mid as \in B\}$  is called the topological  $B$ -divisor of  $a$  and it is denoted by  $\text{Tod}_B a$ . Some properties of the set  $\text{Tod}_B a$  are studied in this paper and some results obtained in the author's previous paper dealing with the topological zero divisors in compact commutative semigroups with zero are amplified. The author also gives some characterizations of the Thierrin radical of an open ideal  $A$  in a compact semigroup  $S$ .

Г. Я. РОТКОВИЧ, Ленинград: *О  $\sigma$ -полных решеточно упорядоченных группах*. Czech. Math. J. 25 (100), (1975), 279—281. (Оригинальная статья.)

Для сингулярной  $l$ -группы  $X$  равносильны следующие условия: (I)  $X$  является полной; (II)  $X$  является  $\sigma$ -полной и дизъюнктно полной. Я. Якубик поставил вопрос: переносится ли этот результат на произвольные  $l$ -группы? В заметке дается на этот вопрос положительный ответ.

M. E. MOORE, Arlington: *A strong complement property of Dedekind domains*. Czech. Math. J. 25 (100), (1975), 282—283. (Original paper.)

I. Reiner in his former paper has shown that any Dedekind domain is completable. In this note it is shown that Dedekind domains are in fact strongly completable, a fortiori, completable.

JORGE MARTINEZ, Gainesville: *Torsion theory for lattice-ordered groups*. Czech. Math. J. 25 (100), (1975), 284—299. (Original paper.)

A class  $\mathcal{T}$  of lattice-ordered groups closed with respect to 1) convex  $l$ -subgroups, 2)  $l$ -homomorphic images and 3) joins of families of convex  $l$ -subgroups in  $\mathcal{T}$ , will be called a torsion class. A torsion class  $\mathcal{T}$  gives rise to a radical satisfying: a)  $\mathcal{T}(C) = C \cap \mathcal{T}(G)$ , for each convex  $l$ -subgroup  $C$ , and b)  $[\mathcal{T}(G)] \phi \subseteq \mathcal{T}(H)$  for each onto  $l$ -homomorphism  $\phi: G \rightarrow H$ .  $\mathbf{T}$ , the lattice of torsion classes admits a binary operation, and each torsion class gives rise to a Loewy-type sequence of torsion classes:  $\mathcal{T}, \mathcal{T}^2, \dots, \mathcal{T}^\sigma, \dots$  for each ordinal  $\sigma$ .  $\mathcal{T}^* = \bigcup_{\sigma} \mathcal{T}^\sigma$  is complete in the sense that it is closed under

extensions, and  $\mathcal{T}^*$  is the smallest such torsion class containing  $\mathcal{T}$ . One interesting property of these radicals turns out to be that  $\mathcal{T}^*(G) \subseteq \mathcal{T}(G)'$ , for each lattice-ordered group  $G$ .

The lattice  $\mathbf{T}$  is Brouwer and thus gives rise to a polarization  $\mathcal{T} \rightarrow \mathcal{T}'$ . A characterization of  $\mathcal{T}'$  is given and also one for  $\mathcal{T}''$ , in terms of quotients of convex  $l$ -subgroups. If  $\mathcal{T}$  is closed under wreath products or direct lexicographic extensions, it is indecomposable, and if it is finitely meet irreducible it is again indecomposable. The possibility of primary decomposition is investigated, and some primary classes are identified.

The notion of a homogeneous lattice-ordered group is introduced:  $G$  is homogeneous if for each torsion class  $\mathcal{T}$   $G \in \mathcal{T}$  or else  $\mathcal{T}(G) = 0$ . This idea is interpreted in terms of the lattice  $\mathbf{T}$ .

F. B. JONES, Riverside: *Connected simple graphs and a selection problem*. Czech. Math. J. 25 (100), (1975), 303—301. (Original paper.)

In this paper, a simple example is given, which yields negative answer to the problem from the former paper by Ceder and Bruckner.

IVAN CHAJDA, Píerov: *Systems of equations and tolerance relations*. Czech. Math. J. 25 (100), (1975), 302—308. (Original paper.)

The purpose of this paper is to give the abstract algebraic foundations of the theory of approximative solutions of stability. Some concepts as exact solution, approximate solution, stable solution are introduced for systems of equations over abstract algebras. Further, theorems characterizing the relation between “accuracy” (expressed by means of tolerance relations) of solution and its stability (also introduced by tolerance relations) are proved. Some examples from numerical analysis, the so called “classical tolerances”, are presented in the paper, however, the whole theory is abstract algebraic only.

IVAN NETUKA, Praha: *Continuity and maximum principle for potentials of signed measures*. Czech. Math. J. 25 (100), (1975), 309—316. (Original paper.)

Using fine topology arguments, the following theorem is proved: Let  $\mu$  be a signed measure with support  $K$  in the  $m$ -dimensional Euclidean space  $R^m$  ( $m > 2$ ) and let  $U\mu$  (the Newtonian potential of  $\mu$ ) be finite in  $R^m$ . If the restriction of  $U\mu$  to  $K$  is continuous on  $K$ , then the potential  $U\mu$  is continuous in the whole space. In fact, an abstract version of the theorem is obtained in the frame of the axiomatic potential theory and an analogue of the classical maximum principle is given for differences of potentials.

ZDENĚK HUSTÝ, Brno: *Algebraische Theorie der Transformation der linearen Differentialgleichungen*. Czech. Math. J. 25 (100), (1975), 317—329. (Übersichtsartikel.)