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CONNECTED SIMPLE GRAPHS AND A SELECTION PROBLEM

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A problem left open by CEDER and BRUCKNER in [1] is the following (as modified by PAUL ERDŐS):

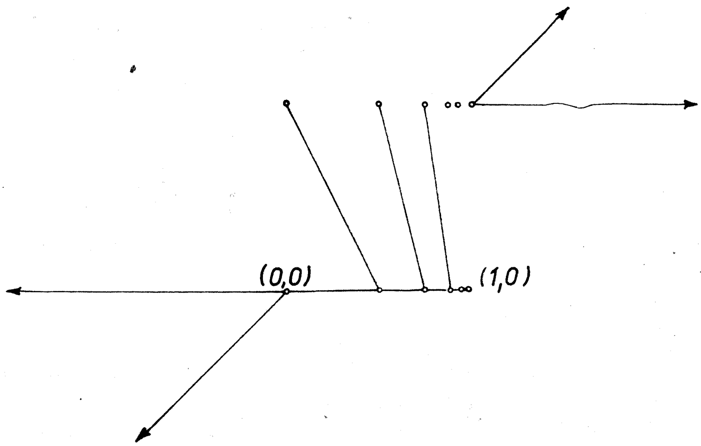
Suppose that M is a connected subset of the plane such that each vertical line intersects M in exactly two points. Does M contain a connected subset M_1 such that every vertical line intersects M_1 in exactly one point? In other words, must there exist a function (from R to R) whose graph lies in M and is connected? The example given below shows that such a selection is not always possible.

Let M be the union of the following plane sets:

$$\{\text{the graph of } y = 0 \text{ for } x \leqq 1\} ;$$
$$\{\text{the graph of } y = 1 \text{ for } x \geqq 1\} ;$$
$$\{\text{the graph of } y = x \text{ for } x \leqq 0 \text{ and } x \geqq 1\} ;$$

and

the closed straight line intervals from $(1 - 1/2^n, 1)$ to $(1 - 1/2^{n+1}, 0)$ for $n = 0, 1, 2, 3, \ldots$.



Clearly, M is connected. Also it is clear that if one is to make a selection of one point for each vertical line which is to produce a connected set, one must choose only those points of M on the x-axis between $x = 0$ and $x = 1$. This is true because, roughly speaking, these points separate M between $-\infty$ and $+\infty$. But having done this, there is no way to connect to the points in M for $x \geqq 1$ and $y \geqq 1$.

References

[1] *A. M. Bruckner* and *J. Ceder,* On jumping functions by connected sets, Czechoslovak Math. J., *22* (1972), 435—448: in particular, Question 1, p. 442.

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