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## ON A PARTIAL COMPLEX STRUCTURE

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Let  $M^3 \subset \mathcal{C}^2$  be a real hypersurface.  $\mathcal{C}^2$  being identified with  $\mathcal{R}^4$  endowed with an endomorphism  $J: \mathcal{R}^4 \rightarrow \mathcal{R}^4$  satisfying  $J^2 = -\text{id}$ ., we get, on  $M^3$ , a structure consisting of a field of tangent planes  $\tau_m = T_m(M^3) \cap JT(M^3)$  and the restrictions  $J_m: \tau_m \rightarrow \tau_m$  of  $J$  to  $\tau_m$ ; see [3] and [4] respectively. Such a structure is called the partial complex structure. An attempt to solve the equivalence problem for the structures of this type has been made by E. CARTAN [1]; unfortunately, his treatment is not a very clear one. In what follows, I am going to present a more simple method for solving the mentioned problem.

Let  $M$  be a 3-dimensional manifold; in what follows, all the considered manifolds and maps are supposed to be of class  $C^\infty$ .

**Definition.** Let  $G$  be the group of matrices of the type

$$(1) \quad \begin{pmatrix} \alpha & -\beta & 0 \\ \beta & \alpha & 0 \\ \gamma & \delta & \varphi \end{pmatrix}; \quad \alpha, \beta, \gamma, \delta, \varphi \in \mathcal{R}; \quad (\alpha^2 + \beta^2)\varphi \neq 0.$$

A  $G$ -structure  $B_G$  on  $M$  is called a *partial complex structure*.

Let  $(v_1, v_2, v_3), (w_1, w_2, w_3)$  be two sections of  $B_G$  over a domain  $U \subset M$ ; we have

$$(2) \quad v_1 = \alpha w_1 - \beta w_2, \quad v_2 = \beta w_1 + \alpha w_2, \quad v_3 = \gamma w_1 + \delta w_2 + \varphi w_3.$$

At each point  $m \in M$  there are induced a plane  $\tau_m = \{v_1, v_2\}$  and an endomorphism  $J_m: \tau_m \rightarrow \tau_m$  given by  $J_m(v_1) = v_2, J_m(v_2) = -v_1$ ; obviously,  $J_m^2 = -\text{id}$ . We are going to suppose that *the field of planes  $\tau$  is non-integrable*.

**Definition.** A vector field  $v$  on  $M$  is called a  $\tau$ -field if  $v_m \in \tau_m$  for each  $m \in M$ . A vector field  $u$  is called an *infinitesimal motion* of  $B_G$  if: (i)  $\mathcal{L}_u v$  is a  $\tau$ -field for each  $\tau$ -field  $v$ , (ii)  $J(\mathcal{L}_u v) = \mathcal{L}_u(Jv)$  for each  $\tau$ -field  $v$ ; here,  $\mathcal{L}_u v = [u, v]$  is the Lie derivative of  $v$ . The structure  $B_G$  is said to be *locally transitive* if, for each  $m \in M$  and  $t \in T_m(M)$ , there is a neighbourhood  $U \subset M$  of  $m$  and an infinitesimal motion  $u$  over  $U$  such that  $u_m = t$ .

Our main problem is to determine all locally transitive  $G$ -structures. Let  $(v_1, v_2, v_3)$ ,  $(w_1, w_2, w_3)$  be two sections of  $B_G$  over  $U \subset M$ . Then

$$(3) \quad \begin{aligned} [v_1, v_2] &= a_1 v_1 + a_2 v_2 + a_3 v_3, & [w_1, w_2] &= A_1 w_1 + A_2 w_2 + A_3 w_3, \\ [v_1, v_3] &= b_1 v_1 + b_2 v_2 + b_3 v_3, & [w_1, w_3] &= B_1 w_1 + B_2 w_2 + B_3 w_3, \\ [v_2, v_3] &= c_1 v_1 + c_2 v_2 + c_3 v_3, & [w_2, w_3] &= C_1 w_1 + C_2 w_2 + C_3 w_3, \end{aligned}$$

the functions  $a_1, \dots, c_3, A_1, \dots, C_3$  satisfying the Jacobi identities

$$(4) \quad \begin{aligned} [v_1, [v_2, v_3]] + [v_2, [v_3, v_1]] + [v_3, [v_1, v_2]] &= 0, \\ [w_1, [w_2, w_3]] + [w_2, [w_3, w_1]] + [w_3, [w_1, w_2]] &= 0. \end{aligned}$$

We get

$$\begin{aligned} [v_1, v_2] &= \{v_1 \beta - v_2 \alpha + (\alpha^2 + \beta^2) A_1\} w_1 + \{v_1 \alpha + v_2 \beta + (\alpha^2 + \beta^2) A_2\} w_2 + \\ &\quad + (\alpha^2 + \beta^2) A_3 w_3 = \\ &= (\alpha a_1 + \beta a_2 + \gamma a_3) w_1 + (\alpha a_2 - \beta a_1 + \delta a_3) w_2 + \varphi a_3 w_3, \\ [v_1, v_3] &= (\cdot) w_1 + (\cdot) w_2 + \{v_1 \varphi + (\alpha \delta + \beta \gamma) A_3 + \alpha \varphi B_3 - \beta \varphi C_3\} w_3 = \\ &= (\cdot) w_1 + (\cdot) w_2 + \varphi b_3 w_3, \\ [v_2, v_3] &= (\cdot) w_1 + (\cdot) w_2 + \{v_2 \varphi + (\beta \delta - \alpha \gamma) A_3 + \beta \varphi B_3 + \alpha \varphi C_3\} w_3 = \\ &= (\cdot) w_1 + (\cdot) w_2 + \varphi c_3 w_3. \end{aligned}$$

From this, we get the existence of sections satisfying  $a_3 = 1$ . Suppose  $a_3 = A_3 = 1$ , i.e.,  $\varphi = \alpha^2 + \beta^2$ . Let us look for the existence of a section  $(w_1, w_2, w_3)$  satisfying  $A_1 = A_2 = B_3 = C_3 = 0$ . This amounts to the existence of  $\alpha, \beta, \gamma, \delta$  such that

$$\begin{aligned} v_1 \beta - v_2 \alpha &= a_1 \alpha + a_2 \beta + \gamma, & 2\alpha v_1 \alpha + 2\beta v_1 \beta + \alpha \delta + \beta \gamma &= (\alpha^2 + \beta^2) b_3, \\ v_1 \alpha + v_2 \beta &= a_2 \alpha - a_1 \beta + \delta, & 2\alpha v_2 \alpha + 2\beta v_2 \beta + \beta \delta - \alpha \gamma &= (\alpha^2 + \beta^2) c_3. \end{aligned}$$

It is easy to see that this system has (at least locally) solutions such that  $\alpha^2 + \beta^2 \neq 0$ . Let  $a_1 = a_2 = b_3 = c_3 = 0, a_3 = 1$  for  $(v_1, v_2, v_3)$ . From the Jacobi identity (4), we get  $v_1 c_1 - v_2 b_1 = v_1 c_2 - v_2 b_2 = c_2 + b_1 = 0$ . A  $\tau$ -field  $v$  is called special if the section  $(v_1 = v, v_2 = Jv, v_3 = [v, Jv])$  has the just described property.

**Theorem.** Let  $B_G$  be a partial complex structure over  $M$ . Let  $v, w$  be its special  $\tau$ -fields, and let

$$(5) \quad \begin{aligned} [v, [v, Jv]] &= av + bJv, & [Jv, [v, Jv]] &= cv - aJv, \\ (Jv)b + va &= 0, & (Jv)a - vc &= 0; \end{aligned}$$

$$(6) \quad [w, [w, Jw]] = Aw + BJw, \quad [Jw, [w, Jw]] = Cw - AJw, \\ (Jw)B + wA = 0, \quad (Jw)A - wC = 0;$$

$$(7) \quad v = \alpha w - \beta Jw.$$

Consider the functions

$$(8) \quad j_1 = (v - Jv \cdot Jv)(c - b) + 8[v, Jv]a - 3(c^2 - b^2), \\ j_2 = (v \cdot Jv + Jv \cdot v)(c - b) + 4[v, Jv](b + c) + 6a(c - b);$$

$J_1$  and  $J_2$  be defined similarly. Then:

(i) We have

$$(9) \quad j_1^2 + j_2^2 = (\alpha^2 + \beta^2)^4 (J_1^2 + J_2^2).$$

(ii) If  $j_1 = j_2 = 0$ ,  $B_G$  is locally transitive. For each point  $m \in M$ , there is its neighbourhood  $U \subset M$  and special  $\tau$ -fields  $v$  over  $U$  satisfying

$$(10) \quad [v, [v, Jv]] = 0, \quad [Jv, [v, Jv]] = 0.$$

Choose such a  $\tau$ -field  $v$ . Further, choose arbitrary real numbers  $R_1, \dots, R_8$ . Then there is exactly one field  $u \in \mathcal{L}(B_G)$  over a suitable neighbourhood  $m \in U_1 \subset U$  such that

$$(11) \quad u_m = R_1 v_m + R_2 (Jv)_m + R_3 [v, Jv]_m, \\ [v, u]_m = R_4 v_m + R_5 (Jv)_m, \\ [v, [v, u]]_m = R_6 v_m + R_7 (Jv)_m + R_8 [v, Jv]_m, \\ [v, [v, [v, u]]]_m = R_8 (Jv)_m + 2R_7 [v, Jv]_m.$$

(iii) Let  $j_1^2 + j_2^2 \neq 0$ . To each point  $m \in M$ , there is its neighbourhood  $U$  and exactly two special  $\tau$ -fields  $v, v' = -v$  over  $U$  satisfying (5) and

$$(12) \quad j_1 = 1, \quad j_2 = 0.$$

$B_G$  being transitive,  $v$  satisfies

$$(13) \quad [v, [v, Jv]] = bJv, \quad [Jv, [v, Jv]] = cv; \\ b, c \in \mathcal{R}, \quad 3(c^2 - b^2) + 1 = 0.$$

For each vector  $t \in T_m(M)$ , there is exactly one field  $u \in \mathcal{L}(B_G)$  — defined over a suitable neighbourhood  $U_1$  of  $m$  — such that  $u_m = t$ .

Proof. Let  $T^c(M) = T(M) \oplus i T(M)$  be the complexification of the tangent bundle  $T(M)$  of  $M$ . The bracket operation in  $T^c(M)$  be introduced, quite naturally, by

$$(14) \quad [v + iv', w + iw'] = [v, w] - [v', w'] + i([v', w] + [v, w']).$$

In  $B_G$ , consider two special sections  $(v_1, v_2, v_3), (w_1, w_2, w_3)$  satisfying

$$(15) \quad [v_1, v_2] = v_3, \quad [v_1, v_3] = av_1 + bv_2, \quad [v_2, v_3] = cv_1 - av_2;$$

$$[w_1, w_2] = w_3, \quad [w_1, w_3] = Aw_1 + Bw_2, \quad [w_2, w_3] = Cw_1 - Aw_2.$$

In  $T^c(M)$ , consider the vector fields

$$(16) \quad V_1 = v_1 + iv_2, \quad V_2 = v_1 - iv_2, \quad V_3 = -2iv_3;$$

$$W_1 = w_1 + iw_2, \quad W_2 = w_1 - iw_2, \quad W_3 = -2iw_3.$$

We get

$$(17) \quad [V_1, V_2] = V_3, \quad [V_1, V_3] = pV_1 + qV_2, \quad [V_2, V_3] = rV_1 - pV_2,$$

$$p = c - b, \quad q = b + c - 2ia, \quad r = -(b + c + 2ia),$$

$$V_2q = -V_1p, \quad V_1r = V_2p;$$

$$[W_1, W_2] = W_3, \quad [W_1, W_3] = PW_1 + QW_2, \quad [W_2, W_3] = RW_1 - PW_2,$$

$$P = C - B, \quad Q = B + C - 2iA, \quad R = -(B + C + 2iA),$$

$$W_2Q = -W_1P, \quad W_1R = W_2P.$$

From (2),

$$(18) \quad V_1 = \varrho W_1, \quad V_2 = \sigma W_2, \quad V_3 = \mu W_1 + \nu W_2 + \varphi W_3;$$

$$\varrho = \alpha + i\beta, \quad \sigma = \alpha - i\beta, \quad \mu = -(\delta + i\gamma), \quad \nu = \delta - i\gamma; \quad \varrho\sigma\varphi \neq 0.$$

Now,

$$(19) \quad \varphi = \varrho\sigma,$$

$$(20) \quad V_1\varrho = -2\varrho\sigma^{-1}\nu, \quad V_2\varrho = -\mu; \quad V_1\sigma = \nu, \quad V_2\sigma = 2\varrho^{-1}\sigma\mu;$$

$$V_2\mu = \varrho r - \varrho\sigma^2 R, \quad V_1\nu = \sigma q - \varrho^2\sigma Q,$$

$$V_1\mu - V_3\varrho = \varrho p - \varrho^2\sigma P, \quad V_2\nu - V_3\sigma = -\sigma p + \varrho\sigma^2 P.$$

It is known [2] that the integrability conditions of (20) imply

$$(21) \quad k_1 = \varrho^3\sigma K_1, \quad k_2 = \varrho\sigma^3 K_2$$

with

$$(22) \quad k_1 = V_1 V_1 p - 2V_3 q - 3pq, \quad k_2 = V_2 V_2 p - 2V_3 r + 3pr$$

and similar definition of  $K_1$  and  $K_2$  respectively. We get

$$(23) \quad k_1 = j_1 + ij_2, \quad k_2 = j_1 - ij_2,$$

$j_1$  and  $j_2$  being given by (8). Thus (21) reduce to  $k_1 = \varrho^3 \bar{\varrho} K_1$ ; the equation  $k_1 \bar{k}_1 = \varrho^4 \bar{\varrho}^4 K_1 \bar{K}_1$  is exactly (9). The equation  $k_1 = \varrho^3 \bar{\varrho}$  has solutions  $\varrho \neq 0$  for each  $k_1 \neq 0$ ; the equation  $1 = \varrho^3 \bar{\varrho}$  has exactly two solutions  $\varrho = \pm 1$ . Thus (i) and the first part of (iii) have been proved.

Suppose  $j_1 = j_2 = 0$ , i.e.,  $k_1 = k_2 = 0$ , and consider the system (19) + (20),  $P = Q = R = 0$ . According to [2], this system is completely integrable, and we have proved the first part of (ii).

Now, consider the structure  $B_G$ , given by a section  $(v_1, v_2, v_3)$  satisfying

$$(24) \quad [v_1, v_2] = v_3, \quad [v_1, v_3] = 0, \quad [v_2, v_3] = 0.$$

Let  $u \in \mathcal{L}(B_G)$ ,

$$(25) \quad u = Av_1 + Bv_2 + Cv_3.$$

From

$$[v_1, u] = v_1 A \cdot v_1 + v_1 B \cdot v_2 + (v_1 C + B) v_3,$$

$$[v_2, u] = v_2 A \cdot v_1 + v_2 B \cdot v_2 + (v_2 C - A) v_3,$$

we get

$$(26) \quad v_1 A - v_2 B = v_2 A + v_1 B = v_1 C + B = v_2 C - A = 0.$$

For  $D := v_1 A$ ,  $E := v_2 A$ , we have

$$(27) \quad v_1 A = D, \quad v_2 A = E;$$

$$v_1 B = -E, \quad v_2 B = D; \quad v_1 C = -B, \quad v_2 C = A.$$

From (27<sub>5,6</sub>),

$$(28) \quad v_3 C = 2D;$$

the integrability conditions of (27<sub>1</sub>) + (27<sub>2</sub>), (27<sub>3</sub>) + (27<sub>4</sub>), (27<sub>5</sub>) + (28) and (27<sub>6</sub>) + (28) are

$$v_3 A = v_1 E - v_2 D, \quad v_3 B = v_1 D + v_2 E,$$

$$0 = 2v_1 D + v_3 B, \quad 0 = 2v_2 D - v_3 A$$

respectively; for  $3F := v_1E$ ,  $3G := v_2E$ , they become

$$(29) \quad \begin{aligned} v_3A &= 2F; \quad v_3B = 2G; \\ v_1D &= -G, \quad v_2D = F; \quad v_1E = 3F, \quad v_2E = 3G. \end{aligned}$$

The integrability conditions of  $(27_1) + (29_1)$ ,  $(27_2) + (29_1)$ ,  $(27_3) + (29_2)$ ,  $(27_4) + (29_2)$ ,  $(29_3) + (29_4)$  and  $(29_5) + (29_6)$  being

$$\begin{aligned} 2v_1F &= v_3D, \quad 2v_2F = v_3E, \quad 2v_1G = -v_3E, \quad 2v_2G = v_3D, \\ v_3D &= v_1F + v_2G, \quad v_3E = 3v_1G - 3v_2F, \end{aligned}$$

we have

$$(30) \quad v_3D = 2H; \quad v_3E = 0; \quad v_1F = H, \quad v_2F = 0; \quad v_1G = 0, \quad v_2G = H$$

for  $H := v_1F$ . The integrability conditions of  $(29_3) + (30_1)$ ,  $(29_4) + (30_1)$ ,  $(29_5) + (30_2)$ ,  $(29_6) + (30_2)$ ,  $(30_3) + (30_4)$  and  $(30_5) + (30_6)$  are

$$\begin{aligned} 2v_1H &= -v_3G, \quad 2v_2H = v_3F, \quad v_3F = 0, \quad v_3G = 0, \\ v_3F &= -v_2H, \quad v_3G = v_1H; \end{aligned}$$

from these, we get

$$(31) \quad v_3F = 0; \quad v_3G = 0; \quad v_1H = 0, \quad v_2H = 0.$$

The integrability conditions of  $(30_3) + (31_1)$ ,  $(30_4) + (31_1)$ ,  $(30_5) + (31_2)$ ,  $(30_6) + (31_2)$  and  $(31_3) + (31_4)$  reduce to

$$(32) \quad v_3H = 0.$$

Thus the system (27)–(32) is completely integrable. For  $u \in \mathcal{L}(B_G)$  given by (25), we get

$$(33) \quad \begin{aligned} [v_1, u] &= Dv_1 - Ev_2, \quad [v_1, [v_1, u]] = -Gv_1 - 3Fv_2 - Ev_3, \\ [v_1, [v_1, [v_1, u]]] &= -3Hv_2 - 6Fv_3, \end{aligned}$$

this completing the proof of (ii).

Finally, let  $B_G$  be transitive with  $j_1^2 + j_2^2 \neq 0$ . Then they are exactly two special sections of  $B_G$  satisfying  $j_1 = 1, j_2 = 0$ ; let  $(v_1, v_2, v_3)$  be one of them. The functions  $a, b, c$  being now invariants of  $B_G$ , they have to be constants, and we get  $-3(c^2 - b^2) = 1, a(c - b) = 0$ . Because of  $c - b \neq 0, a = 0$ , i.e.,

$$(34) \quad \begin{aligned} [v_1, v_2] &= v_3, \quad [v_1, v_3] = bv_2, \quad [v_2, v_3] = cv_1; \\ b, c &\in \mathcal{R}, \quad 3(c^2 - b^2) + 1 = 0. \end{aligned}$$

Let  $u \in \mathcal{L}(B_G)$ ,  $u$  being given by (25). From

$$\begin{aligned} [v_1, u] &= v_1A \cdot v_1 + (v_1B + bC)v_2 + (v_1C + B)v_3, \\ [v_2, u] &= (v_2A + cC)v_1 + v_2B \cdot v_2 + (v_2C - A)v_3, \end{aligned}$$

we get

$$v_1A - v_2B = v_2A + v_1B + (b + c)C = v_1C + B = v_2C - A = 0.$$

For  $D := v_1A$ ,  $E := v_1B + bC$ ,

$$(35) \quad \begin{aligned} v_1A = D, \quad v_2A = -E - cC; \quad v_1B = E - bC, \quad v_2B = D; \\ v_1C = -B, \quad v_2C = A. \end{aligned}$$

The integrability condition of (35<sub>5</sub>) and (35<sub>6</sub>) is

$$(36) \quad v_3C = 2D.$$

The integrability conditions of (35<sub>1</sub>) + (35<sub>2</sub>), (35<sub>3</sub>) + (35<sub>4</sub>), (35<sub>5</sub>) + (36) and (35<sub>6</sub>) + (36) are

$$\begin{aligned} v_3A + v_1E + v_2D = cB, \quad v_3B - v_1D + v_2E = bA, \\ 2v_1D + v_3B = bA, \quad v_3A - 2v_2D = cB. \end{aligned}$$

For  $3F := v_1E$ ,  $3G := v_2A$ , we get

$$(37) \quad \begin{aligned} v_3A = -2F + cB; \quad v_3B = -2G + bA; \\ v_1D = G, \quad v_2D = -F; \quad v_1E = 3F, \quad v_2E = 3G. \end{aligned}$$

The integrability conditions of (35<sub>1</sub>) + (37<sub>1</sub>), (35<sub>2</sub>) + (37<sub>1</sub>), (35<sub>3</sub>) + (37<sub>2</sub>), (35<sub>4</sub>) + (37<sub>2</sub>), (37<sub>3</sub>) + (37<sub>4</sub>) and (37<sub>5</sub>) + (37<sub>6</sub>) are

$$\begin{aligned} 2v_1F + v_3D = (b + c)E, \quad 2v_2F - v_3E = 2cD, \\ 2v_1G + v_3E = 2bD, \quad 2v_2G + v_3D = -(b + c)E, \\ v_3D + v_1F + v_2G = 0, \quad v_3E - 3v_1G + 3v_2F = 0. \end{aligned}$$

For  $H := v_1F - bE$ , they are

$$(38) \quad \begin{aligned} v_3D = -2H + (c - b)E; \quad v_3E = 2(b - c)D; \\ v_1F = H + bE, \quad v_2F = \frac{1}{3}(3b + 5c)D; \\ v_1G = \frac{1}{3}(5b + 3c)D, \quad v_2G = H - cE. \end{aligned}$$



The integrability conditions of  $(37_3) + (38_1)$ ,  $(37_4) + (38_1)$ ,  $(37_5) + (38_2)$ ,  $(37_6) + (38_2)$ ,  $(38_3) + (38_4)$  and  $(38_5) + (38_6)$  are

$$(39) \quad 2v_1H + v_3G = (3c - 2b)F, \quad 2v_2H - v_3F = (2c - 3b)G;$$

$$(40) \quad v_3F = -\frac{1}{3}(b + 2c)G; \quad v_3G = -\frac{1}{3}(2b + c)F;$$

$$(41) \quad v_2H = \frac{1}{3}(7c - 5b)G, \quad v_1H = \frac{1}{3}(5c - 7b)F.$$

Substituting from (40) and (41) into (39), we get

$$(42) \quad bF = 0, \quad cG = 0.$$

Let  $b \neq 0 \neq c$ , i.e.,  $F = G = 0$ . From  $(38_{4,5})$ ,  $D = 0$ . The equations  $(38_{3,6})$  imply  $(b + c)E = 0$ ;  $b + c = 0$  being impossible because of (34), we have  $E = 0$ . We get  $H = 0$  from  $(38_3)$ . Thus we obtain the completely integrable system

$$(43) \quad \begin{aligned} v_1A &= 0, & v_2A &= -cC, & v_3A &= cB, \\ v_1B &= -bC, & v_2B &= 0, & v_3B &= bA, \\ v_1C &= -B, & v_2C &= A, & v_3C &= 0. \end{aligned}$$

Next, suppose  $b \neq 0, c = 0$  (the case  $b = 0, c \neq 0$  being analogous). Then  $F = 0$  because of  $(42_1)$ . We get  $D = 0$  and  $G = 0$  from  $(38_4)$  and  $(40_1)$  respectively. From  $(38_{1,3})$ ,  $v_3D + v_1F = -H$ ,  $v_3D + 2v_1F = bE$ , i.e.,  $E = H = 0$ . Thus we obtain the system (43) with  $c = 0$ . This proves the second part of (iii) and the Theorem.

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