Czechoslovak Mathematical Journal

Štefan Porubský

On coverings of almost Dedekind domains

Czechoslovak Mathematical Journal, Vol. 26 (1976), No. 1, 145–153

Persistent URL: `http://dml.cz/dmlcz/101381`

Czechoslovak Mathematical Journal, 26 (101) 1976, Praha

ON COVERINGS OF ALMOST DEDEKIND DOMAINS

ŠTEFAN PORUBSKÝ, Bratislava

(Received July 15, 1974)

In [3] the so-called Mycielski's problem concerning the minimal bound of the number of arithmetic sequences in exactly covering systems of arithmetic sequences is presented. An ingenious solution of this problem has been given by ZNÁM in [6]. In [5] this problem is extended to principal ideals domains (PID). The primary purpose of this paper is to present the generalization of this problem to covering systems on almost Dedekind domains. Another generalization of Mycielski's problem can be found in [2].

1. PRELIMINARIES

1.0. Throughout this paper all rings under consideration are *commutative integral domains with identity*. "$\subseteq$" denotes the set-theoretic inclusion, "$\subset$" proper inclusion and $|A|$ is the cardinal of A. Recall that an ideal A of a ring R with $A \subset R$ is called *genuine*. A non-zero genuine ideal is called *proper*.

1.1. Lemma. *For any two ideals A, B of a ring R and $a, b \in R$, the residue classes $a + A$, $b + B$ intersect if and only if $A + B$ contains the principal ideal $(a - b)$.*

1.2. Definition. An integral domain R with identity is said to be *almost Dedekind* if, given any maximal ideal M of R, the localization R_M is a Dedekind domain.

1.3. Lemma [1; 29.4]. *Let R be an integral domain with identity which is not a field. The following statements about R are equivalent:*

a) *R is almost Dedekind.*

b) *R is one-dimensional and primary ideals of R are prime powers.*

1.4. Let R be a ring with identity and $\{M_\lambda\}_{\lambda \in \Lambda}$ the set of its maximal ideals. Then, given an ideal A of R we have

$$A = \bigcap_{\lambda \in \Lambda} A^{e_\lambda c_\lambda},$$

where e_λ and c_λ denote extension and contraction with respect to the ring of quotients R_{M_λ} [1; 3.10]. On the other hand, if R is a one-dimensional integral domain then every proper prime ideal P_λ is either a minimal prime ideal of A or A is not a subset of P_λ. In the former case $A^{e_\lambda c_\lambda}$ is the isolated P_λ-primary component of A, i.e. it is the unique minimal P_λ-primary ideal containing A. The latter case yields $A^{e_\lambda c_\lambda} = R$. Therefore, if R is an almost Dedekind domain then 1.3.b gives

$$A = \bigcap_{\lambda \in \Lambda} P^{n_\lambda(A)}, \quad n_\lambda(A) \geqq 0 .$$

The exponents $n_\lambda(A)$ are uniquely determined because in the almost Dedekind domains we have $\bigcap_{n=0}^{\infty} B^n = (0)$ for every genuine ideal B [1; 29.5]. The exponents n_λ just introduced have the following properties:

a) *$A \subseteq B$ if and only if $n_\lambda(A) \geqq n_\lambda(B)$ for every $\lambda \in \Lambda$.*
b) $n_\lambda(A + B) = \min \{n_\lambda(A), n_\lambda(B)\}$.
c) $n_\lambda(A \cap B) = \max \{n_\lambda(A), n_\lambda(B)\}$.

Since e_λ and c_λ preserve the inclusion, part a) is evident. Now, b) is a consequence of a) and the fact that the isolated P_λ-primary component is the minimal P_λ-primary ideal containing the given ideal. As to c), it follows from $(A \cap B)^{e_\lambda c_\lambda} = A^{e_\lambda c_\lambda} \cap B^{e_\lambda c_\lambda}$.

1.5. Lemma. *For any three ideals A, B, C of a ring R with $C \subseteq B \subseteq A + C$ the residue-class rings $(A \cap B)/(A \cap C)$ and B/C are isomorphic.*

Proof. Define the homomorphism f:

$$f : A \cap B \to B/C , \quad x \mapsto x + C .$$

The mapping f is onto if and only if the residue class $x + C$ meets the set $A \cap B$ for every $x \in B$, that is, if and only if $x + C$ and A intersect for every $x \in B$, i.e. if and only if $B \subseteq A + C$. But then $(A \cap B)/\ker f$ and B/C are isomorphic and our lemma follows.

1.6. Definitions. Let f be a function defined on a ring R. A function f is said to be *periodic* if there is a non-zero ideal A in R such that

$$x - y \in A \quad \text{implies} \quad f(x) = f(y) .$$

If the role of the ideal A is essential in our consideration we say that f is *periodic over A*. Further, f is said to be *A-periodic* if f is periodic over A and there is no ideal $B \supset A$ such that f is periodic over B.

1.7. Lemma. *A function f periodic over an ideal A is B-periodic for a uniquely determined ideal $B \supseteq A$.*

Proof. A standard application of Zorn's lemma to the system $\mathcal{S}$ of the ideals over which f is periodic gives that $\mathcal{S}$ contains at least one maximal ideal. On the other hand, $\mathcal{S}$ is closed under the sum of ideals and therefore $\mathcal{S}$ contains only one maximal element and the conclusion of Lemma follows.

1.8. Lemma. *Let A, B be two genuine ideals in an almost Dedekind domain which have no genuine prime overideal in common. Then A and B are comaximal.*

Proof. Two ideals are comaximal if their radicals are comaximal. Since the radical $r(A)$ of any A equals the intersection of the minimal prime ideals of A, and A and B have no common genuine prime overideal, $r(A)$ and $r(B)$ are intersections of disjoint sets of prime ideals. Then we have

$$n_\lambda(r(A) + r(B)) = \min \{n_\lambda(r(A)), n_\lambda(r(B))\} = 0 ,$$

that is

$$r(A) + r(B) = \bigcap_{\lambda \in \Lambda} P_\lambda^0 = R .$$

2. COVERING SYSTEMS

2.0. Definitions. Let A_i be proper ideals in a ring R, $a_i \in R$ for $i \in I$. A system of residue classes

$$a_i + A_i , \quad i \in I \tag{1}$$

is said to be *covering* (on R) if

$$\bigcup_{i \in I} (a_i + A_i) = R .$$

Covering system (1) is called *exactly covering* (on R) if the covering is disjoint, that is if

$$i \neq j \quad \text{implies} \quad (a_i + A_i) \cap (a_j + A_j) = \emptyset .$$

Covering system (1) is called *reduced* if none of its residue classes can be removed without violating the covering property of (1), i.e. if

$$\bigcup_{i \neq j} (a_i + A_i) \subset R$$

for every $j \in I$.

2.1. Let (1) be a system of residue classes (not necessarily covering) in a ring R. Define the function $m(x)$ in the following manner:

$$m(x) = \left|\{i \in I : x \in a_i + A_i\}\right| \quad \text{for} \quad x \in R .$$

The function $m(x)$ will be called *covering function* of system (1). It can happen that the function m is periodic. Then m is A^*-periodic for a uniquely determined ideal A^*. This ideal A^* will be called the *ideal of periodicity* of system (1).

The concept of the ideal of periodicity generalizes that of the period of a system of congruences defined in [4].

If the covering function of system (1) is not periodic then $\bigcap_{i \in I} A_i = (0)$. On the other hand, $\bigcap_{i \in I} A_i$ is non-zero if I is a finite set.

The covering function of a covering system need not be periodic in general as is shown by the following example on the ring of integers

$$1 + (2), \quad 0 + (p),$$

where p runs over all rational primes.

3. COVERING OF ALMOST DEDEKIND DOMAINS

3.0. Henceforth we shall suppose R to be an almost Dedekind domain and $\{P_\lambda\}_{\lambda \in \Lambda}$ to be the set of all its distinct proper prime ideals.

3.1. Let (1) be a reduced covering system on R possessing a periodic covering function m and let A^* be its ideal of periodicity. Fix an arbitrary $i_0 \in I$. Without loss of generality we may suppose $m(a_{i_0}) = 1$. Then

$$(a_i + A_i) \cap (a_{i_0} + A_{i_0} \cap A^*) = \emptyset \tag{2}$$

for every $i \neq i_0$ in I. Put

$$n_\lambda(A_{i_0} \cap A^*) = \alpha_\lambda \quad \text{for} \quad \lambda \in \Lambda,$$

i.e., $P_\lambda^{\alpha_\lambda}$ is the isolated P_λ-primary component of $A_{i_0} \cap A^*$. If

$$n_\lambda(A_{i_0}) = \beta_\lambda \quad \text{for} \quad \lambda \in \Lambda$$

then 1.4.b yields $\beta_\lambda \leqq \alpha_\lambda$ for each $\lambda \in \Lambda$. Finally put

$$Q_\lambda = \bigcap_{\substack{\nu \neq \lambda \\ \nu \in \Lambda}} P_\nu^{\alpha_\nu}$$

for each $\lambda \in \Lambda$.

3.2. Lemma. *Let $\lambda \in \Lambda$ and let $0 \leqq \gamma < \alpha_\lambda$ be an integer with*

$$(a_{i_0} + Q_\lambda \cap P_\lambda^\gamma) \cap (a_i + A_i) \neq \emptyset \tag{3}$$

for $i \in I$ and $i \neq i_0$. Then $n_\lambda(A_i) > \gamma$.

Proof. 1.1 and (3) imply

$$(a_{i_0} - a_i) \subseteq Q_\lambda \cap P_\lambda^\gamma + A_i .$$

On the other hand, (2) yields

$$(a_{i_0} - a_i) \nsubseteq Q_\lambda \cap P_\lambda^{\alpha_\lambda} + A_i .$$

If $0 \leqq n_\lambda(A_i) \leqq \gamma$ $(<\alpha_\lambda)$ then $n_\lambda(Q_\lambda \cap P_\lambda^\gamma + A_i) = n_\lambda(A_i) = n_\lambda(Q_\lambda \cap P_\lambda^{\alpha_\lambda} + A_i)$. Moreover, we have $n_\nu(Q_\lambda \cap P_\lambda^\gamma + A_i) = \min\{n_\nu(Q_\lambda \cap P_\lambda^\gamma), n_\nu(A_i)\}$ for $\nu \neq \lambda$. But $n_\nu(Q_\lambda \cap P_\lambda^\gamma) = n_\nu(Q_\lambda)$ because of 1.4.c, i.e. $Q_\lambda \cap P_\lambda^\gamma + A_i = Q_\lambda \cap P_\lambda^{\alpha_\lambda} + A_i$ which is in contradiction with the above inclusions.

3.3. According to 1.3.b,

$$P_\lambda \supset P_\lambda^2 \supset P_\lambda^3 \supset \ldots \supset P_\lambda^{\alpha_\lambda - 1}$$

is the set of P_λ-primary ideals lying over $P_\lambda^{\alpha_\lambda}$ in R for each $\lambda \in \Lambda$.

Let $Y_{\lambda,\delta}$ denote a fixed set of the representatives of all distinct non-zero residue classes of the residue-class ring $(Q_\lambda \cap P_\lambda^\delta)/(Q_\lambda \cap P_\lambda^{\delta+1})$, where δ is a non-negative integer and $\lambda \in \Lambda$ $(P_\lambda^0 = R)$. Thus if $y_{\lambda,\delta} \in Y_{\lambda,\delta}$ then $y_{\lambda,\delta} \in Q_\lambda \cap P_\lambda^\delta$ but $y_{\lambda,\delta} \notin$ $\notin Q_\lambda \cap P_\lambda^{\delta+1}$.

3.4. Lemma. *No pair of elements of the form*

$$a_{i_0} + y_{\lambda,\delta}$$

with $y_{\lambda,\delta}$ ranging over $Y_{\lambda,\delta}$ for $\delta = 0, 1, \ldots, \alpha_\lambda - 1$ and $\lambda \in \Lambda$ belongs to the same residue class of the system

$$a_i + A_i\,, \quad i \neq i_0\,, \quad i \in I\,.$$

Proof. Distinguish the following two cases:

I.
$$a_{i_0} + y_{\lambda,\delta} \in a_j + A_j\,,$$
$$a_{i_0} + x_{\lambda,\delta'} \in a_j + A_j\,,$$

where $j \neq i_0$, $\delta \leqq \delta'$ and $y_{\lambda,\delta} \in Y_{\lambda,\delta}$, $x_{\lambda,\delta'} \in Y_{\lambda,\delta'}$. By subtraction we get

$$y_{\lambda,\delta} - x_{\lambda,\delta'} = m\,, \quad m \in A_j\,.$$

Lemma 3.2 gives $n_\lambda(A_j) > \delta'$, i.e. $A_j \subseteq P_\lambda^{n_\lambda(A_j)} \subset P_\lambda^{\delta'}$, thus $m \in P_\lambda^{n_\lambda(A_j)}$ and also $m \in P_\lambda^{\delta'}$. On the other hand, since $x_{\lambda,\delta'} \in Q_\lambda \cap P_\lambda^{\delta'}$ and $x_{\lambda,\delta'} \notin Q_\lambda \cap P_\lambda^{\delta'+1}$, it follows $x_{\lambda,\delta'} \in P_\lambda^{\delta'}$ and $x_{\lambda,\delta'} \notin P_\lambda^{\delta'+1}$. Similarly, $y_{\lambda,\delta} \in P_\lambda^\delta$ and $y_{\lambda,\delta} \notin P_\lambda^{\delta+1}$.

Let $\delta < \delta'$. Then $y_{\lambda,\delta} \notin P_\lambda^{\delta'}$ and therefore $y_{\lambda,\delta} - x_{\lambda,\delta'} \notin P_\lambda^{\delta'}$. This contradicts the facts that $m = y_{\lambda,\delta} - x_{\lambda,\delta'}$ and $m \in P_\lambda^{\delta'}$.

Let $\delta = \delta'$. Since $y_{\lambda,\delta} + P_\lambda^{\delta+1} \neq x_{\lambda,\delta} + P_\lambda^{\delta+1}$ ($x_{\lambda,\delta}$ and $y_{\lambda,\delta}$ are distinct elements from $Y_{\lambda,\delta}$), it follows that $y_{\lambda,\delta} - x_{\lambda,\delta} \notin P_\lambda^{\delta+1}$. But $n_\lambda(A_j) \geqq \delta + 1$, that is $m \in P_\lambda^{n_\lambda(A_j)} \subseteq P_\lambda^{\delta+1}$ and the contradiction is evident.

II.
$$a_{i_0} + y_{\lambda,\delta} \in a_j + A_j\,,$$
$$a_{i_0} + x_{\nu,\delta'} \in a_j + A_j$$

with $j \neq i_0$ and $\lambda \neq \nu$. Again

$$y_{\lambda,\delta} - x_{\nu,\delta'} = m\,, \quad m \in A_j\,. \tag{4}$$

Similarly as in the previous case $m \in P_\nu^{n_\nu(A_j)} \subset P_\nu^{\delta'}$. Since $x_{\nu,\delta'} \in P_\nu^{\delta'}$ and $x_{\nu,\delta'} \notin P_\nu^{\delta'+1}$, we have $x_{\nu,\delta'} \notin P_\nu^{n_\nu(A_j)}$. On the other hand, $y_{\lambda,\delta} \in Q_\lambda \subseteq P_\lambda^{\alpha_\nu}$ whenever $\nu \neq \lambda$. If $P_\nu^{\alpha_\nu} \subseteq P_\nu^{n_\nu(A_j)}$ then we immediately get a contradiction with (4). In case $P_\nu^{n_\nu(A_j)} \subset P_\nu^{\alpha_\nu}$ both m and $y_{\lambda,\delta}$ are in $P_\nu^{\alpha_\nu}$. But $x_{\nu,\delta'} \notin P_\nu^{\delta'+1}$ and therefore $x_{\nu,\delta'} \notin P_\nu^{\alpha_\nu}$ ($\delta' \leqq \alpha_\nu - 1$), again a contradiction.

3.5. If $x \in Y_{\lambda,\delta}$ for $\delta = 0, 1, \ldots, \beta_\lambda - 1$ then $x \notin P_\lambda^{\beta_\lambda}$ and therefore $x \notin A_{i_0}$, i.e. none of the elements of the set

$$a_{i_0} + Y_{\lambda,\delta} = \{a_{i_0} + x : x \in Y_{\lambda,\delta}\} \tag{5}$$

with $\lambda \in \Lambda$ and $\delta = 0, 1, \ldots, \beta_\lambda - 1$ belongs to the class $a_{i_0} + A_{i_0}$. But it is evident that the elements of (5) with $\delta \geqq \beta_\lambda$ belong to $a_{i_0} + A_{i_0}$. This fact will be of importance in our further results and therefore we define

$$m_{i_0}(x) = \left|\{i \in I - i_0 : x \in a_i + A_i\}\right| \quad \text{for} \quad x \in R\,.$$

For instance, if $A^* = A_{i_0}$ or $A^* = R$ then $m(x) = m_{i_0}(x)$ for all elements x in sets (5) with $\delta = 0, 1, \ldots, \beta_\lambda - 1, \lambda \in \Lambda$.

3.6. Theorem. *Suppose R is an almost Dedekind domain and $\{P_\lambda\}_{\lambda\in\Lambda}$ is the set of all proper prime ideals in R. Let (1) be a reduced covering system on R possessing a periodic covering function m. Given any $i_0 \in I$ choose a_{i_0} in such a way that $m(a_{i_0}) = 1$. Then*

$$|I| \geqq 1 + \sum_{\lambda\in\Lambda'} \sum_{\delta=0}^{\alpha_\lambda-1} \sum_{x\in Y_{\lambda,\delta}} m_{i_0}(a_{i_0} + x)\,, \tag{6}$$

where $\Lambda' = \{\lambda \in \Lambda : \alpha_\lambda \neq 0\}$ while the other symbols are defined as in the preceding paragraphs.

The proof follows immediately from 3.4 and 3.5.

3.7. Corollary. *Under the hypotheses of* 3.6 *it holds*

$$|I| \geqq 1 + \sum_{\lambda\in\Lambda'} \sum_{\delta=0}^{\beta_\lambda-1} \left[|P_\lambda^\delta/P_\lambda^{\delta+1}| - 1\right]. \tag{7}$$

Proof. Because $m_{i_0}(x) \geqq 1$ for $x \in a_{i_0} + Y_{\lambda,\delta}$ with $\delta = 0, 1, \ldots, \beta_\lambda - 1$, $\lambda \in \Lambda'$, we have

$$\sum_{x \in Y_{\lambda,\delta}} m_{i_0}(a_{i_0} + x) \geqq \left|(Q_\lambda \cap P_\lambda^\delta)/(Q_\lambda \cap P_\lambda^{\delta+1})\right| - 1 .$$

Using 1.8 and 1.5 we get

$$\left|(Q_\lambda \cap P_\lambda^\delta)/(Q_\lambda \cap P_\lambda^{\delta+1})\right| = \left|P_\lambda^\delta/P_\lambda^{\delta+1}\right| .$$

3.8. Now we specialize down to the case of some Dedekind domains. In this case the ideal A_{i_0} can be written in the form $A_{i_0} = \prod_{\lambda \in \Lambda} P_\lambda^{\beta_\lambda}$, where only a finite number of β's is non-zero. Since R is Noetherian, every P_λ is finitely generated. Moreover suppose that every P_λ with $\lambda \in \Lambda'$ is the direct sum of principal ideals

$$P_\lambda = (b_{\lambda,1}) \oplus \ldots \oplus (b_{\lambda,t_\lambda}) .$$

Then we have

$$P_\lambda^\delta = \bigoplus_{i=1}^{v(\lambda,\delta)} (c_i) ,$$

where $c_i = \prod_j b_{\lambda,j}^{e_{i,j}}$ with $e_{i,j} \geqq 0$ and $\sum_j e_{i,j} = \delta$, while $v(\lambda, \delta) = \binom{t_\lambda + \delta - 1}{\delta}$ is the number of combinations of t_λ elements taken δ at a time, when repetitions are allowed.

If the elements x_i range indepently over a fixed set of representatives of the residue-class ring R/P_λ then the elements of the form

$$\sum_{i=1}^{v(\lambda,\delta)} x_i c_i$$

with c_i's as above, form a set of all representatives of the residue-class ring $P_\lambda^\delta/P_\lambda^{\delta+1}$. Thus

$$\left|P_\lambda^\delta/P_\lambda^{\delta+1}\right| = v(\lambda, \delta) . \left|R/P_\lambda\right|$$

and therefore

$$|I| \geqq 1 + \sum_{\lambda \in \Lambda'} \sum_{\gamma=0}^{\beta_\lambda - 1} \left[\binom{t_\lambda + \delta - 1}{\delta} . \left|R/P_\lambda\right| - 1\right]$$

in this case. In particular, if R is a PID then

$$|I| \geqq 1 + \sum_{\lambda \in \Lambda'} \beta_\lambda(\left|R/P_\lambda\right| - 1) .$$

In the case of the ring of integers this inequality was proved in [7].

Since the modification of all obtained results is obvious if some of the appearing cardinals are infinite, it does not seem necessary to append explicitly their formal rewritting.

3.9. Consider the following reduced covering system on the ring of integers

$$0 + (2)\,,\quad 0 + (3)\,,\quad 1 + (4)\,,\quad 5 + (6)\,,\quad 7 + (12)$$

in order to show that the estimates (6) and (7) are different in general. The estimate (6) yields different results depending on the choice of representatives in $Y_{\lambda,\delta}$ and on the choice of a_{i_0} for which $m(a_{i_0}) = 1$. In the system above we have $A^* = (12) = (2)^2 \,.\, (3)$. Consider the class $0 + (2)$. If $Y_{1,0} = \{9\}$, $Y_{1,1} = \{6\}$, $Y_{2,0} = \{4, 8\}$ and $a_{i_0} = 8$ or 2, then the right-hand side of (6) is 4 or 3, respectively. On the other hand, the right-hand side of (7) is 2 in this case.

3.10. Let us turn to exactly covering systems. In this case $A^* = R$ and $m(x) = 1$ for every $x \in R$, and therefore the right-hand sides of (6) and (7) yield the same result. In the case of PID's and the ring of integers, this common estimate was proved for exactly covering systems in [5] and [6], respectively. In these papers this estimate is shown to be the best possible. We show that the estimate (6) or (7) is the best possible for exactly covering systems also in our case. More precisely, we show (similarly as in [5] and [6]) that any residue class $a_{i_0} + A_{i_0}$ is contained in such an exactly covering system for which the bound (7) is attained.

To this purpose, suppose that Λ' is well-ordered. Using the transfinite induction we can construct a decreasing chain of ideals consisting of the subchains

$$\bigcap_{\nu<\lambda} P_\nu^{\alpha_\nu} = A^{(\lambda)} \supset A^{(\lambda)} \cap P_\lambda \supset A^{(\lambda)} \cap P_\lambda^2 \supset \ldots \supset A^{(\lambda)} \cap P_\lambda^{\alpha_\lambda}$$

for every $\lambda \in \Lambda'$, where $A^{(\lambda')} = R$ for the first element λ' of Λ'. This chain terminates in $\bigcap_{\lambda\in\Lambda'} P_\lambda^{\alpha_\lambda} = \bigcap_{\lambda\in\Lambda} P_\lambda^{\alpha_\lambda} = A_{i_0}$. If $D_1 \supset D_2$ are two adjacent terms in this chain then after replacing each D_1 by the non-zero residue classes $x + D_2$ of D_1/D_2 we get an exactly covering system on R from this chain. But then the translation $x \mapsto x + a_{i_0}$ gives the required exactly covering system because

$$\left|(A^{(\lambda)} \cap P_\lambda^\delta)/(A^{(\lambda)} \cap P_\lambda^{\delta+1})\right| = \left|P_\lambda^\delta/P_\lambda^{\delta+1}\right|$$

according to 1.5 and 1.8.

3.11. Problem. *Does an analogue of Theorem* 3.6 *hold also for reduced covering systems not possessing a periodic covering function?*

References

[1] *Gilmer, R.:* Multiplicative ideal theory I, II. Queen's Papers on Pure and Applied Math. No. *12*, Ontario 1968.

[2] *Korec, I.:* On a generalization of Mycielski's and Znám's conjectures about coset decomposition of Abelian groups. Fund. Math. *85*, 41—48 (1974).

[3] *Mycielski, J., Sierpinski, W.:* Sur une propriété des ensembles linéaires. Fund. Math. *58*, 143—147 (1966).

[4] *Porubský, Š.:* Covering systems and generating functions. Acta Arith. *26*, 223—231 (1975).

[5] *Porubský, Š., Znám, Š.:* On covering of rings by their residue classes. Acta Fac. Rerum Natur. Univ. Comenian. Math. *27*, 45—51 (1972).

[6] *Znám, Š.:* On Mycielski's problem on system of arithmetical progressions. Colloq. Math. *25*, 201—204 (1966).

[7] *Znám, Š.:* On properties of systems of arithmetic sequences. Acta Arith. *26*, 279—283 (1975).

Author's address: 886 25 Bratislava, ul. Obrancov mieru 49, ČSSR (Matematický ústav SAV).