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NEWS AND NOTICES

AWARD OF THE NATIONAL PRIZE TO M. FIEDLER AND V. PTÁK

On 21st April, 1978 Professor MIROSLAV FIEDLER, Dr.Sc. and Professor VLASTIMIL PTÁK, Dr.Sc. of the Mathematical Institute of the Czechoslovak Academy of Sciences received the National Prize from the hands of the President of the Czech National Council. A number of these prizes is awarded each year in recognition of outstanding achievements, and they received the award jointly for a collection of papers on matrix theory.

Both studied mathematics at Charles University, Prague, on its re-opening after the Second World War, and both have been members of the Mathematical Institute, where they are active to this day, ever since its founding. Professor Fiedler is now Head of the Department of Numerical Methods, Graph Theory and Mathematical Logic, while Professor Pták is Head of the Department of Functional Analysis.

Professor Fiedler began to publish in the early fifties. His earliest articles are devoted to algebraic curves and the geometry of simplexes, and he has several times returned subsequently to the latter topic. The main concern of his investigations from the second half of the fifties was numerical methods for the solution of algebraic equations and systems of linear equations and he is still an expert in the field of numerical methods. At the end of the fifties his first papers on graph theory appeared; at that time he was principally inspired by applications to economics. For the past twenty years his main field of activity has been matrix theory, in which he is one of the world's foremost authorities. Around sixty out of his over eigthy publications to date are devoted to sundry aspects of linear algebra. He combines algebraic, combinatorial and geometrical ideas and generally obtains results by an ingenious exploitation of relatively elementary methods. His is mathematics carried out at a high level, but drawing its inspiration from and finding application in practicality.

Professor Pták's main interest is functional analysis. His first papers, on semigroups, came out around 1950. There followed a series of articles on topological vector spaces, and since the middle of the fifties he has contributed to the most diverse branches of functional analysis and neighbouring disciplines (topology, matrix theory and analysis). So far he has published over a hundred papers. In them are contained results of fundamental significance in their respective areas (for example, hermitian algebras, critical exponents of operators, topological vector spaces and closed graph theorems), and in recognition of some of these he was awarded the State Prize of Klement Gottwald in 1966. His work is characterized by sound and natural motivation in the choice of questions to study and an elegant exposition pointing to far-reaching consequences.

For many years he has conducted a regular seminar which has become a well known institution to both Czechoslovak and foreign mathematicians. An outgrowth of the seminar is a regular Summer School, which also often enjoys visits from abroad. Participants of the seminar are familiar with his interest in matrices: as the acme of abstraction is reached, proceedings are liable to be interrupted by a request for the significance in finite dimensions to be explained.

The intersection of the interests of Professors Fiedler and Pták is evidently the theory of matrices. They have long shared a study, and it is not surprising that this has led to a synthesis of the discrete, geometrical, algebraic and functional analytic approaches to their common interest, and resulted in 16 joint papers. Outstanding results from these papers are to be found in the main monographs on the subject which have appeared in recent years (Faddejev-Faddejeva, Gantmacher, Householder, Marcus-Minc, Seneta, Todd, Varga) and are cited in hundreds of articles in learned journals.

Let us briefly sketch the contribution made in the collection of papers for which the prize was awarded, even though it is hard to separate it from the context of the rest of the authors' work. Professor Fiedler in particular has further investigated some of the matters in a number of independent papers.

Let us begin with papers which are mainly concerned with numerical mathematics. In [4] there occurs the first ever investigation of the effect of splitting a symmetric matrix on the rate of convergence of the appropriate iterative method of Gauss-Seidel type. In [5] it is shown how to construct, for a given symmetric matrix A, a sequence of unitary matrices U_k such that $U_k A U_k^*$ converges quadratically (under certain assumptions) to a diagonal matrix. Thus, in effect, an iterative method for computing the spectrum of a symmetric matrix is proposed. [10] also has numerical applications: essentially it deals with the reduction of the inversion of large matrices to that of smaller ones. The main application is the inversion of ill-conditioned Leontief matrices. Most of the other papers have some implications for numerical mathematics.

Various questions (the location of eigenvalues, the convergence of iterative methods and the analysis of electric circuits) lead to the investigation of matrices with non-positive off-diagonal entries (Fan, Koteljanskij, Ostrowski). In [7], which is among the most frequently cited of the whole collection, earlier results were elaborated and important new results deduced, and their significance for spectral theory, the convergence of iterative processes in linear algebra and matrix inequalities demonstrated. The class K introduced here (that is, the class of real square matrices with non-positive off-diagonal entries and with positive principal minors) was further studied in [13], [14], [15] and [16]. [13] contains quantitative improvements of some of the results of [7], particularly relating to applications to spectral questions and also investigates connections with theorems on the convergence of relaxation methods. The results of [4], subsequently extended by R. S. Varga to unsymmetric matrices, are here generalized to the wider class of processes of Gauss-Seidel type. [16] was inspired by the theorem of Koteljanskij on estimates of the determinant of a matrix in terms of the determinant of a majorizing matrix of the class K. The theorem is proved in a natural way and substantially strengthend. One can observe certain resemblances between properties of positive definite matrices and matrices of the class K. On the other hand, the two classes are so different that it seems as though any similarity must be accidental. In [14] the relationship is explained: the cause is to be found in properties of bilinear forms. Another class having connections with the class K is that of diagonally dominant matrices. [15] continues Ostrowski's studies of classes of matrices which are diagonally similar to matrices with a dominant diagonal, and characterizes the class of matrices which are diagonally similar to a given matrix having a dominant-diagonal in a weaker sense.

Criteria of regularity for a square matrix are found in a further series of papers. When applied to the matrix $A - \lambda I$, these yield regions containing eigenvalues of A. Thus, for example, the classical Hadamard theorem on the regularity of diagonally dominant matrices gives rise to the classical Gershgorin circles.

There is an extensive literature on these questions and criteria known before the publication of [6] generally involved the values of entries of a matrix. Much more general estimates, in terms only of the norm of the off-diagonal part of a matrix, were established in [6]. The results are valid for a wide class of norms, including the commonest ones. Criteria of a different type are obtained in [8]. A finite-dimensional space is decomposed into the direct sum of subspaces on each of which some norm is given. These induce operator norms on the blocks of a matrix partitioned according to this decomposition. The criterion asserts that a matrix is regular provided that a certain matrix formed out of these norms belongs to K. By making various choices of direct decomposition and norms we obtain a profusion of special criteria of regularity and corresponding localization for eigenvalues theorems, including previously known results (Gershgorin, Ostrowski, Fan, Brauer). [1] is also devoted to this question. In [2], [11] and [12] the concrete estimates established in [8] are improved yet further. The first two of these also develop the following idea: if a square matrix A is partitioned into blocks

$$A = \begin{pmatrix} \alpha & a \\ b & B \end{pmatrix}$$

where α is a number, it is to be expected that, as long as the vectors a and b are small, there will be an eigenvalue of A near α. In [12] the problem is generalized: the matrix is partitioned into 2×2 block form and spectrum of the matrix is estimated in terms of the spectra of the diagonal blocks. On the basis of this idea an iterative procedure for the calculation of eigenvalues is proposed.

Some properties of matrices depend only on the distribution of zero and non-zero entries and can therefore be studied by purely combinatorial methods. In [3] the combinatorial structure of powers of a matrix is used to infer from the classical Perron-Frobenius theorem on the spectral radius of a non-negative matrix further information about the eigenvalues of a non-negative matrix in relation to its imprimitivity index. The Perron-Frobenius theorem also inspired [17]. According to [3] a consequence of this theorem is that a doubly stochastic matrix has -1 as an eigenvalue only if it has an even imprimitivity index; that is, only when it has block diagonal form (up to the same permutation of rows and columns). In [17] an estimate is found for the distance from -1 to the remaining eigenvalues of a doubly stochastic matrix which does not enjoy this property in terms of its distance from the matrices which do enjoy it and a measure of its irreducibility. A combinatorial approach is often to be found in other papers mentioned here. The notion of distance from a space of matrices also occurs in [9], in the study of the approximations of a finite-dimensional space by singular transformations. It is an interesting fact that the distance of a matrix A from the set of matrices of rank at most r is equal to the $(r + 1)$-th (according to the magnitude) eigenvalue of AA^*.

The last two papers contribute to the theory of cones in finite-dimensional spaces. In [18] a natural notion of diagonal of a convex set is introduced, and diagonals of polyhedral cones are investigated, particularly in relation to the linear dependence of extremal rays. The results obtained here are applied to the study of cones of linear operators in [19]. The main result is that in a finite-dimensional space an extremal operator can have any rank r other than $r = 2$.

We warmly congratulate Professors Fiedler and Pták on their distinction and look forward to further fruits of their collaboration.

Pavla Vrbová and *Antonín Vrba,* Praha

LIST OF PRIZE-WINNING PAPERS

[1] *M. Fiedler:* Some estimates of spectra of matrices, Symp. PICC, Roma (1960), 33—36.

[2] *M. Fiedler:* Some estimates of the proper values of matrices, J. SIAM, *13* (1965), 1—5.

[3] *V. Pták:* Об одной комбинаторной теореме и ее применении к неотрицательным матрицам (On a combinatorial theorem and its application to non-negative matrices), Czech. Math. J. *83* (1958), 487—495.

[4] *M. Fiedler, V. Pták:* Über die Konvergenz des verallgemeinerten Seidelschen Verfahrens zur Lösung von Systemen linearer Gleichungen, Math. Nachr. *15* (1956), 31—38.

[5] *M. Fiedler, V. Pták:* O jedné iterační metodě diagonalisace symetrických matic (On an iterative method of diagonalizing symmetric matrices), Čas. pěst. mat. *85* (1960), 18—36.

[6] *M. Fiedler, V. Pták:* Some inequalities for the spectrum of a matrix, Mat.-fyz. čas. SAV *10* (1960), 148—166.
[7] *M. Fiedler, V. Pták:* On matrices with non-positive off-diagonal elements and positive principal minors, Czech. Math. J. *12* (1962), 382—400.
[8] *M. Fiedler, V. Pták:* Generalized norms of matrices and the location of the spectrum, Czech. Math. J. *12* (1962), 558—571.
[9] *M. Fiedler, V. Pták:* Sur la meilleure approximation des transformations linéaires par des transformations de rang préscrit, C. R. Acad. Sci. *254* (1962), 3805—3807.
[10] *M. Fiedler, V. Pták:* On aggregation in matrix theory and its application to numerical inverting of large matrices, Bull. Acad. Polon. *11* (1963), 757—759.
[11] *M. Fiedler, V. Pták:* Оценки и итерационные методы для нахождения простого собственного числа почти разложимой матрицы (Estimates and iterative methods for finding a simple eigenvalue of an almost decomposable matrix), DAN SSSR *151* (1963), 790—793.
[12] *M. Fiedler, V. Pták:* Estimates and iteration procedures for proper values of almost decomposable matrices, Czech. Math. J. *14* (1964), 593—608.
[13] *M. Fiedler, V. Pták:* Some results on matrices of class K and their application to the convergence rate of iteration procedures, Czech. Math. J. *16* (1966), 260—273.
[14] *M. Fiedler, V. Pták:* Some generalizations of positive definiteness and monotonicity, Num. Math. *9* (1966), 163—172.
[15] *M. Fiedler, V. Pták:* Diagonally dominant matrices, Czech. Math. J. *17* (1967), 420—433.
[16] *M. Fiedler, V. Pták:* Cyclic products and an inequality for determinants, Czech. Math. J. *19* (1969), 428—451.
[17] *M. Fiedler, V. Pták:* A quantitative extension of the Perron-Frobenius theorem for doubly stochastic matrices, Czech. Math. J. *25* (1975), 339—353.
[18] *M. Fiedler, V. Pták:* Diagonals of convex sets, Czech. Math. J. *28* (1978), 25—44.
[19] *M. Fiedler, V. Pták:* The rank of extreme positive operators on polyhedral cones, Czech. Math. J. *28* (1978), 45—55.

SIXTIETH ANNIVERSARY OF BIRTHDAY OF PROFESSOR KAREL SVOBODA

RNDr. Karel Svoboda, Professor of Mathematics at the Purkyně University at Brno, reached sixty years of age on Dezember 9, 1978. The scientific interest of Professor Svoboda has been concentrated on Algebraic Geometry.

A more detailed analysis of scientific and educational activity of Professor Svoboda may be found in the article „Šedesát let prof. RNDr. Karla Svobody" by A. Švec, Čas. pěst. mat. 103 (1978), 421—423.

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