Motupalli Satyanarayana

Correction to my paper on structure and ideal theory of commutative semigroups

_Czechoslovak Mathematical Journal_, Vol. 29 (1979), No. 4, 662–663

Persistent URL: [http://dml.cz/dmlcz/101645](http://dml.cz/dmlcz/101645)

**Terms of use:**

© Institute of Mathematics AS CR, 1979

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these _Terms of use._

This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project _DML-CZ: The Czech Digital Mathematics Library_ [http://dml.cz](http://dml.cz)
CORRECTION TO MY PAPER ON
STRUCTURE AND IDEAL THEORY OF COMMUTATIVE SEMIGROUPS

M. SATYANARAYANA, Bowling Green

(Received March 1, 1979)

Professor B. PONDEŠIČEK has kindly drawn my attention to some errors in my paper [1]. These errors have crept in because of the neglect of verifying the theorems in trivial cases. The following are the corrected versions of Theorems 1.6 and 1.7 in [1]. Here we number them in the same way for ready reference.

**Theorem 1.6.** Let $S$ be a U-semigroup which is not a group. If $P$ is the union of all its proper prime ideals, then $P \neq \emptyset$ and $S = x \cup xS$ for every $x \in S \setminus P$. The converse holds if $P \neq S$.

**Proof.** If $S$ does not contain any proper prime ideals, then for every $a$ in $S$, $\sqrt{(a \cup aS)} = S$, which implies $S = a \cup aS$ by U-semigroup property. Then $S$ becomes a group, which is not true by an assumption. Thus $P$ is non-empty. Now, if $x \in S \setminus P$, $S = \sqrt{(x \cup xS)}$, which implies $S = x \cup xS$. Conversely let $S$ be not a group and let $A$ be an ideal different from $S$. If $x \in A \setminus P$, then $S = x \cup xS$ by assumption and so $A = S$, which is a contradiction. Therefore $A \subseteq P$ we claim now that $\sqrt{(A)} = S$. If possible, let $\sqrt{(A)} = S$. Then if $x \in s$, $x^n \in A$ for some natural number $n$ and so $x^n \in P$, which is a prime ideal and thus $x \in P$. Therefore $S = P$, which is a contradiction. Thus $S$ is a U-semigroup.

**Theorem 1.7.** Let $S$ be a semigroup which is not the union of all its proper prime ideals but contains maximal ideals. Then the following are equivalent:

i) $S = S^2$,

ii) $S$ contains a unique maximal ideal which is prime.

**Proof.** (i) $\Rightarrow$ (ii). Let $T = \{a : \sqrt{(aS^1)} = S\}$. If $T = \emptyset$, then for every $a \in S$, $\sqrt{(aS^1)} = S$ and so $S$ contains no proper prime ideals. But maximal ideals are prime by [2]. Hence this case is inadmissible. If $T \neq S$, then $T$ is the unique maximal ideal. For, let $M$ be any maximal ideal. Since $S = S^2$, $M$ is a prime ideal and so $\sqrt{(M)} = M$. 662
Now if \( a \in M \setminus T \), then \( S = \sqrt{(a \cup aS)} \subseteq \sqrt{(M)} = M \). Thus \( M \subseteq T \) and so \( M = T \). The only other possibility is \( T = S \). Since \( S \) is not the union \( P \) of its prime ideals, we have then for \( x \in S \setminus P \), \( \sqrt{(x \cup xS)} = S \), which is not true since \( T = S \).

(ii) \( \Rightarrow \) (i) follows by Schwartz’s result [2].

References


Author’s address: Bowling Green State University, Department of Mathematics and Statistics, Bowling Green, Ohio 43403, U.S.A.