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ON REPRESENTATIONS OF TOLERANCE ORDERED COMMUTATIVE SEMIGROUPS

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In this paper we shall give an algebraic representation and a categorial representation of tolerance ordered commutative semigroups. This investigation was started by V. Trnková [1] and [2] who considered the representations of non-ordered commutative semigroups. In [3] J. Adámek and V. Koubek studied the representations of ordered commutative semigroups.

By a *tolerance ordered commutative semigroup* $\langle S, +, \leqq, \sim \rangle$ we mean an ordered commutative semigroup $\langle S, +, \leqq \rangle$ on which there exists a *tolerance* (i.e., reflexive and symmetric) relation $\sim$ satisfying the following conditions:

(1) If $x \sim u$ ano $y \sim v$, then $x + y \sim u + v$.

(2) If $x \sim y$, $x \leqq u$ and $y \leqq v$, then $u \sim v$.

Let $\mathscr{S} = \langle S, +, \leqq, \sim \rangle$, $\mathscr{P} = \langle P, +, \precsim, \approx \rangle$ be two tolerance ordered commutative semigroups. A mapping $h : S \to P$ is said to be *an isomorphic mapping of $\mathscr{S}$ into $\mathscr{P}$* if h is an injective homomorphism of the semigroup $\langle S, + \rangle$ into the semigroup $\langle P, + \rangle$ satisfying the following conditions for $x, y \in S$:

(3) $x \leqq y$ if and only if $h(x) \precsim h(y)$;

(4) $x \sim y$ if and only if $h(x) \approx h(y)$.

We shall say that $\mathscr{S}$ is a *tolerance ordered subsemigroup of $\mathscr{P}$* (write $\mathscr{S} \subseteqq \mathscr{P}$) if $S \subseteqq P$ and the embedding of S into P is an isomorphic mapping of $\mathscr{S}$ into $\mathscr{P}$.

Proposition 1. *Let a, b be two elements of a tolerance ordered commutative semigroup $\mathscr{S} = \langle S, +, \leqq, \sim \rangle$ such that $a \sim b$. Then there exists a tolerance ordered commutative semigroup $\mathscr{P} = \langle P, +, \precsim, \approx \rangle$ with $\mathscr{S} \subseteqq \mathscr{P}$ and* card $P = \aleph_0 .$card S *such that $z \precsim a$, $z \precsim b$* for some $z \in P$.

Proof. Let $\mathscr{S} = \langle S, +, \leqq, \sim \rangle$ be a tolerance ordered commutative semigroup and let $a, b \in S$ and $a \sim b$. By N we denote the additive semigroup of non-negative integers. We can suppose that $0 \in N \setminus S$. Put $Z = S \cup \{0\}$ with $x + 0 = x = 0 + x$ for all $x \in Z$. Define $0 \leqq 0$ and $0 \sim 0$ and suppose that there exists no element x of S such that either $0 \leqq x$ or $x \leqq 0$ or $0 \sim x$. It is easy to show that $\langle Z, +, \leqq, \sim \rangle$

is a tolerance ordered commutative semigroup. Put $P = Z \times N$. Evidently, card $P = \aleph_0$. card S.

Define an operation $+$ in $P : (s, m) + (t, n) = (s + t, m + n)$ for $s, t \in Z$ and $m, n \in N$. It is clear that $\langle P, + \rangle$ is a commutative semigroup. For any $s \in S$ we put $\varphi(s) = (s, 0)$. Then φ is an isomorphic mapping of the semigroup $\langle S, + \rangle$ into the semigroup $\langle P, + \rangle$.

Define a relation $\lesssim$ on $P : (s, m) \lesssim (t, n)$ for $s, t \in Z$ and $m, n \in N$ if and only if $m = m_1 + m_2 + n$ and $s + m_1 a + m_2 b \leqq t$ for some $m_1, m_2 \in N$. (Notice that $0x = 0$ and $kx = (k - 1)x + x$ for $x \in Z$ and $k - 1 \in N$.) It is clear that the relation $\lesssim$ is reflexive. We shall show that $\lesssim$ is transitive. Let $s, t, u \in Z$, $m, n, p \in N$, $(s, m) \lesssim (t, n)$ and $(t, n) \lesssim (u, p)$. Then $m = m_1 + m_2 + n$, $s + m_1 a + m_2 b \leqq t$, $n = n_1 + n_2 + p$ and $t + n_1 a + n_2 b \leqq u$ for some $m_1, m_2, n_1, n_2 \in N$. Hence we have $m = (m_1 + n_1) + (m_2 + n_2) + p$, $s + (m_1 + n_1) a + (m_2 + n_2) b \leqq u$ and so $(s, m) \lesssim (u, p)$. Now we shall prove that the relation $\lesssim$ is antisymmetric. Suppose that $(s, m) \lesssim (t, n)$ and $(t, n) \lesssim (s, m)$, where $s, t \in Z$ and $m, n \in N$. Then $m = m_1 + m_2 + n$, $s + m_1 a + m_2 b \leqq t$, $n = n_1 + n_2 + m$ and $t + n_1 a + n_2 b \leqq s$ for some $m_1, m_2, n_1, n_2 \in N$. Hence we have $m_1 = m_2 = n_1 = n_2 = 0$ and so $m = n$, $s = t$. Therefore, $(s, m) = (t, n)$. Finally, we shall show that the order $\lesssim$ is compatible with $+$. Let $(s, m), (t, n), (u, p) \in P$ and $(s, m) \lesssim (t, n)$. Then $m = m_1 + m_2 + n$ and $s + m_1 a + m_2 b \leqq t$ for some $m_1, m_2 \in N$. Hence we have $m + p = m_1 + m_2 + (n + p)$, $(s + u) + m_1 a + m_2 b \leqq t + u$ and so $(s, m) + (u, p) \lesssim (t, n) + (u, p)$. Thus $\langle P, +, \lesssim \rangle$ is an ordered commutative semigroup. It is easy to show that for $s, t \in S$ we have $s \leqq t$ if and only if $\varphi(s) = (s, 0) \lesssim (t, 0) = \varphi(t)$. This implies that φ is an isomorphic mapping of the ordered semigroup $\langle S, +, \leqq \rangle$ into the ordered semigroup $\langle P, +, \lesssim \rangle$.

Define a relation $\approx$ on $P : (s, m) \approx (t, n)$ for $s, t \in Z$ and $m, n \in N$ if and only if there exist $(s_1, p), (t_1\ p) \in P$ such that $(s_1, p) \lesssim (s, m)$, $(t_1, p) \lesssim (t, n)$ and $s_1 \sim t_1$. Clearly, $\approx$ is a tolerance relation on P. We shall show that $\approx$ is compatible with $+$ (i.e., $\approx$ satisfies (1)). Let $(s, m), (t, n), (u, p), (v, r) \in P$ and $(s, m) \approx (t, n)$, $(u, p) \approx (v, r)$. Then there exist $(s_1, k), (t_1, k), (u_1, l), (v_1, l) \in P$ such that $(s_1, k) \lesssim (s, m)$, $(t_1, k) \lesssim (t, n)$, $(u_1, l) \lesssim (u, p)$, $(v_1, l) \lesssim (v, r)$, $s_1 \sim t_1$ and $u_1 \sim v_1$. Hence we have $(s_1 + u_1, k + l) \lesssim (s + u, m + p)$, $(t_1 + v_1, k + l) \lesssim (t + v, n + r)$, $s_1 + u_1 \sim t_1 + v_1$ and so $(s, m) + (u, p) \approx (t, n) + (v, r)$. It is easy to show that the relation $\approx$ satisfies (2) nad so $\langle P, +, \lesssim, \approx \rangle$ is a tolerance ordered commutative semigroup. Now we shall prove that for $s, t \in S$ we have $s \sim t$ if and only if $(s, 0) \approx (t, 0)$. Evidently, $s \sim t$ implies that $(s, 0) \approx (t, 0)$. Suppose that $(s, 0) \approx (t, 0)$. Then there exist $(s_1, k), (t_1, k) \in P$ such that $(s_1, k) \lesssim (s, 0)$, $(t_1, k) \lesssim (t, 0)$ and $s_1 \sim t_1$. This implies that $k = k_1 + k_2 + k_3$ for some $k_1, k_2, k_3 \in N$ such that either

$$x = s_1 + k_1 a + k_2 a + k_3 b \leqq s\,, \quad y = t_1 + k_1 a + k_2 b + k_3 b \leqq t$$

or

$$x = s_1 + k_1 a + k_2 b + k_3 b \leqq s\,, \quad y = t_1 + k_1 a + k_2 a + k_3 b \leqq t\,.$$

Since by hypothesis $a \sim b$, we have $x \sim y$ and so $s \sim t$. Hence φ is an isomorphic mapping of the tolerance ordered semigroup S into the tolerance ordered semigroup P. We put $z = (0, 1)$. It is clear that $z \lesssim (a, 0) = \varphi(a)$ and $z \lesssim (b, 0) = \varphi(b)$. This concludes the proof.

Let $\langle Q, +, \lesssim \rangle$ be an arbitrary ordered commutative semigroup. We can define a compatible tolerance $\approx$ on Q in a natural way. For $x, y \in Q$ we put $x \approx y$ if and only if there exists $z \in Q$ such that $z \lesssim x$ and $z \lesssim y$. Clearly, $\langle Q, +, \lesssim, \approx \rangle$ is a tolerance ordered commutative semigroup. We shall write $\approx \; = \tau(\lesssim)$.

Proposition 2. *For every tolerance ordered commutative semigroup* $\mathscr{S} = \langle S, + \leqq, \sim \rangle$ *there exists a tolerance ordered commutative semigroup* $\mathscr{Q} = \langle Q, +, \lesssim, \tau(\lesssim) \rangle$ *such that* $\mathscr{S} \subseteq \mathscr{Q}$ *and* card $Q = \aleph_0$. card S.

The proof is a simple adaptation of the proof of Theorem 1.3 [3] and proceeds in two steps by iterating Proposition 1.

(I). For $\mathscr{S}$ there exists a tolerance ordered commutative semigroup $\mathscr{S}^* = \langle S^*, +, \lesssim, \approx \rangle$ with $\mathscr{S} \subseteq \mathscr{S}^*$, card $S^* = \aleph_0$. card S and whenever $x \sim y$ in $S(!)$, then exists z in $\mathscr{S}^*$ such that $z \lesssim x$ and $z \lesssim y$.

Proof. By C we denote the set of all couples (x, y) in $\mathscr{S}$ with $x \sim y$ (i.e., $C = \sim$ on S) and we choose a bijective mapping $m : \alpha \to C$, where $\alpha =$ card C. Define a chain of semigroups $\mathscr{S}_i = \langle S_i, +, \leqq, \sim \rangle$, where i is an ordinal $< \alpha$, i.e., $i \in \alpha$. Put $\mathscr{S}_0 = \mathscr{S}$. Given $\mathscr{S}_i$, then according to Proposition 1 there exists a tolerance ordered commutative semigroup $\mathscr{S}_{i+1}$ with respect to the couple $m(i) = (x, y)$ in S such that $\mathscr{S}_i \subseteq \mathscr{S}_{i+1}$, card $S_{i+1} = \aleph_0$. card S_i and $z \leqq x$, $z \leqq y$ for some $z \in S_{i+1}$. Given $\mathscr{S}_j$, $j < i$, for a limit ordinal i, we put $S_i = \bigcup_{j<i} S_j$. This is a tolerance ordered commutative semigroup $\mathscr{S}_i$; $+$, $\leqq$ and $\sim$ are defined in the obvious inductive way. The tolerance ordered commutative semigroup $\mathscr{S}^*$ with $S^* = \bigcup_{i<\alpha} S_i$ satisfies the condition (I).

(II). Using the symbol * as in (I) we define a sequence of tolerance ordered commutative semigroups $\mathscr{Q}_n = \langle Q_n, +, \leqq, \sim \rangle$ such that $\mathscr{Q}_0 = \mathscr{S}$ and $\mathscr{Q}_{n+1} = (\mathscr{Q}_n)^*$ for any $n \in N$. We can prove by an analogous argument as in (I) that $\mathscr{Q} = \langle Q, +, \lesssim, \approx \rangle$ with $Q = \bigcup_{n=0}^{\infty} Q_n$ is a tolerance ordered commutative semigroup, $\mathscr{S} \subseteq \mathscr{Q}$ and card $Q = \aleph_0$. card S. We shall show that $\approx \; = \tau(\lesssim)$. It follows from (2) that $\tau(\lesssim) \subseteq \approx$. Let $x \approx y$ in $\mathscr{Q}$. Then $x \sim y$ in $\mathscr{Q}_n$ for some $n \in N$ and so there exists z in $\mathscr{Q}_{n+1}$ such that $z \leqq x$ and $z \leqq y$. Therefore $x \, \tau(\lesssim) \, y$ in $\mathscr{Q}$ and thus we have $\mathscr{Q} = \langle Q, +, \lesssim, \tau(\lesssim) \rangle$.

Now, we shall prove an algebraic representational result. Let α be an arbitrary cardinal. Denote by N^α the additive semigroup of all functions $f : \alpha \to N$, and by $\exp N^\alpha$ the set of all non-void subsets of N^α. For $A, B \in \exp N^\alpha$ we put $A + B = \{f + g;\ f \in A \text{ and } g \in B\}$. Then $\langle \exp N^\alpha, +, \subseteq, \tau(\subseteq) \rangle = \mathscr{N}_\alpha$ is a tolerance ordered commutative semigroup (via inclusion). It is clear that $A \,\tau(\subseteqq)\, B$ if and only if $A \cap B \neq \emptyset$.

Theorem 1. *($\mathcal{N}_\alpha$ are universal tolerance ordered commutative semigroups.) For every tolerance ordered commutative semigroup $\mathcal{S} = \langle S, +, \leqq, \sim \rangle$ there exists an isomorphic mapping h of $\mathcal{S}$ into $\mathcal{N}_\alpha$, where $\alpha = \aleph_0 \,.\, \operatorname{card} S$.*

Proof. Given $\mathcal{S}$, then according to Proposition 2 there exists a tolerance ordered commutative semigroup $\mathcal{Q} = \langle Q, +, \leqq, \tau(\leqq) \rangle$ such that $\mathcal{S} \subseteqq \mathcal{Q}$. It follows from the theorems of 1.3 [3] that $\langle Q, +, \leqq \rangle$ is an ordered subsemigroup of an ordered semigroup $\langle R, +, \lesssim \rangle$. There exists an injective homomorphism h of $\langle R, + \rangle$ into $\langle \exp N^\alpha, + \rangle$, where $\alpha = \aleph_0 \,.\, \operatorname{card} Q = \aleph_0 \,.\, \operatorname{card} S$, such that $x \lesssim y$, if and only if $h(x) \subseteqq h(y)$ for any $x, y \in R$. If $x \,\tau(\lesssim)\, y$ in R, then there is $z \in R$ such that $z \lesssim x$ and $z \lesssim y$ and so $h(z) \subseteqq h(x)$ and $h(z) \subseteqq h(y)$. Then $h(z) \subseteqq h(x) \cap h(y) \neq \emptyset$ and so $h(x) \,\tau(\subseteqq)\, h(y)$ in $\exp N^\alpha$. Conversely, if $h(x) \cap h(y) \neq \emptyset$ then it follows from the construction of h in the second theorem of 1.3 [3] that there is $z \in R$ such that $h(z) \subseteqq h(x) \cap h(y)$. Then $z \lesssim x$ and $z \lesssim y$. Putting $\mathcal{R} = \langle R, +, \lesssim, \tau(\lesssim) \rangle$ we see that h is an isomorphic mapping of $\mathcal{R}$ into $\mathcal{N}_\alpha$. To prove our theorem, it suffices to show that $\mathcal{S} \subseteqq \mathcal{R}$.

It is clear that $\mathcal{Q} \subseteqq \mathcal{R}$ if and only if $\tau(\lesssim) \cap (Q \times Q) \subseteqq \tau(\leqq)$. By way of contradiction, we assume that there exist $a, b \in Q$ such that $a \,\tau(\lesssim)\, b$ and a non $\tau(\leqq)\, b$. Putting $W = \{w \in R;\ w \lesssim a \text{ and } w \lesssim b\}$ we obtain that

$$(5) \qquad W \neq \emptyset = W \cap Q\,.$$

It follows from part (II) of the first theorem of 1.3 [3] that $R = \bigcup_{n=0}^{\infty} R_n$, where $R_0 = Q$ and $R_n \subseteqq R_{n+1}$ for any $n \in N$. According to (5) there exists $m \in N$ such that

$$(6) \qquad W \cap R_{m+1} \neq \emptyset = R_m \cap W\,.$$

By part (I) of the first theorem of 1.3 [3] we have $R_{m+1} = \bigcup_{i<\alpha} Q_i$ for a certain ordinal α, where $Q_0 = R_m$ and $Q_i \subseteqq Q_j$ for arbitrary ordinals $i \leqq j < \alpha$. It follows from (6) that there exists an ordinal β such that $0 < \beta < \alpha$, $W \cap Q_\beta \neq \emptyset$ and $W \cap Q_i = \emptyset$ for any ordinal $i < \beta$. If β is a limit number, then it follows from (I) of 1.3 [3] that $Q_\beta = \bigcup_{i<\beta} Q_i$ and so $W \cap Q_j \neq \emptyset$ for some $j < \beta$, which is a contradiction. If β is an isolated number, then there exists an ordinal γ such that $\beta = \gamma + 1$. It is clear that $a, b \in Q_\gamma$. Since $W \cap Q_\beta \neq \emptyset$, we have $z \lesssim a$, $z \lesssim b$ for some $z \in Q_\beta$. It follows from (c) of 1.2 [3] that $x \lesssim a$, $x \lesssim b$ for some $x \in Q_\gamma$ and so $W \cap Q_\gamma \neq \emptyset$, which is a contradiction. Consequently, $\mathcal{Q} \subseteqq \mathcal{R}$. Since $\mathcal{S} \subseteqq \mathcal{Q}$, we have $\mathcal{S} \subseteqq \mathcal{R}$.

Note 1. Putting $\sim\ = \tau(\leqq)$ in Theorem 1 we obtain Adámek-Koubek's Theorem (see [3]):

For every ordered commutative semigroup $\mathcal{S} = \langle S, +, \leqq \rangle$ there exists an *injective homomorphism* h *of* $\langle S, +\rangle$ *into* $\langle \exp N^\alpha . +\rangle$ $(\alpha = \aleph_0 . \operatorname{card} S)$ *such that* $x \leqq y$ *if and only if* $h(x) \subseteqq h(y)$ *for all* $x, y \in S$.

By a *tolerance commutative semigroup* $\langle S, +, \sim\rangle$ we mean a commutative semigroup $\langle S, +\rangle$ on which there exists a tolerance relation $\sim$ satisfying the condition (1).

Corollary 1. *For every tolerance commutative semigroup* $\mathscr{S} = \langle S, +, \sim\rangle$ *there exists an injective homomorphism* h *of* $\langle S, +\rangle$ *into* $\langle \exp N^\alpha, +\rangle$ $(\alpha = \aleph_0 . \operatorname{card} S)$ *such that* $x \sim y$ *if and only if* $h(x) \cap h(y) \neq \emptyset$ *for all* $x, y \in S$.

The proof follows from Theorem 1 when we put $\leqq\ = \mathrm{id}_S$.

Note 2. It is clear that $\mathrm{id}_S = \tau(\mathrm{id}_S)$ and so Theorem 1 implies Trnková's Theorem (see [1]):

For every commutative semigroup $\mathscr{S}$ *there exists an injective homomorphism* h *of* $\mathscr{S}$ *into* $\langle \exp N^\alpha, +\rangle$ $(\alpha = \aleph_0 . \operatorname{card} S)$ *such that* $x \neq y$ *if and only if* $h(x) \cap h(y) = \emptyset$.

Finally, we shall show a categorial representation of tolerance ordered commutative semigroups.

Let $\mathscr{K}$ be a category. Denote by $\coprod$ (or $\vee$) the sum and by $\prod$ (or $\times$) the product of objects in $\mathscr{K}$. We write $A \cong B$ if A, B are isomorphic objects. An object A is said to be a summand of an object B if $A \vee X \cong B$ holds for an object X. We shall say that objects A and B have a common nontrivial summand if there exist objects C, X and Y such that $A \cong C \vee X$, $B \cong C \vee Y$ and C is not isomorphic to a sum of the empty collection.

A category $\mathscr{K}$ is said to be *distributive* if it has all sums and finite products and if any collections $\{A_i\}_{i\in I}$ and $\{B_j\}_{j\in J}$ of objects satisfy

$$(\coprod_{i\in I} A_i) \times (\coprod_{j\in J} B_j) \cong \coprod_{(i,j)\in I\times J} A_i \times B_j .$$

(See [2].)

Let A be an object in a distributive category. By A^0 we mean a product of the empty collection. Put $A^{n+1} = A^n \times A$ for any $n \in N$. A collection $\{A_i\}_{i\in I}$ of objects in a distributive category $\mathscr{K}$ is said to be t-independent if the following implication holds.

Let $f_j \in N^I$ $(j \in J)$ and $g_k \in N^I$ $(k \in K)$. If the objects $\coprod_{j\in J} \prod_{i\in I} A_i^{f_j(i)}$, $\coprod_{k\in K} \prod_{i\in I} A_i^{g_k(i)}$ have a common nontrivial summand, then $f_a = g_b$ for some $a \in J$ and some $b \in K$.

Theorem 2. *If a distributive category* $\mathscr{K}$ *with products has arbitrarily large t-independent collections of objects, then for every tolerance ordered commutative semigroup* $\mathscr{S} = \langle S, +, \leqq, \sim\rangle$ *there exists a collection* $\{T_s\}$ $(s \in S)$ *of S-indexed objects in* $\mathscr{K}$ *such that for* $x, y \in S$ *we have*

(i) $T_x \ncong T_y$ *if* $x \neq y$;

(ii) $T_x \times T_y \cong T_{x+y}$;

(iii) T_x *is a summand of* T_y *if and only if* $x \leqq y$;
(iv) T_x, T_y *have a common nontrivial summand if and only if* $x \sim y$.

Proof. Put $\alpha = \aleph_0 \,.\, \operatorname{card} S$. Then there exists a t-independent collection $\{A_i\}_{i \in I}$ of objects in $\mathscr{K}$, where $\alpha \leqq \operatorname{card} I$. It follows from Theorem 1 that there exists an isomorphic mapping of $\mathscr{S}$ into $\mathscr{N}_\alpha$. It is easy to show that there exists an isomorphic mapping of $\mathscr{N}_\alpha$ into $\mathscr{N}_I = \langle \exp N^I, +, \subseteqq, \tau(\subseteqq)\rangle$ and so there exists an isomorphic mapping h of $\mathscr{S}$ into $\mathscr{N}_I$. We can see that every t-independent collection of objects is independent in the sense of [3] and so it follows from Theorem 2.4 [3] that there exists a collection $\{T_s\}$ $(s \in S)$ of S-indexed objects in $\mathscr{K}$ satisfying the conditions (i), (ii), (iii) and

(iv′) if $x \sim y$ for $x, y \in S$, then T_x and T_y have a common nontrivial summand.

To prove our theorem it suffices to show that the following implication holds:

(iv″) If T_x and T_y have a common nontrivial summand, then $x \sim y$ in $\mathscr{S}$.

Suppose that T_x and T_y have a common nontrivial summand. According to the proof of Theorem 2.4 [3] we have

$$T_x = \coprod_{\gamma} \coprod_{f \in X} \prod_{i \in I} A_i^{f(i)}\,, \quad T_y = \coprod_{\gamma} \coprod_{g \in Y} \prod_{i \in I} A_i^{g(i)}\,,$$

where $X = h(x)$, $Y = h(y)$, $\gamma = \operatorname{card} N^I$ and the symbol $\coprod_\gamma A$ means the sum of γ copies of A. Since the collection $\{A_i\}_{i \in I}$ is t-independent, we have $X \cap Y \neq \emptyset$ and so $h(x)\, \tau(\subseteqq)\, h(y)$ in $\mathscr{N}_I$. Hence, by (4), we have $x \sim y$ in $\mathscr{S}$.

Corollary 2. *If a distributive category* $\mathscr{K}$ *with products has arbitrarily large t-independent collections of objects, then for every tolerance commutative semigroup* $\mathscr{S} = \langle S, +, \sim \rangle$ *there exists a collection* $\{T_s\}$ $(s \in S)$ *of* S*-indexed objects in* $\mathscr{K}$ *such that (i), (ii) and (iv) from Theorem 2 hold for* $x, y \in S$.

Note 3. The following categories are distributive with products and have arbitrarily large t-independent collections of objects: completely regular topological spaces, universal algebras with two unary operations (see [2]), posets, symmetric graphs (see [4]) and some others.

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