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NEWS AND NOTICES

KLEMENT GOTTWALD STATE PRIZE '85 AWARDED FOR RESEARCH IN COMBINATORICS

RNDr. Jaroslav Nešetřil, CSc. from the Department of Cybernetics, Computer Science and Operational Research, and RNDr. Vojtěch Rödl, CSc. from the Department of Mathematics, Nuclear and Physics Engineering Faculty of the Czech Technical University, were awarded Klement Gottwald State Prize for results in the theory of combinatorial structures on 30 April 1985.

The State Prizes have been granted since 1951, the year 1985 thus being the 35th year of their existence. Therefore, and also because the '85 laureates are the first laureates in Mathematics born after the year 1945 (J. Nešetřil 1946, V. Rödl 1949), a brief survey may be of interest.

State Prizes for Mathematics were awarded in the years 1951—55, 1959, 1964, 1966, 1968, 1972—74, 1979, 1985. The domains of Mathematics for which the State Prizes were granted cover almost all branches of Mathematics intensively cultivated in Czechoslovakia, including applications. First of the fourteen prizes in Mathematics was bestowed in 1951 upon Academician E. Čech for his lifework especially in topology. The whole lifework was taken into account also in 1952 when Academician V. Jarník received the State Prize for his results in the number theory. The other prizes were granted for recent results: two in algebra, theory of ordinary differential equations, topology, and numerical mathematics; one in combinatorics, functional analysis, differential geometry, and mathematical statistics.

Let us now turn back to the '85 laureates. Having started to publish in the early seventies, they have published more than 100 papers each. Each of them substantially contributed to many fields of combinatorial mathematics, and their results rank them among the most prominent world specialists. The theory of generalized decompositions, the so-called Ramsey theory, is the domain in which their joint import is most highly appreciated, both of them being generally considered co-creators of the theory. It was for the set of 35 research papers from this field that they were granted the State Prize. Their results have already found applications in number theory, computer science, topology and set theory.

In 1930 an English mathematician, F. P. Ramsey, proved an assertion that later substantially affected the development of combinatorial mathematics:

For every positive integers n, p there exists a positive integer $m > n$ such that for an arbitrary decomposition

$$[X]^2 = R_1 \cup R_2 \cup \ldots \cup R_p$$

of all non-ordered pairs of an m-element set X into p parts there is an n-element subset $Y \subseteqq X$ such that $[Y]^2 \subseteqq R_i$ for some $i \leqq p$. If $p = 2$, the theorem asserts that each graph with at least m vertices contains either the complete n-vertex subgraph (K_n) or an n-element set of independent vertices.

Six years later, Hungarian mathematicians P. Erdös and G. Szekeres independently proved Ramsey's theorem and used it to prove the following result from elementary geometry: For every positive integer n there is m such that among arbitrary m points in the plane, of which no three are collinear, there are n points that are vertices of a convex n-gon.

In the sixties and early seventies, the progress of combinatorial mathematics brought along further theorems of Ramsey's type, asserting that sufficiently large structures contain large substructures with higher degree of organization. J. Nešetřil and V. Rödl started with problems of some leading specialists and found answers, using methods which opened new prospects to the theory. They developed several original methods of proofs which are now parts of monographs on combinatorics. In order to avoid the introduction of technially demanding notions, we present several characteristic results that can be formulated in a simple way.

Let us denote by $\mathscr{G}_p^n$ the class of all graphs (V, E) possessing the property that for an arbitrary decomposition of edges $E = E_1 \cup \ldots \cup E_p$ at least one of the graphs (V, E_i) contains K_n.

The Ramsey theorem implies that the classes $\mathscr{G}_p^n$ are nonempty. The solution of the well known problem of P. Erdös and A. Hajnal given by Nešetřil and Rödl substantially generalizes the results of several authors and shows that for every $n \geqq 3$ and $p \geqq 2$ there exist infinitely many minimal (with respect to inclusion) graphs in $\mathscr{G}_p^n$ containing a maximal complete subgraph of size n. This result inspired J. Baumgartner and A. Taylor to introduce the Nešetřil-Rödl order of ultrafilters in βN.

Another well regarded result of Nešetřil and Rödl is the proof of edge-decomposition property of an important class of hypergraphs — the class $S(k, l)$ of all partial Steinerian systems. They proved that for every hypergraph $G \in S(k, l)$ there is $(W, F) \in S(k, l)$ such that for an arbitrary decomposition $F = F_1 \cup F_2$ there is an induced substystem $G' = (V', E') \subseteqq (W, F)$ such that G' is isomorphic with G and $E' \subseteqq F_i$ for some $i \leqq 2$.

Using this result they disproved the generally accepted conjecture that graphs not containing cycles of lengths 3 and 4 do not possess the edge-decomposition property.

The following notion is characteristic for several papers.

Let $\mathscr{K}$ be a class of relation or set systems and let $A \in \mathscr{K}$. The class $\mathscr{K}$ is said to be A — Ramseayan if for each $B \in \mathscr{K}$ and every positive integer p there exists $C \in \mathscr{K}$ such that for an arbitrary decomposition

$$\binom{C}{A} = C_1 \cup \ldots \cup C_p$$

of the class of all subobjects of the object C isomorphic with A there is a subobject $B' \in \binom{C}{B}$ such that $\binom{B'}{A} \subseteqq C_i$ for some $i \leqq p$. For a number of naturally defined classes (e.g. partial ordering, the class of all hypergraphs) Nešetřil and Rödl succeeded in finding characterizations of those objects A for which the classes in question are A-Ramseyan. A class $\mathscr{K}$ is called Ramseyan if it is A-Ramseyan for all $A \in \mathscr{K}$. Very general assumptions on the class of relation systems $\mathscr{K}$ guarantee that the following properties are equivalent:

(a) the class $\mathscr{K}$ is Ramseyan:

(b) there exists such a finite set $\mathscr{A}$ of irreducible relation systems that $\mathscr{K}$ consists of all relation systems containing no induced substructure isomorphic with a structure from $\mathscr{A}$.

(A precise formulation can be found in Bull. A.M.S. *83* (1977), 127—128, the full proof was given in J. Comb. Theory A *34* (1983), 183—201.)

A characteristic feature of the methods of proofs elaborated by Nešetřil and Rödl is their applicability to the solution of many further problems in combinatorial number theory, topology, computer science and mathematical logic. From combinatorial number theory let us mention the simple proof of Erdös' theorem on multiplicative bases of positive integers, and the solution of Erdös-Newman's conjecture and of Erdös' problem on $B^{(2)}$ sequences, which is a variant of Pisier's problem on Sidon's sequences, and a generalization of Van der Warden's theorem on arithmetic progressions.

Besides their successful research, we should mention also the activity of Nešetřil and Rödl in mathematics teaching. More than ten now very successful young mathematicians started their work under their supervision in the Combinatorial Seminar at the Faculty of Mathematics and Physics.

J. Nešetřil and V. Rödl, having successfully solved difficult problems, have reached a very general formulation of Ramseyan problems and completely solved them for relation systems. Similarly perhaps to all the preceding laureates they found their own way in science. They go forward with the zest and resolution of youth. On behalf of the Czechoslovak mathematical community we congratulate them and wish them many further successes in mathematical research.

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