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ON A PROBLEM OF B. ZELINKA

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In [1], p. 178 B. Zelinka has posed the following problem:

*Does there exist a commutative semigroup such that each tolerance on its element set is compatible with it?* We solve his problem affirmatively by producing an example of such a semigroup.

For definitions and notations we refer to [1].

**Example.** Let  $S = \{e, a, b, c\}$ . The multiplication table of the semigroup  $\langle S, * \rangle$  is given by the following table:

*	e	a	b	c
e	e	e	e	e
a	e	e	e	e
b	e	e	e	e
c	e	e	e	e

Clearly one can check that  $\langle S, * \rangle$  is a commutative semigroup. Let  $T$  be any tolerance relation on  $\langle S, * \rangle$ . Clearly  $T \supseteq \{(e, e), (a, a), (b, b), (c, c)\}$ . Now note that the product of any two elements is  $e$  in  $S$ . Hence if  $x T y$  and  $u T v$  where  $x, y, u, v \in S$  then clearly  $x * u T y * v$  since  $(e, e) \in T$ . Hence every tolerance relation is compatible.

In fact for every integer  $n > 2$ , there exists a semigroup  $\langle S, * \rangle$  satisfying the conditions posed in the problem. Take

$$S = \{x_1 = e, x_2, x_3, \dots, x_n\}.$$

Define  $x_i * x_j = e$  for all  $i, j = 1, 2, \dots, n$ .

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*Reference*

- [1] *Zelinka, B.*: Tolerances in Algebraic Structures, Czechoslovak Math. J. 25 (1975), 175–178.

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