Czechoslovak Mathematical Journal

Dimitrios A. Kandilakis; Nikolaos S. Papageorgiou

Correction on "The properties of the Aumann integral with applications to differential inclusions and control systems"

Czechoslovak Mathematical Journal, Vol. 40 (1990), No. 4, 695–696

Persistent URL: http://dml.cz/dmlcz/102422

Terms of use:

© Institute of Mathematics AS CR, 1990

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.

This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* http://dml.cz

Czechoslovak Mathematical Journal, 40 (115) 1990, Praha

CORRECTION ON "THE PROPERTIES OF THE AUMANN INTEGRAL WITH APPLICATIONS TO DIFFERENTIAL INCLUSIONS AND CONTROL SYSTEMS"

by D. KANDILAKIS and N. S. PAPAGEORGIOU

Czechoslovak Math. Jour. *39* (1989), pp. 1—15

As it was pointed to us by Dr. J. Davy, theorem 4.1 of our paper [3] is incorrect as it stands. Stronger hypotheses on the orientor field $F(t, x)$ are needed. The counterexample in p. 397 of [1] shows that the continuity hypothesis on $F(t, \cdot)$ cannot be relaxed if we want to have peripheral attainability.

The next result presents a correct infinite dimensional version of theorem 4.1 of [3], partially extends theorem 7.3 of Davy and also gives us more information about the extremal solutions.

Let $T = [0, b]$, X a separable Banach space and let $S(x_0)$ be the solution set of $\dot{x}(t) \in F(t, x(t))$ a.e., $x(0) = x_0$, while $S_e(x_0)$ denotes the solution set of $\dot{x}(t) \in \in \operatorname{bd} F(t, x(t))$ a.e., $x(0) = x_0$, with $\operatorname{bd} F(t, x(t))$ denoting the boundary of $F(t, x)$).

Theorem. *If $F: T \times X \to P_{kc}(X)$ is a multifunction s.t.*

(1) *$t \to F(t, x)$ is measurable,*

(2) *$h(F(t, x'), F(t, x)) \leqq k(t) \|x' - x\|$ a.e. with $k(\cdot) \in L^1_+$,*

(3) *$|F(t, x)| \leqq a(t) + b(t) \|x\|$ a.e. with $a(\cdot), b(\cdot) \in L^1_+$,*

(4) *$\gamma(F(t, B)) \leqq r(t) \gamma(B)$ a.e. for all $B \subseteq X$ nonempty, bounded with $r(\cdot) \in L^1_+$ and with $\gamma(\cdot)$ being the ball measure of noncompactness,*

then $S(x_0) = \overline{S_e(x_0)}$ the closure taken in $C(T, X)$ and if $z \in R(t) = S(x_0)(t)$, then there exist "bang-bang" solutions $y_n(\cdot)$ s.t. $y_n(t) \to z$.

Proof. From theorems 3.1 and 4.1 of [4] we know that $S(x_0) \in P_k(C(T, X))$. Let $x(\cdot) \in S(x_0)$. Then by definition $x(t) = x_0 + \int_0^t f(s)\, ds$, $t \in T$, $f \in S^1_{F(\cdot, x(\cdot))}$. Given $\varepsilon > 0$ and using theorem 2.1 of Chuong [2] we can find $f_1 \in S^1_{\operatorname{bd} F(\cdot, x(\cdot))}$ s.t. $\|x - z_1\|_\infty < \varepsilon$, where $z_1(t) = x_0 + \int_0^t f_1(s)\, ds$, $t \in T$. Through Aumann's selection theorem, we can find $f_2: T \to X$ measurable s.t. $d(f_1(t)_1 \operatorname{bd} F(t, z_1(t))) = \|f_1(t) - f_2(t)\|$ $t \in T$. Let $z_2(t) = x_0 + \int_0^t f_2(s)\, ds$. Then $\|z_2(t) - x(t)\| \leqq \leqq \|z_2(t) - z_1(t)\| + \|z_1(t) - x(t)\| \leqq \int_0^t \|f_2(s) - f_1(s)\|\, ds + \varepsilon \leqq \varepsilon(\int_0^t k(s)\, ds + 1)$. Suppose we have obtained $f_1 \ldots f_n \in L^1(X)$ s.t.

$$\|f_{m+1}(t) - f_m(t)\| \leqq \varepsilon\, k(t) \frac{1}{(m - 1)!} (\textstyle\int_0^t k(s)\, ds)^{m-1}$$

and

$$f_{m+1}(t) \in \mathrm{bd}F(t, z_m(t)) \text{ a.e.}, \quad z_m(t) = x_0 + \int_0^t f_m(s)\,ds,$$
$$m = 1, 2, \ldots, n-1.$$

Then we can write

$$\|z_{m+1}(t) - z_m(t)\| \leqq \int_0^t \|f_{m+1}(s) - f_m(s)\|\,ds \leqq \varepsilon \int_0^t \frac{k(s)}{(m-1)!} \left(\int_0^s k(r)\,dr\right)^{m-1} ds =$$

$$= \frac{\varepsilon}{m!} \left(\int_0^t k(s)\,ds\right)^m \Rightarrow \|z_{m+1}(t) - x(t)\| \leqq \varepsilon \sum_{q=1}^{m+1} \frac{1}{q!} \left(\int_0^t k(s)\,ds\right)^q \leqq \varepsilon \exp \|k\|_1 .$$

Again Aumann's theorem gives us $f_{n+1} \in S^1_{\mathrm{bd}F(\cdot, z_n(\cdot))}$.

$$\|f_{n+1}(t) - f_n(t)\| \leqq h(\mathrm{bd}F(t, z_n(t)), \mathrm{bd}F(t, z_{n-1}(t))) \leqq$$

$$\leqq k(t) \|z_n(t) - z_{n-1}(t)\| \leqq \frac{\varepsilon}{(n-1)!} k(t) \left(\int_0^t k(s)\,ds\right)^{n-1}$$

and this completes the induction.

Clearly $f_n(t) \to^s \hat{f}(t)$ in X and $\hat{f} \in L^1(X)$. Also $z_n(t) \to^s z(t) = x_0 + \int_0^t \hat{f}(s)\,ds$ and $\hat{f}(t) \in \lim \mathrm{bd}F(t, z_n(t)) = \mathrm{bd}F(t, z(t))$ a.e. (hypothesis $H(F)$ (5)) $\Rightarrow z(\cdot) \in S(x_0)$. Thus in the limit we have $\|z - x\|_\infty \leqq \varepsilon \exp \|k\|_1$. Since $\varepsilon > 0$ was arbitrary we conclude that $S(x_0) = \overline{S_e(x_0)}$ the closure in $C(T, X)$. So if $z \in R(t)$, then $z = y(t) \in S(x_0)$. So we can find "bang-bang" solutions $y_n(\cdot) \in S_e(x_0)$ s.t. $y_n(t) \to z$.

Q.E.D.

References

[1] *J. Davy:* Properties of the solution set of a generalized differential equation. Bull. Austr. Math. Soc. *6* (1972), pp. 379—398.

[2] *P. V. Chuong:* Some results on density of extreme selections for measurable multifunctions. Math. Nachr. *126* (1986), pp. 312—326.

[3] *D. Kandilakis - N. S. Papageorgiou:* On the properties of the Aumann integral with applications to differential inclusions and control systems. Czechosl. Math. J. *39* (1989), pp. 1—15.

[4] *N. S. Papageorgiou:* On multivalued semilinear evolution equations. Boll. Unione. Mat. Ital. *3* (1989), pp. 1—16.

Authors' address: University of California, 1015 Department of Mathematics, Davis, CA 95616, U.S.A.